# Multiple solution of linear algebraic systems by an iterative method with recomputed preconditioner in the analysis of microstrip structures

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**Abstract.** A multiple solution of linear algebraic systems with dense matrix by iterative methods is considered. To accelerate the process, the recomputing of the preconditioning matrix is used. A priory condition of the recomputing based on change of the arithmetic mean of the current solution time during the multiple solution is proposed. To confirm the effectiveness of the proposed approach, the numerical experiments using iterative methods BiCGStab and CGS for four different sets of matrices on two examples of microstrip structures are carried out. For solution of 100 linear systems the acceleration up to 1.6 times, compared to the approach without recomputing, is obtained.

**Keywords:** Multiple solution, linear algebraic system, iterative method, recomputed preconditioning, capacitance matrix, microstrip line, modal filter.
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## INTRODUCTION

There are tasks that require multiple solution of linear algebraic systems of type $\mathbf{A}_k\mathbf{x}_k=\mathbf{b}_k$ for $k=1\div m$ [1], for example, a time-consuming computation of the capacitance matrix for strip structures with changing parameters ($\mathbf{A}_k$ – square and dense matrix of order $N$, $\mathbf{b}_k = \mathbf{b}_1$) [2]. To calculate the capacitance matrix derived from the same structure, simultaneously changing the value of the permittivity of dielectrics, the block LU decomposition is used [3]. This algorithm is based on the fact that only the diagonal entries in the right lower corner of the matrix vary. In the general case, changes in sizes of a structure result in variation of irregularly located matrix entries [4], so it is preferable to use iterative methods to accelerate solutions. For example, the iterative method (BiCGStab) having been used to accelerate the solution showed a significant speed up with respect to Gaussian elimination [5]. To accelerate the iterative process the two methods were used: the initial guess of the current linear algebraic system is the computed solution of the previous one; the use of the implicit preconditioning matrix **M** computed from the first coefficient matrix for current system solution. However, the effectiveness of preconditioning decreases with increase of difference between the first and the current matrix [2]. To solve this problem it is suggested to recompute the matrix **M** when the rate of convergence in solving the current linear system is too slow [6]. An alternative approach consists in updating of the matrix **M** instead of its total recomputation. When the implicit preconditioning is used, both matrixes **L** and **U** shall be updated. This paper does not include that approach, as it is not effective for multiple computation [1]. The presence of optimal threshold value (wherein the time of linear system solution is minimal) is shown. However, it is not possible to determine a priori when to recompute the matrix **M**. Thus, a search for a priori condition of the recomputation is urgent.

## AN ITERATIVE ALGORITHM FOR SOLVING MULTIPLE LINEAR ALGEBRAIC SYSTEMS WITH RECOMPUTED PRECONDITIONER

To determine when to recompute the preconditioner, we propose to use the arithmetic mean time of linear systems solution $k$ ($\overline{T}$), which is equal to the ratio of the total time of these systems solution ($T_\Sigma$) to their number $k$:

$$\overline{T} = \frac{T_\Sigma}{k}. \tag{1}$$

$T_\Sigma$ is expressed as follows:

$$T_\Sigma = T_{PR} + \sum_{k=1}^{m} T_k \,, \tag{2}$$

here $T_{PR}$ is the time of calculating of the preconditioning matrix **M** from the matrix of the first linear system, $T_k$ is the time of the $k$th linear system solution, $m$ is the number of linear systems.

Using (1) and (2) we consider the change in the arithmetic mean solving time ($\overline{T}$) as a function of $k$ as follows:

$$\overline{T}(k) = \frac{T_\Sigma(k)}{k} = \frac{T_{PR}}{k} + \frac{\sum_{j=1}^{k} T_j}{k} = f(k) + g(k) \,,$$

here $f(k)$ is the function that depends on $T_{PR}$, and $g(k)$ is the function that depends on the time of solution of the $j$th linear system excluding $T_{PR}$. For small $k$ the condition $f(k)>g(k)$ is satisfied, since $T_{PR}>T_j$ because LU decomposition is used as a way of preconditioner forming [5, 6]. For large $k$ $f(k)<g(k)$ because:

$$\lim_{k\to\infty}\left(\frac{T_{PR}}{k}\right) = 0 \,,\ \lim_{k\to\infty}\frac{\sum_{j=1}^{k} T_j}{k} > 0 \,.$$

The existence of an extremum in function $\overline{T}(k)$ follows from these conditions.

## NUMERICAL EXPERIMENTS

To confirm this fact we used the simple structure consisting of a conductor on a dielectric substrate above the perfectly conducting plane (Figure 1), which was modeled in the TALGAT software [8] based on the method of moments [4]. As an iterative method, we used BiCGStab [7]. The number of segments at the boundaries of the structure was not changed, whence all the systems were of the same order $N$ = 1600. For all systems $b$ is a unit vector. As the initial guess, the computed solution of the previous system was used; for the first system we used the unit vector as the initial guess (similarly [5]). The matrix **M** is obtained using the LU-decomposition. Iterations were continued until the relative residual norm became smaller than $10^{-8}$. We used a personal computer (parallelization was not exploited, i.e., one core of the processor was busy) with the following parameters: platform – Intel(R) Core (TM) i7; processor frequency – 2.80 GHz, memory – 12 Gb; number of cores – 8; operating system – Windows 8x64. Obtained dependencies $T_k$, $f(k)$, $g(k)$ and $\overline{T}(k)$ show the presence of extremum of the function $\overline{T}(k)$ (for $k$ = 54), as expected (Figure 2). Therefore, it is proposed to perform recomputing of the matrix **M** at the time when the value of the function $\overline{T}(k)$ begins to increase.


