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I - V
ANNUAL INTERNATIONAL
MICROELECTRONICS OLYMPIAD OF ARMENIA
TESTS AND PROBLEMS

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Welcome!

The problem book that you hold in your hands is the latest contribution of Armenia to the advancement of microelectronics. I am sure you will find it very modern, and useful. This is the book of tests and problems of the Annual International Microelectronics Olympiad of Armenia. At the same time, it will also be a very valuable resource for students, Masters and PhDs of microelectronics and similar engineering disciplines, providing them an opportunity to assess and improve their knowledge, as well as to develop solution skills of tests and problems.

The results of all the Olympiads have witnessed that the book has served its purpose and has become “a table book” for participants.

This is already the fourth problem book and contains tests and problems of the previous I-V Olympiads. It is a unique contribution in training highly qualified specialists which is a key area for the development of Armenia’s IT sector. It will also contribute to the realization of the goals of the next Annual International Microelectronics Olympiads of Armenia, contributing to the ascension of Armenian microelectronics to a state-of-the-art and leading branch of industry.

It is my sincere hope this problem book will take its notable place in your professional library.



With best wishes,

Rich Goldman,

Vice president for Corporate Marketing and Strategic Alliances of Synopsys Inc.,

CEO of Synopsys Armenia CJSC,

President of Org. Committee of Annual International Microelectronics Olympiad of Armenia,

Honorable Doctor of SEUA

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PREFACE

During the last five years the First (September 22-25, 2006), the Second (September 18-25, 2007), the Third (September 16-29, 2008), the Fourth (September 15-30, 2009) and the Fifth (September 28-October 28, 2010) Microelectronics Olympiads of Armenia (<http://www.microolymp.am>) took place. The goals of these Olympiads are to: stimulate the further development of microelectronics in Armenia and participant countries, discover young, talented resources (University students and specialists of microelectronics area up to 30 years), increase interest towards microelectronics among them, and understand the level of knowledge in the field of microelectronics among young specialists to make necessary adjustments to educational programs. The success of the Microelectronics Olympiads of Armenia which evolved from the local competition to the annual and international event, was mainly conditioned by the financial support and huge organizational efforts of its general sponsors: Synopsys Armenia CJSC (CEO Rich Goldman, Director Hovik Musayelyan) and Vivacell-MTS CJSC (General Manager Ralph Yirikyan), also sponsors: USAID CAPS project (director Artak Ghazaryan), Enterprise Incubator Foundation (Director Bagrat Yengibanryan), ACBA Credit Agricole bank (President Stepan Gishyan), Unicomp CJSC (Director Armen Baldryan), Microsoft RA LTD (General Manager Grigor Barseghyan), Arminco CJSC (Manager Andranik Aleksanyan), Union of Manufacturers & Businessmen of Armenia (UMBA) (Executive President Arsen Ghazaryan), Viasphere Technopark CJSC (General Manager Aram Vardanyan), Union of Technology Enterprises (ITE) (Executive Director Karen Vardanyan), "Yerkir Media" TV Company (Director of media and political part Gegham Manukyan), "ARKA" Agency (Director Konstantin Petrosov), "Mediastyle" LTD (General Manager Vardan Asatryan), "EKONOMIKA" Magazine (Editor-in-Chief Lyusya Mehrabyan), "Delovoy Express" Weekly (Editor-in-Chief Eduard Naghdalyan), "168 Hours" Daily (Editor-in-Chief Satenik Seyranyan) Public Radio of Armenia (Executive Director Armen Amiryan). Universities' assistance was great in providing participants: State Engineering University of Armenia (Rector Vostanik Marukhyan, Vice-rector Ruben Aghgashyan), Yerevan State University (Rector Aram Simonyan, Vice-rector Alexander Grigoryan), Russian-Armenian (Slavonic) University (Rector Armen Darbinian, Vice-rector Gagik Sargsyan), European Regional Academy of Armenia (Rector Andranik Avetisyan, Vice-rector Kristina Sargsyan), Gyumri Information Technologies Center (Acting Director Amalya Eghoyan), Jordan University of Science and Technology (President Wajih M. Owais) and Princess Sumaya University for Technology (President Issa Batarseh).

I express my deep gratitude to the above mentioned people as well as the other members of Organizing Committee of the Microelectronics Olympiads of Armenia – Andranik Hovhannisyan (Deputy Director of Synopsys Armenia CJSC), Academician Vladimir Harutyunyan (Head of the Microelectronics Chair at Yerevan State University), Academician Harutyun Terzyan (State Engineering University of Armenia Professor), Academician Eduard Ghazaryan (Head of the General and Theoretical Physics Chair at Russian-Armenian (Slavonic) University), Andranik Aleksanyan (General Manager of Arminco CJSC), members of Program Committee Dr. David Parent (Associate Professor of San-Jose State University (USA, CA)), Prof. Sergey Umnyashkin (Vice-Rector of Moscow Institute of Electronic Technology (Russia)), Prof. Vladimir Hahanov (Dean of Computer Engineering Faculty of Kharkov National University of Radioelectronics (Ukraine)), Prof. Predrag Petković (Professor of University of Niš (Serbia)), Prof. Marin Hristov (Professor of Technical University of Sofia (Bulgaria)), as well as Gayane Markosyan who accomplished the PR of the Olympiads (SEUA Associate Professor, Senior University Program Coordinator of Synopsys Armenia CJSC).

The Olympiads are held in two stages. The first stage, which entailed a test involving a number of basic tasks, is held simultaneously and in the local places in participating countries. The top contestants advance to the second stage, which take place in Armenia and involve a challenging contest and complex engineering tasks requiring advanced solutions. The set of the test questions and problems of I-V Annual Microelectronics Olympiads of Armenia along with their solutions are included in this book. Test questions and problems are related to VLSI Design and EDA areas and are classified according to their basic sections: Digital integrated circuits, Analog integrated circuits, RF integrated circuits, Semiconductor physics and electronic devices, Semiconductor technology, Numerical methods and optimization, Discrete mathematics and theory of combinations, Object-oriented programming, Nanoelectronics.

The problem book, first of all, is for future participants of the Annual International Microelectronics Olympiad of Armenia of the coming years. I hope it will contribute to the increase of knowledge level of potential participants in the Olympiad. At the same time, it can also be useful for other students, Masters, PhDs and engineers of the above mentioned areas.

The problem book also contains similar test questions and problems that are the variations of the corresponding type of tests and problems given to the participants during the Olympiads.

The problem book, in its future publications, will be extended with the test questions and problems of the coming Olympiads.

You can send your remarks to microelectronics_olympiad@synopsys.com.

I express my special gratitude to the below mentioned coauthors of the problem book most of whom are also members of Olympiad Program Committee:

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- whom I also express my deep gratitude.

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TESTS AND PROBLEMS

1. DIGITAL INTEGRATED CIRCUITS

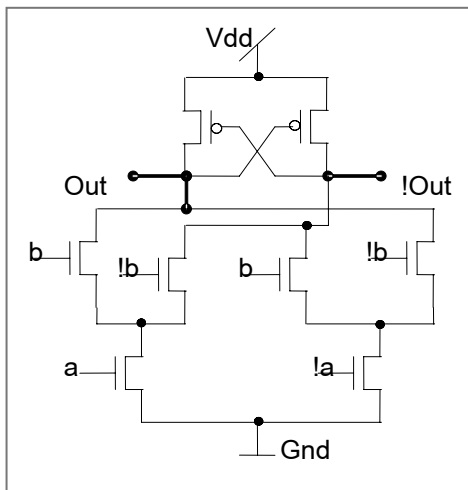
a) Test questions

- 1a1.** There is a tri-state buffer where internal delays can be ignored. Right after z state is set, the output voltage level will be:
- A. $V_{DD}/2$ where V_{DD} is supply voltage
 - B. High or low, depending on the state before z state is set
 - C. Indefinite
 - D. High
 - E. Low

- 1a2.** In CMOS ICs, PMOS transistor is usually configured as:
- A. No potential is given to substrate
 - B. Substrate is connected to source
 - C. Substrate is connected to drain
 - D. The highest potential is given to substrate
 - E. The lowest potential is given to substrate

- 1a3.** There is a JK flip-flop. Mark the prohibited input combination.
- A. $J=1, K=1$
 - B. $J=1, K=0$
 - C. $J=0, K=1$
 - D. $J=0, K=0$
 - E. No prohibited combination

- 1a4.** What logic function is implemented by the presented circuit?



- A. AND
- B. XOR-XNOR
- C. AND-NAND
- D. OR-NOR
- E. MUX-MUXI

- 1a5.** Which is the Canonical Disjunctive Normal Form (CDNF) of the function described by the following truth table?

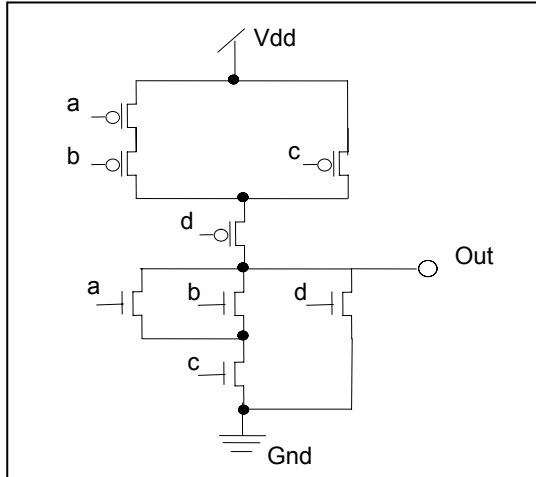
Inputs			Output
a	b	c	y
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

- A. $y=!a\&b\&!c+!a\&b\&c+a\&b\&!c+a\&b\&c$
- B. $y=!a\&!b\&!c+!a\&!b\&c+a\&!b\&!c+a\&!b\&c$
- C. $y=(a+b+c)\&(a+b+!c)\&(a+!b+c)\&(a+!b+!c)$
- D. $y=(!a+!b+!c)\&(a+!b+c)\&(a+!b+!c)\&(a+!b+c)$
- E. $y=(a+!b+c)\&(a+!b+!c)\&(a+!b+c)\&(a+!b+!c)$

- 1a6.** Threshold voltage of a MOS transistor is called the voltage which is necessary to be applied between the gate and the source:
- A. For $1\mu A$ current flow through drain
 - B. Of the transistor for current flow through drain which is 10 times more than leakage current of transistor

- C. Of the transistor for average concentration of charge carriers that maintain transistor's conductance be equal to average concentration of majority charge carriers in substrate in channel formation place
- D. Of the transistor for average concentration of charge carriers that maintain transistor's conductance be equal to average concentration of minority charge carriers in substrate in channel formation place
- E. Of the transistor for the transistor to be saturated

1a7. What formula describes the circuit?



- A. $Out = !((!a+!b)&c+!d)$
- B. $Out = !((a+b)&c+d)$
- C. $Out = !((a&b+c)&d)$
- D. $Out = ((!a+!b)&c+!d)$
- E. $Out = !((!a&!b+!c)&!d)$

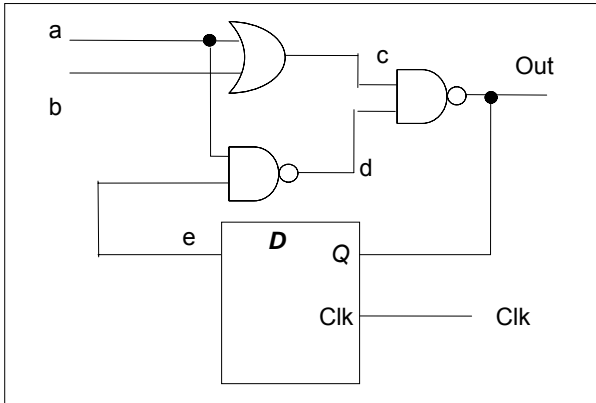
1a8. Which statement is correct?

- A. The operation of charge-coupled devices' (CCD) is based on processes occurring in bipolar transistors
 - B. CCD frequency internal limit is influenced by thermo generation of charge carriers
 - C. CCD frequency parameters do not depend on the degree of semiconductor's surface energetic levels
 - D. CCD frequency properties do not depend on the type of a semiconductor
 - E. CCDs are static devices
- 1a9. Which of the shown answers more contributes to the successful solution of placement issue of a cell?
- A. Maximum distance between high frequency circuits
 - B. Maximum distance between low frequency circuits
 - C. Maximum proximity of more related cells
 - D. A. and C. together
 - E. B. and C. together
- 1a10. One of the rules of concurrent modeling is
- A. $Lc(y)$ list defined for an external output line composes the set of testable faults which can be found by the given input set (vector)
 - B. Single stuck-at fault (SSF) model assume that there is only one fault in tested logic circuit
 - C. Two types of stuck-at logic faults - stuck-at-1 fault (SA1 or $s@1$) and stuck-at-0 fault (SA0 or $s@0$)
 - D. $Lc(y)$ list defined for an external output line composes the set of testable faults which can be found by an output set (vector)
 - E. None
- 1a11. In state-of-the-art integrated circuits the minimum width of interconnect transmission lines is limited by
- A. Resolution of the lithography process
 - B. Mutual agreement of customer and manufacturer
 - C. Desire of designer
 - D. Technological method to get thin layers
 - E. Phenomena of electromigration
- 1a12. The increase of logic circuit organization's parallelism mostly leads to
- A. The increase of performance
 - B. The increase of the number of primary outputs
 - C. The decrease of the number of logic cells
 - D. A. and B. together
 - E. B. and C. together

- 1a13.** The $f(x_1, x_2, x_3)$ function takes 1 value on 0, 3, 5, 6 combinations. What class does the given function belong to?
- Constant 0
 - Constant 1
 - Linear and selfdual
 - Selfdual
 - Monotone
- 1a14.** The operation of a Gunn diode is based on:
- The effect of semiconductor inversion in strong electrical field
 - The appearance of negative differential impedance in strong electrical field
 - The tunnel effect in strong electrical field
 - The rectifying properties of p-n junction
 - The contact effects between metal and semiconductor
- 1a15.** Which of the answers more contributes to the increase of fan-out?
- Increase of cells' input resistance
 - Increase of cells' output resistance
 - Decrease of cells' output resistance
 - A. and B. together
 - A. and C. together
- 1a16.** Which of the following statements is wrong for synchronous FSM?
- Memory element competition, static and dynamic risks in combinational circuits are dangerous
 - The abstract presentation of the automaton is used to design a circuit
 - Synchronization of asynchronous input signals are required with clock signals
 - All FFs trigger at the same clock signal
 - The wrong answer is missing
- 1a17.** The common emitter configuration compared with the common base configuration:
- Increases frequency properties
 - Increases the collector junction's resistance
 - Increases collector junction's breakdown voltage
 - Increases the current gain
 - Decreases the thermal component of collector current
- 1a18.** Which of following methods of interconnect designing more contributes to speed increase?
- Increase of interconnect layers
 - Decrease of total length of interconnects
 - Decrease of length of signal processing critical path
 - B. and C. together
 - A. and C. together
- 1a19.** Among the following principles, which is wrong for concurrent simulation?
- Classification of vectors according to quality, move of low quality vectors in the beginning of simulation, elimination of detected faults – it increases simulation speed, reduces simulation time
 - Any subset of faults is simulated, extrapolation of fault coverage is executed according to obtained results
 - IC simulation, estimation of vectors' quality - 01 or 10 toggle node number, toggle coverage
 - For the given vector there is strict correlation between detectable fault number and 01, 10 toggle number
 - None
- 1a20.** There is an inverter which has a passive capacitive load. If supply voltage increases,
- Rise will increase, fall will decrease
 - Fall will increase, rise will decrease
 - Output transition time will decrease
 - Output transition time will increase, as during switching the load must be charged by larger ΔU voltage
 - Output transition time will remain the same, as ΔU will increase, but charging current will also increase
- 1a21.** The transfer characteristic of MOS transistor is the dependence of:
- Drain voltage on gate-source voltage
 - Drain voltage on drain-source voltage
 - Drain current on drain-source voltage
 - Drain current on gate-source voltage
 - Gate current on gate-source voltage