**FIGURE 1**. Cross section of the structure modeled in the TALGAT software

In order to test the proposed condition we formed 100 matrices for two structures. For structure 1 (Figure 1) the matrices of orders $N$ = 1600 and 3200 were obtained by varying the height of the conductor ($t$) in the range of 6, 7 ... 106 microns. For the structure 2 (Figure 3), which is a modal filter [9], the matrices of orders $N$ = 2001 and 3001 were obtained by varying the gap ($s$) in the range of 100, 101 ... 200 microns.

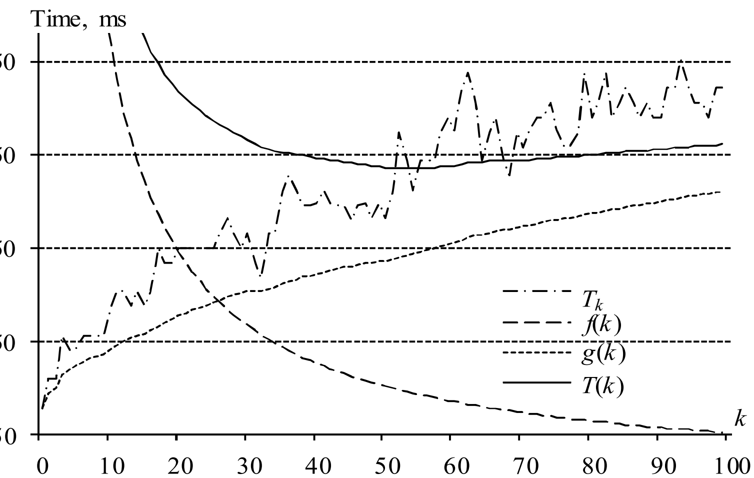


**FIGURE 2.** Dependencies $(T_k)$, $f(k)$, $g(k)$, $\overline{T}(k)$

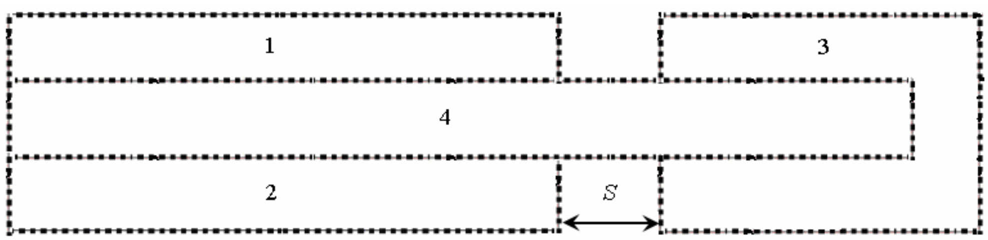


**FIGURE 3.** Cross section of the modal filter in the TALGAT software. Three conductors (1, 2, 3) with the dielectric (4)

To confirm the effectiveness of the proposed approach in combination with various methods, the CGS method [10] is used. The test results of the proposed algorithm with respect to the solution with the initial preconditioner are given in Table 1.

**TABLE 1.** Speed up of 100 linear systems solution by BiCGStab and CGS methods with recomputed preconditioner.

| **Method** | **Structure 1** | | **Structure 2** | |
|---|---|---|---|---|
| | ***N*=1600** | ***N*=3200** | ***N*=2001** | ***N*=3001** |
| BiCGStab | 1.51 | 1.15 | 1.60 | 1.46 |
| CGS | 1.09 | 1.19 | 1.41 | 1.23 |

Test results show that the using of the recomputed preconditioner based on the increase of the arithmetic mean of the time gives acceleration for all structures, matrix orders and iterative methods. The maximum acceleration (1.6) was obtained by using the BiCGStab method for structure 2 and $N = 2001$. The minimum acceleration (1.09) was obtained by using the CGS method for structure 1 and $N = 1600$. The presence of acceleration shows the potential of the proposed approach. Thus, the proposed algorithm allows to determine adaptively when to recompute the preconditioner for acceleration of the multiple solution with arbitrary changes of matrix. Another advantage is independence from iterative method, preconditioning, and parameters of iterative process.

In conclusion, it is worth noting that the authors developed the algorithm for adaptive preconditioner recomputation based on the arithmetic complexity of iterative methods, which has been tested. Also, we carry on work to create an algorithm for multiple solution of linear systems with the reference template matrix for creating a preconditioner, which will allow to get acceleration that is close to maximum without the recomputation. In addition, we consider the possibility of adjusting preconditioner instead of recomputation. In the near future the results of these studies will be tested and can be presented in extended version of this paper.

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