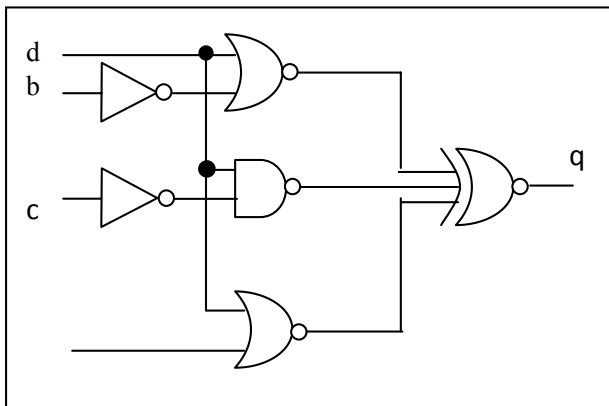
- 1a22.** Latch-up phenomena is proper to
 A. *ECL circuits*
 B. *Only CMOS circuits*
 C. *N-MOS and CMOS circuits*
 D. *P-MOS and CMOS circuits*
 E. *All bipolar circuits*
- 1a23.** Which one is a prohibited input combination for RS latch?
 A. *R=0, S=1*
 B. *R=1, S=0*
 C. *R=0, S=0*
 D. *R=1, S=1*
 E. *There is no prohibited combination*
- 1a24.** Define in which state will Johnson's 6 bit counter go, after the 10th pulse is applied. Initial state is 000111.
 A. *011110*
 B. *001011*
 C. *101010*
 D. *011010*
 E. *The correct answer is missing*
- 1a25.** How many pins does the bipolar transistor have?
 A. *1- emitter,*
 B. *2- emitter and base,*
 C. *2- base and collector,*
 D. *2- emitter and collector;*
 E. *3- emitter, base and collector,*
- 1a26.** What semiconductor material is mostly used in integrated circuits?
 A. *Ge*
 B. *Si*
 C. *GaAs*
 D. *Fe*
 E. *Zn*
- 1a27.** From the following statements which is wrong for DRAM?
 A. *Usually one MOS transistor is used to keep 1 bit in memory*
 B. *FF is as a memory cell*
 C. *Address inputs are multiplexed*
 D. *DRAMs are considered energy dependent*
 E. *The wrong answer is missing*
- 1a28.** In logic design level the following is designed:
 A. *A stand alone device which is divided up to such multibit blocks as registers, counters, etc.*
 B. *A stand alone logic gate or FF, which consists of electronic components – transistors, diodes, etc.*
 C. *A stand alone semiconductor component, e.g. a transistor*
 D. *A digital device the components of which are separate logic gates and FFs*
 E. *A general system which consists of RAM memory device, datapath devices, etc.*
- 1a29.** In component level of the design the following is designed:
 A. *A separate semiconductor device, e.g. a transistor*
 B. *A separate device, which is being disassembled until diverse components, such as registers, calculators, etc.*
 C. *A separate logic gate or FF which consists of electronic components – transistors, diodes, etc.*
 D. *A general system which consists of operative memory device, numerical device, etc.*
 E. *A digital device which consists of separate logic gates and FFs*
- 1a30.** In order to prevent latch-up in CMOS circuits it is necessary to:
 A. *Increase the parasitic capacitances between the buses of output buffer's parasitic bipolar transistors*
 B. *Increase the gain of parasitic bipolar transistors*
 C. *Put the drains of n and p transistors as close as possible*
 D. *Put n+ guard ring around n+ source/drain*
 E. *Put p+ guard ring around n+ source/drain and put n+ guard ring around p+ source/drain*

1a31. Identify the synchronous model of the following circuit (TP-delays, TF-transition time, "p"-previous state of a flip-flop).



- A. $c = a \& b$,
 $d = !(a | e)$,
 $Out = !(c \& d)$,
 $e = \text{posedge}(Clk ? Out: 'p')$, $TP = 0.1n$
- B. $c = a | b$
 $d = !(a | e)$,
 $Out = !(c \& d)$,
 $e = \text{posedge}(Clk) ? Out: 'p'$
- C. $c = a \& b$,
 $b = !(a | e)$,
 $Out = !(c \& d)$,
 $e = !(posedge(Clk) ? Out: 'p')$, $TF = 0.1n$
- D. $c = a \& b$, $TP = 0.1n$,
 $d = !(a | e)$, $TP = 0.1n$,
 $Out = !(c \& d)$, $TP = 0.1n$,
 $e = \text{posedge}(Clk) ? Out: 'p'$, $TP = 0.1n$
- E. $c = a \& b$, $TP = 0.1n$,
 $d = a | e$, $TP = 0.1n$,
 $Out = c \& d$, $TP = 0.1n$,
 $e = !(posedge(Clk) ? Out: 'p')$, $TP = 0.1n$

1a32. In case of which switching the occurrence of dynamic hazard exists in the following circuit?

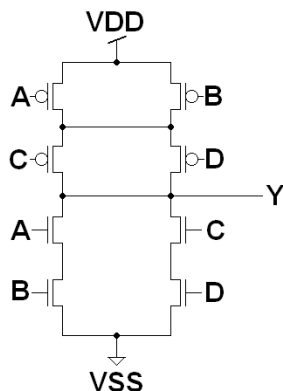


- A. $a = 0-1$, $b = 1$,
 $c = 0$, $d = 1$
- B. $a = 0$, $b = 0-1$,
 $c = 0$, $d = 0$
- C. $a = 0$, $b = 1$,
 $c = 0$, $d = 0-1$
- D. $a = 1$, $b = 0$,
 $c = 1$, $d = 0-1$
- E. $a = 0-1$, $b = 0$,
 $c = 1$, $d = 0$

1a33. In digital circuits, PMOS transistor's

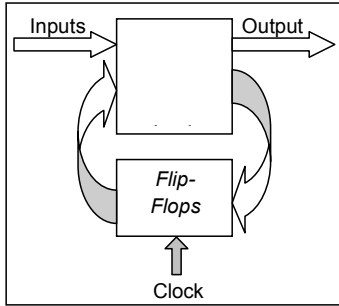
- A. Delays do not depend on the supply voltage
- B. Threshold voltage is proportional to the delay of transistor
- C. Delays do not depend on temperature
- D. The highest potential is usually applied to substrate
- E. Dynamic power dissipation depends only on transistor resistance

1a34. For the following circuit mention the input which will have the maximum input-to-output delay.



- A. A
- B. C
- C. D
- D. B
- E. All are equal

- 1a35. In the following digital system, where setup time of FF is T_{SU} , delay T_{CLKQ} , and delay of logic part T_{LOGIC} , the highest clock frequency will be



- A. $f=1 / T_{LOGIC}$
 B. $f=1 / (T_{LOGIC}+T_{SU})$
 C. $f=1 / (T_{LOGIC}+T_{CLKQ}+T_{SU})$
 D. $f= T_{LOGIC}+T_{CLKQ}+T_{SU}$
 E. $f=1 / (T_{LOGIC}+2*T_{CLKQ}+T_{SU})$

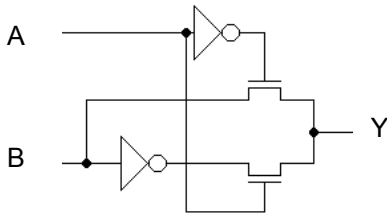
- 1a36. Which is the sum of the following two signed hexadecimal numbers if the addition is performed by saturation adder $81H + FEH$?

- A. $7FH$
 B. FFH
 C. $00H$
 D. $80H$
 E. $77H$

- 1a37. Which of the following is the binary representation of 3.25 decimal number in 4bit, 4bit fixed point format?

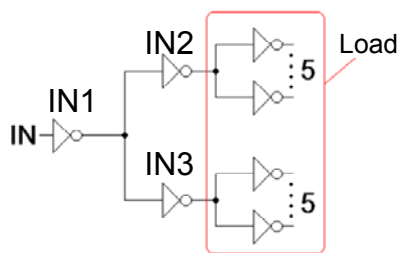
- A. 01110101
 B. 10001010
 C. 00110100
 D. 00111010
 E. 01111001

- 1a38. XOR logic circuit is presented. Which is the disadvantage of the circuit?



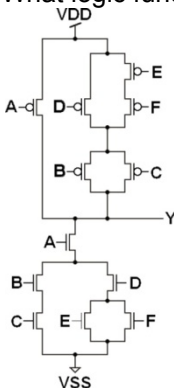
- A. *Input capacitances are large*
 B. *Delays are large*
 C. *Output 1 level is degraded*
 D. *Output 0 level is degraded*
 E. *Power consumption is large*

- 1a39. Buffer circuit composed by I1, I2, I3 identical inverters is shown. The I2 and I3 are loaded by 5 similar inverters. What expression is the average delay of the buffer defined by?



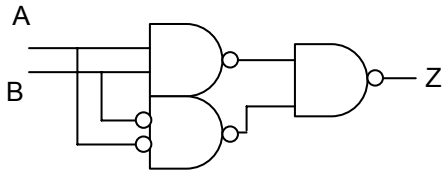
- A. $(R_p + R_n) \cdot (2 \cdot C_{out} + 7 \cdot C_{in})$
 B. $(R_p + R_n) \cdot (5 \cdot C_{out} + 7 \cdot C_{in})$
 C. $(R_p + R_n) \cdot (C_{out} + 7 \cdot C_{in})$
 D. $2 \cdot (R_p + R_n) \cdot (2 \cdot C_{out} + 7 \cdot C_{in})$
 E. $(R_p + R_n) \cdot (2 \cdot C_{out} + 5 \cdot C_{in})$

- 1a40. What logic function is realized in the circuit?



- A. $A \cdot (B \cdot C + D \cdot (E + F))$
 B. $A \cdot (B + C + D \cdot (E + F))$
 C. $A + B \cdot C + D \cdot (E + F)$
 D. $\overline{A \cdot (B \cdot C + D \cdot (E + F))}$
 E. $A + B \cdot C + D \cdot (E + F)$

1a41. What formula does the given circuit describe?



- A. $Z=A+B+(\!A\&\!B)$
- B. $Z=(\!A\&B)+(\!B\&A)$
- C. $Z=\!A\&B$
- D. $Z=A\&(\!B)$
- E. $Z=A+\!B$

1a42. Saturation condition of a NMOS transistor looks like:

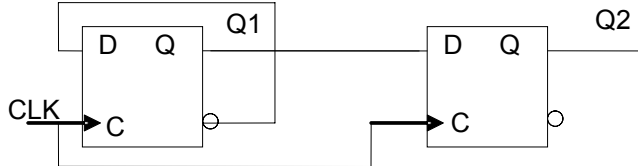
- A. $V_{DS} \leq V_{GS} - V_{THN}$
- B. $V_{DS} \geq V_{GS} - V_{SB}$
- C. $V_{GS} \geq V_{DS} - V_{THN}$
- D. $V_{DS} \geq V_{GS} - V_{THN}$
- E. $V_{DS} \geq V_{GS} + V_{THN}$

1a43. Conjunctive Normal Form (CNF) of the function in the following truth table has the following view:

a	b	c	Y
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

- A. $(\!a+\!b+\!c)\&(\!a+b+c)\&(\!a+b+c)\&(\!a+\!b+c)$
- B. $(a+\!b+\!c)\&(\!a+b+c)\&(\!a+b+c)\&(\!a+\!b+\!c)$
- C. $(\!a+\!b+c)\&(\!a+\!b+c)\&(\!a+b+c)\&(\!a+\!b+c)$
- D. $(a+b+c)\&(\!a+b+c)\&(\!a+b+\!c)\&(\!a+\!b+c)$
- E. $\!a\&\!b\&\!c+\!a\&b\&\!c+a\&\!b\&c+a\&b\&c$

1a44. In what state will the below shown automaton go, after applying four pulses if the initial state is Q1Q2=1x?



- A. 01
- B. 10
- C. 00
- D. 11
- E. x1

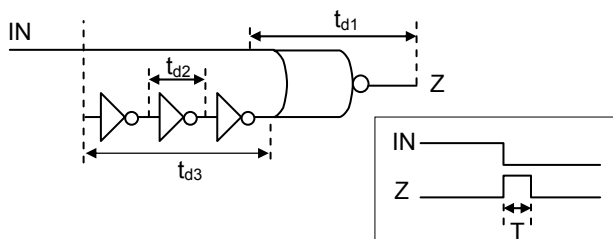
1a45. In digital circuits, for an NMOS transistor

- A. The lowest potential is usually given to substrate
- B. Delays do not depend on the supply voltage
- C. Threshold voltage is proportional to the delay of transistor
- D. Delays do not depend on temperature
- E. Dynamic power dissipation depends only on transistor resistance

1a46. Which is the sum of the following two signed hexadecimal numbers if the addition is performed by saturation adder 12H + 70H?

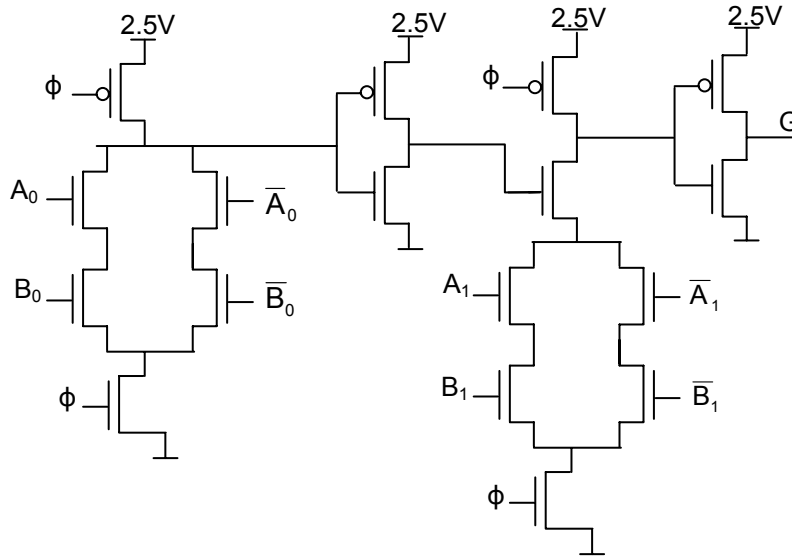
- A. 7FH
- B. FFH
- C. 00H
- D. 80H
- E. 82H

1a47. T duration of the short pulse, obtained on the output of the given circuit, is mainly defined by?



- A. $td1$
 - B. $td3$
 - C. $td2$
 - D. $td1 + td2$
 - E. $td1 + td3$
- delays

- 1a48. If A and B are interpreted as two-bit binary words $A = \{A_1, A_0\}$ and $B = \{B_1, B_0\}$, what interpretation can be applied to output G?



- A. Scalar product:
 $G = (A_0 \& B_0) + (A_1 \& B_1)$
 B. Modulo-2 sum:
 $G = (A_0 \oplus B_0) \oplus (A_1 \oplus B_1)$
 C. Equality:
 $G = (A = B)$
 D. Non-equality:
 $G = (A \neq B)$
 E. Logic sum:
 $G = A_0 + B_0 + A_1 + B_1$

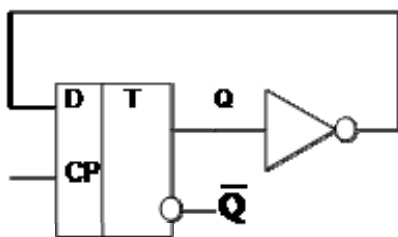
- 1a49. Considering that NOR2 cell's inputs are independent and equally distributed, which is the probability of output switching?

- A. 0.25
 B. 0.375
 C. 0.5
 D. 0.75
 E. 0.875

- 1a50. Assuming $k_n = 2k_p$, in what case will the resistances from the output of NOR2 cell to VDD and VSS be equal?

- A. $W_p = W_n$
 B. $W_p = 2W_n$
 C. $W_p = 4W_n$
 D. $W_p = 6W_n$
 E. $W_p = 8W_n$

- 1a51. The figure shows the circuit of a frequency divider. What expression gives the minimum period of clock pulses?



- A. $T_{min} = t_{su} + t_{c2q} + t_{pinv}$
 B. $T_{min} = t_{c2q} + t_{pinv}$
 C. $T_{min} = t_{su} + t_{hd} + t_{c2q} + t_{pinv}$
 D. $T_{min} = t_{hd} + t_{c2q} + t_{pinv}$
 E. $T_{min} = t_{su} + t_{hd} + t_{pinv}$

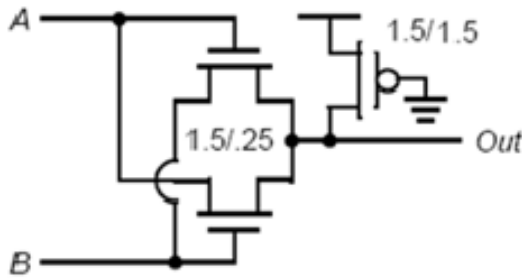
- 1a52. How many digits does the thermometer code have which is obtained after modifying 4-bit binary code?

- A. 16
 B. 4
 C. 15
 D. 8
 E. 32

- 1a53. For the same size inverter what version has minimum leakage current?

- A. PMOS low-vt, NMOS high-vt
 B. PMOS standard-vt, NMOS high-vt
 C. PMOS high-vt, NMOS standard-vt
 D. PMOS high-vt, NMOS low-vt
 E. PMOS low-vt, NMOS standard-vt

1a54. What logic function does the circuit implement?



- A. $OUT=A+B$
- B. $OUT=A\&B$
- C. $OUT=!A+!B$
- D. $OUT=!A\&!B$
- E. $OUT=A\oplus B$

1a55. Which is the basic consequence of MOS transistor's degradation due to warm carriers?

- A. The increase of threshold voltage
- B. The decrease of threshold voltage
- C. The increase of channel resistance
- D. The decrease of channel resistance
- E. The decrease of drain-package disruption voltage

1a56. Assuming $k_n=rk_p$, in what case will the resistances from the output of 3NAND cell to VDD and VSS be equal?

- A. $W_p/W_n=r$
- B. $W_p/W_n=r/9$
- C. $W_p/W_n=r/6$
- D. $W_p/W_n=r/3$
- E. $W_p/W_n=2r/3$

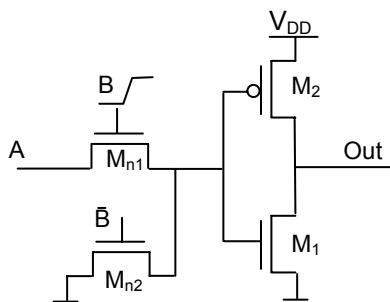
1a57. Which one of the given expressions is wrong?

- A. $A\oplus B = !A\oplus B$
- B. $1\oplus B\oplus A = B\oplus A$
- C. $A\oplus B = !A\oplus !B$
- D. $A\oplus B = !A\oplus !B$
- E. $!A\oplus B = !(A\oplus B)$

1a58. What is the minimum number of transistors in a pass gate implemented 1:4 multiplexer, assuming that normal and complemented select variables are available:

- A. 16
- B. 12
- C. 8
- D. 6
- E. 4

1a59. What function does the circuit implement?



- A. $OUT=A+B$
- B. $OUT=A\&B$
- C. $OUT=!A+!B$
- D. $OUT=!A\&!B$
- E. $OUT=A\oplus B$

1a60. C capacitance is connected to the end of interconnect line with L length, line parameters are c [F/m], r [Ohm/m]. By what formula is the signal delay time in the line given?

- A. $0.7rc$
- B. $0.7Lr(C+c)$
- C. $0.7Lr(C+Lc/2)$
- D. $0.7Lr(C+Lc)$
- E. $0.7Lr(C/2+Lc/2)$

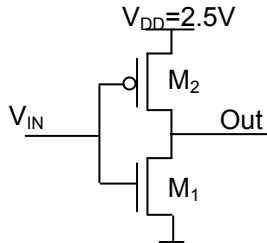
1a61. By increasing the metal line length of interconnect in IC, the delay increases

- A. Linearly
- B. By square law
- C. By 3/2 law

- D. By 2/3 law
- E. By cubic law

- 1a62. In what case is the short connection current missing in CMOS inverter?
- A. $V_{tp} = V_{tn}$
 - B. $V_{tp} + V_{tn} = 0$
 - C. $V_{tp} + V_{tn} < V_{DD}$
 - D. $|V_{tp}| + V_{tn} < V_{DD}$
 - E. $|V_{tp}| + V_{tn} < V_{DD}/2$

- 1a63. Which answer is true if $V_{IN} = V_{DD}$?



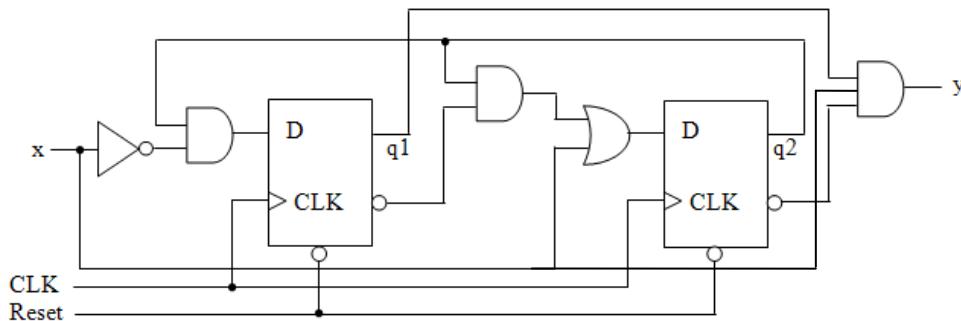
- A. M1-saturated, M2-linear
- B. M1-linear, M2-linear
- C. M1-linear, M2-saturated
- D. M1-saturated, M2-saturated
- E. None

- 1a64. In what case is the short connection current missing in a CMOS inverter?
- A. $V_{tp} = V_{tn}$
 - B. $V_{tp} + V_{tn} = 0$
 - C. $V_{tp} + V_{tn} < V_{DD}$
 - D. $|V_{tp}| + V_{tn} < V_{DD}$
 - E. $|V_{tp}| + V_{tn} < V_{DD}/2$

- 1a65. What equation describes JK flip-flop function?
- A. $Q^+ = Q \& J + !Q \& K$
 - B. $Q^+ = !Q \& J + Q \& K$
 - C. $Q^+ = !Q \& J + Q \& !K$
 - D. $Q^+ = Q \& !J + !Q \& K$
 - E. $Q^+ = Q \& J + !Q \& !K$

- 1a66. At passing from one technology to the other, transistors are scaled by $S < 1$ coefficient due to which the gate capacitance of a transistor with minimal sizes, depending on S :
- A. Increases linearly
 - B. Decreases linearly
 - C. Increases by square law
 - D. Decreases by square law
 - E. Does not change

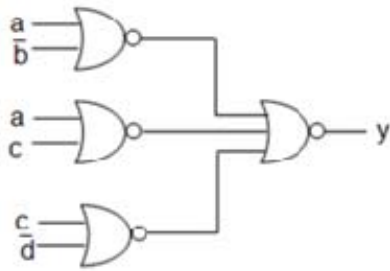
- 1a67. Given a circuit of synchronous FSM, detector of an input sequence, perform FSM analysis.



Determine which input sequence the given FSM detects. The end of the previous word may be the beginning of the next one.

- A. 0110
- B. 1001
- C. 0100
- D. 1101
- E. The correct answer is missing

1a68. Given a circuit in NOR basis. What function does the given circuit implement?

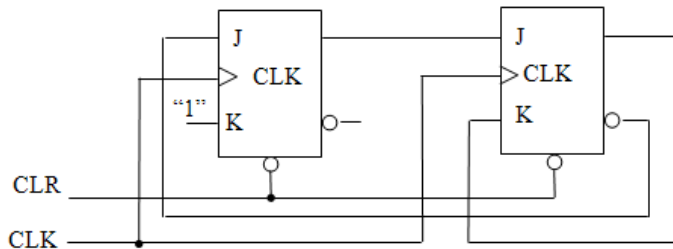


- A. $\bar{a} \cdot b + \bar{a} \cdot \bar{c} + \bar{c} \cdot d$
- B. $a \cdot c + \bar{b} \cdot c + a \cdot \bar{d}$
- C. $b \cdot c \cdot \bar{d} + \bar{a} \cdot b + c \cdot d$
- D. $\bar{a} \cdot b \cdot \bar{d} + a \cdot \bar{b} + c \cdot d$
- E. The correct answer is missing

1a69. What is the difference of electrical “short” or “long” interconnects at most characterized by?

- A. Interconnect width
- B. Signal power
- C. Signal edge increase
- D. Current power
- E. A. and B. together

1a70. Analyze an FSM circuit, built on JK-flip-flops. Which of the listed functions realizes the given circuit?



- A. It is a modulo-4 binary up counter
- B. It is a modulo-3 binary up counter
- C. It is a modulo-4 binary down counter
- D. It is a modulo-3 binary down counter
- E. The correct answer is missing

1a71. In case of the shown element base, what is the order of power reduction of the circuit? a) bipolar; b) CMOS; c) N-MOS

- A. a - c - b
- B. b - c - a
- C. a - b - c
- D. b - a - c
- E. c - b - a

1a72. It is required to construct 64:1 multiplexer using 8:1 multiplexer. How many 8:1 multiplexers are needed?

- A. 8 MUX 8:1
- B. 9 MUX 8:1
- C. 10 MUX 4:1
- D. 11 MUX 8:1
- E. The correct answer is missing

1a73. Which of the below listed criteria of organizing interconnects more contributes to the increase of performance?

- A. Increase of the number of interconnects layers
- B. Similarity of interconnects length
- C. Increase of the number of vias
- D. Reduction of critical path of signal processing
- E. A. and B. together

1a74. The logic of which circuit presented by VHDL code is senseless or wrong?

- A.

```
process (clock)
begin
  Y <= A and B;
end process;
```
- B.

```
process (A)
begin
  A <= A + 1;
end process;
```
- C.

```
process (A)
begin
```

```

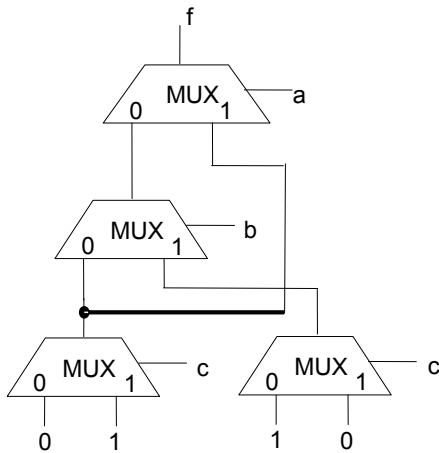
    Y <= A + 1;
  end process;
D. process (A, B)
  begin
    Y <= A and B;
  end process;
E. process (reset_n, clock)
  begin
    if (reset_n = '0') then
      Y <= 0;
    elsif (clock'EVENT and clock = '1') then
      Y <= A and B;
    end if;
  end process;

```

1a75. Given $F(x_1, x_2, x_3) = x_1 \oplus x_2 \oplus x_2 \cdot x_3$ function. Which of the given expressions corresponds to the given function?

- A. $F = x_1 \cdot \bar{x}_2 + x_1 \cdot x_3 + \bar{x}_1 \cdot x_2 \cdot \bar{x}_3$
- B. $F = x_1 \cdot x_2 + \bar{x}_1 \cdot x_3 + \bar{x}_1 \cdot x_2 \cdot \bar{x}_3$
- C. $F = \bar{x}_1 \cdot \bar{x}_2 + x_1 \cdot x_2 + x_2 \cdot \bar{x}_3$
- D. $F = x_1 \cdot \bar{x}_2 + x_2 \cdot x_3 + x_1 \cdot \bar{x}_3 \cdot x_2$
- E. $F = x_1 \cdot \bar{x}_2 + x_1 \cdot x_2 + x_2 \cdot \bar{x}_3$

1a76. Which of the functions below is realized in the given circuit on multiplexer?



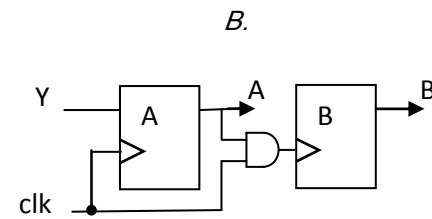
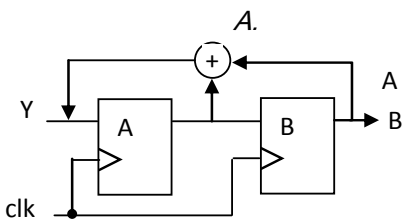
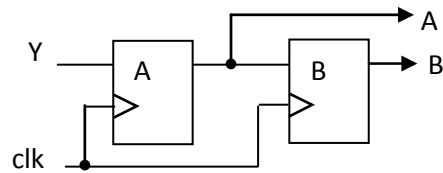
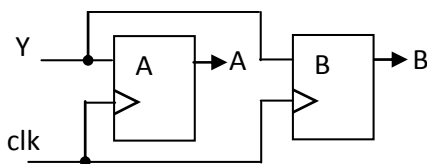
- A. $b \cdot c + a \cdot c + \bar{a} \cdot b \cdot \bar{c}$
- B. $a \cdot b + b \cdot \bar{c} + a \cdot \bar{c}$
- C. $a \cdot \bar{b} + a \cdot b \cdot c + \bar{b} \cdot c$
- D. $a \cdot b + a \cdot b \cdot c + \bar{b} \cdot c$
- E. $b \cdot c + a \cdot c + b \cdot c$

1a77. Which circuit does the use of the following assignment correspond to?

```

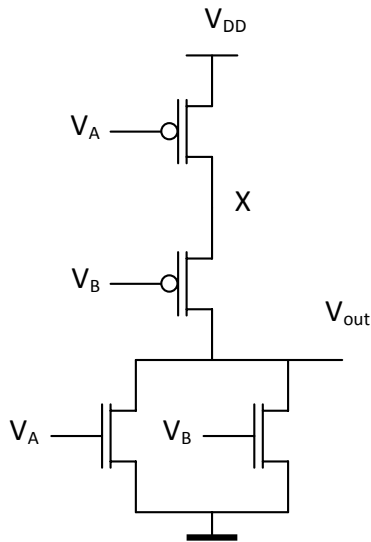
always @(posedge clk)
begin
  A <= Y;
  B <= A;
end

```



E. None of the circuits corresponds to the assignment.

1a78. In case of what transition of an input signal, the circuit output does not switch and X node switches?



- A. $AB=00 \rightarrow 01$
- B. $AB=00 \rightarrow 11$
- C. $AB=01 \rightarrow 10$
- D. $AB=11 \rightarrow 00$
- E. $AB=10 \rightarrow 00$

1a79. For what purpose is Low-Doped-Drain (LDD) region created?

- A. To increase threshold voltage
- B. To increase saturation voltage
- C. To increase gate-source break-down voltage
- D. To reduce gate capacitance
- E. To increase gate's oxide break-down voltage

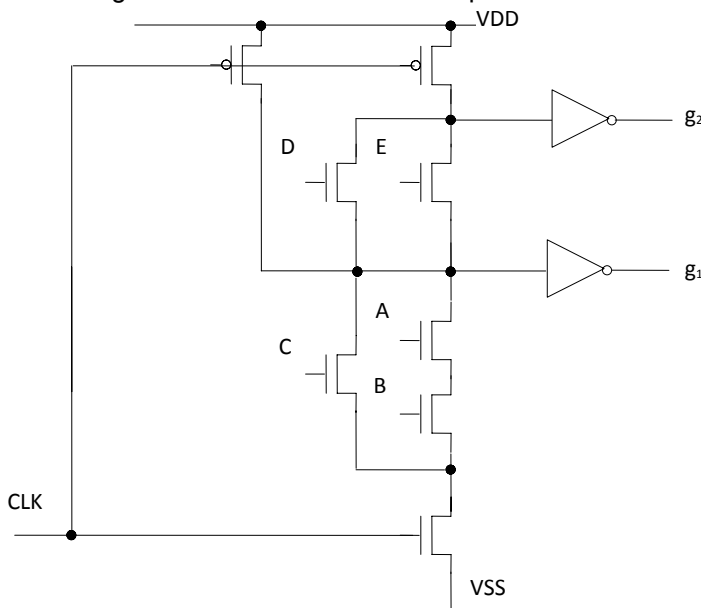
1a80. The first stage of 12-input 2-stage decoder is implemented by 3AND cells, and the second one - by 4AND cells. How many 3AND cells are there in the first stage?

- A. 16
- B. 24
- C. 32
- D. 8
- E. 64

1a81. Considering that memory array has equal number of 1-bit cells in lines and columns, define how many word lines 10 address bit memory has if the word length is 4 bits.

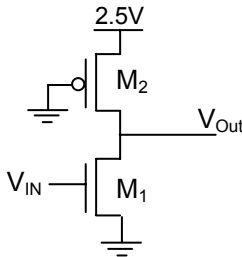
- A. 32
- B. 64
- C. 128
- D. 10242
- E. 256

1a82. What logic function does the circuit implement?



- A. $g_1=(A+B)C,$
 $g_2=(A+B)C+ED$
- B. $g_1=AB+C,$
 $+C)(E+D)$
- C. $g_1=AD+C,$
 $g_2=(AB+C)ED$
- D. $g_1=AB \oplus C,$
 $g_2=(AB \oplus C)ED$
 $g_1=AB+C, g_2=(AB+C)$
 $(E \oplus D)$

1a83. Which answer is correct if $V_{IN}=1.25V$, $V_{tp}=-1V$; $V_{tn}=1V$, $V_{SP}=1.25V$?



- A. *M1-in saturation mode, M2- in linear mode*
- B. *M1-in linear mode, M2-in linear mode*
- C. *M1- in linear mode, M2-in saturation mode*
- D. *M1-in saturation mode, M2-in saturation mode*
- E. *None*

1a84. Assuming that the inputs of XOR2 cell are independently distributed evenly, what is the probability of output switching?

- A. *0.25*
- B. *0.375*
- C. *0.5*
- D. *0.75*
- E. *0.875*

1a85. What is the main consequence of MOS transistor degradation due to “hot” carriers?

- A. *Increase of threshold voltage*
- B. *Decrease of threshold voltage*
- C. *Increase of channel resistance*
- D. *Decrease of channel resistance*
- E. *Decrease of drain-bulk breakdown voltage*

1a86. What does the threshold voltage of MOS transistor depend on?

- A. *Channel length*
- B. *Concentration of substrate dopant atoms*
- C. *Diffusion depth of source and drain*
- D. *Gate voltage*
- E. *Drain voltage*

1a87. In what case is the crowbar current missing in a CMOS inverter?

- A. $V_{tp}=V_{tn}$
- B. $V_{tp}+V_{tn}=0$
- C. $V_{tp}+V_{tn}<V_{DD}$
- D. $|V_{tp}|+V_{tn}<V_{DD}$
- E. $|V_{tp}|+V_{tn}<V_{DD}/2$

1a88. Assuming $k_n=2.5k_p$, in what case will the resistances from the output of NOR2 cell to VDD and VSS be equal?

- A. $W_p=4W_n$
- B. $W_p=W_n$
- C. $W_p=2W_n$
- D. $W_p=10W_n$
- E. $W_p=5W_n$

1a89. Assuming $k_n=rk_p$, in what case will the resistances from the output of 4NAND cell to VDD and VSS be equal?

- A. $W_p/W_n=r$
- B. $W_p/W_n=r/8$
- C. $W_p/W_n=r/4$
- D. $W_p/W_n=r/16$
- E. $W_p/W_n=2r$

1a90. C capacitance is connected to the end of interconnect line with L length, line parameters are c [F/m], r [Ohm/m]. By what formula is the signal delay time in the line given?

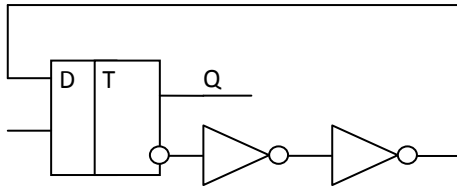
- A. $0.7rc$
- B. $0.7Lr(C+c)$
- C. $0.7Lr(C+Lc/2)$
- D. $0.7Lr(C+Lc)$
- E. $0.7Lr(C/2+Lc/2)$

1a91. What equation describes JK flip-flop transitions?

- A. $Q^+=Q&J + !Q&K$
- B. $Q^+=!Q&J + Q&K$
- C. $Q^+=!Q&J + Q&!K$

- D. $Q^+ = Q \& J + !Q \& K$
 E. $Q^+ = Q \& J + !Q \& !K$

1a92. The figure shows the circuit of a frequency divider.
 What expression gives the minimum period of clock pulses?



- A. $T_{min} = t_{su} + t_{c2q} + 2t_{pinv}$
 B. $T_{min} = t_{c2q} + 2t_{pinv}$
 C. $T_{min} = t_{su} + t_{hd} + t_{c2q} + 2t_{pinv}$
 D. $T_{min} = t_{hd} + t_{c2q} + 2t_{pinv}$
 E. $T_{min} = t_{su} + t_{hd} + 2t_{pinv}$

1a93. Which of the given expressions is wrong?

- A. $A \oplus !B = !A \oplus B$
 B. $1 \oplus !B \oplus A = B \oplus A$
 C. $A \oplus B = !A \oplus !B$
 D. $.A \oplus !B = !A \oplus !B$
 E. $!.A \oplus B = !(A \oplus B)$

1a94. When passing from one technology node to another, transistors are scaled by $S < 1$ coefficient due to which the gate capacitance of a transistor with minimum sizes depends on S (assuming gate oxide thickness does not change):

- A. *Increases by square law*
 B. *Decreases by square law*
 C. *Increases linearly*
 D. *Decreases by cubic law*
 E. *Does not change*

1a95. Before reading from 1T DRAM cell, the bitline should:

- A. *Discharge to VSS*
 B. *Discharge to V_t*
 C. *Discharge to VDD*
 D. *Charge to $VDD/2$*
 E. *Charge to $VDD - V_t$*

1a96. For the same size inverter what version has the minimum input capacitance?

- A. *PMOS low-vt, NMOS high-vt*
 B. *PMOS standard-vt, NMOS high-vt*
 C. *PMOS high-vt, NMOS standard-vt*
 D. *PMOS high-vt, NMOS low-vt*
 E. *PMOS low-vt, NMOS standard-vt*

1a97. What is the minimum number of transistors in a pass gate implemented 1:4 multiplexer, assuming that normal and complemented select variables are available?

- A. 16
 B. 12
 C. 8
 D. 6
 E. 4

1a98. Define in which state 6 bit Johnson's counter will be after the 10th clock pulse. The initial state is 000111.

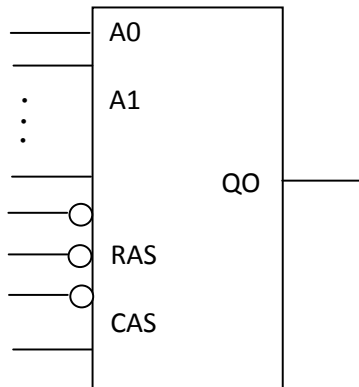
- A. 011110
 B. 011111
 C. 111111
 D. 111101
 E. *The correct answer is missing*

1a99. A fragment of Verilog description is presented below. What value will Y variable take after execution of this fragment?

- ```
reg A; reg [1:0] B,C; reg [2:0] D;
reg [15:0] Y;
A=1'b1; B=2'b01; C=2'b10; D=3'b110;
Y={2{A}, 3{B}, C, 2{D}};
```

- A.  $Y = 10'b\ 1100\_1100\_0111\_1001$
- B.  $Y = 10'b\ 1000\_1111\_1010\_0101$
- C.  $Y = 10'b\ 1101\_0101\_1011\_0110$
- D.  $Y = 10'b\ 1100\_0100\_0111\_1001$
- E. The correct answer is missing

1a100. Define the volume of the IC DRAM presented below.



- A. 256 M bit
- B. 16 K bit
- C. 16 M bit
- D. 32M bit
- E. The correct answer is missing

1a101. Which of the choices represents the number  $-7/256$  as a floating point number with single precision (standard IEEE 754)?

- A. 1284 F000
- B. BCE0 0000
- C. DA00 1000
- D. CA01 1000
- E. The correct answer is missing

1a102. How many address inputs does IC DRAM have with organization 512 Mx1Bit?

- A. 28
- B. 15
- C. 12
- D. 4
- E. The correct answer is missing

1a103. Given  $F(x_1, x_2, x_3) = x_1 \oplus x_1 \cdot x_3 \oplus x_2 \cdot x_3$  function. Which of the mentioned expressions corresponds to the given function?

- A.  $F = x_2 \cdot x_3 + x_1 \cdot \bar{x}_3$
- B.  $F = x_1 \cdot \bar{x}_2 + \bar{x}_1 \cdot x_3 + x_1 \cdot x_2 \cdot \bar{x}_3$
- C.  $F = \bar{x}_1 \cdot \bar{x}_2 + x_1 x_2 + x_2 \cdot \bar{x}_3$
- D.  $F = x_1 \cdot x_2 + x_1 \cdot x_2 + x_2 \cdot x_3$
- E. The correct answer is missing

1a104. Which size does IC ROM have, implementing combinational multiplication of two 8 bit numbers?

- A. 128Kx8
- B. 256x16
- C. 64Kx16
- D. 256Kx16
- E. The correct answer is missing

1a105. How many ICs with 512Kx8 organization are required to implement static memory module of 8M 16 bit words size?

- A. 16
- B. 32
- C. 8
- D. 24
- E. The correct answer is missing

1a106. The following examples are expected to be models of logical shifter (written in Verilog HDL). Which of these examples will serve as pure combinational logic?

```
1)
wire [7:0] my_signal;
 assign my_signal = my_signal << 1;
```

2)  

```
reg [7:0] my_signal;
always @ (*) begin
 my_signal = my_signal << 1;
end
```

3)  

```
reg [7:0] my_signal;
always @ (posedge clock) begin
 my_signal <= my_signal << 1;
end
```

Select the only correct version of these four possible answers:

- A. Only example 1) is the correct one
- B. Only example 2) is the correct one
- C. Only example 3) is the correct one
- D. Only examples 1) and 2) are the correct ones
- E. All examples are correct

**1a107.** The following examples are expected to be models of logical "NOT" cell (written in Verilog HDL). Which of these examples will serve as correct combinational logic?

1) 

```
module not_cell (input_sig, output_sig);
input input_sig;
output output_sig;
 always @ (*) begin
 output_sig = 1'b1;
 if (input_sig == 1'b1) begin
 output_sig = 1'b0;
 end
 end
end
endmodule
```

2) 

```
module not_cell (input_sig, output_sig);
input input_sig;
output output_sig;
 always @ (*) begin
 if (input_sig == 1'b1) begin
 output_sig = 1'b0;
 end
 else begin
 output_sig = 1'b1;
 end
 end
end
endmodule
```

3) 

```
module not_cell (input_sig, output_sig);
input input_sig;
output output_sig;
 always @ (input_sig) begin
 if (input_sig == 1'b1) begin
 output_sig = 1'b0;
 end
 else begin
 output_sig = 1'b1;
 end
 end
end
endmodule
```

Select the only correct version of these four possible answers:

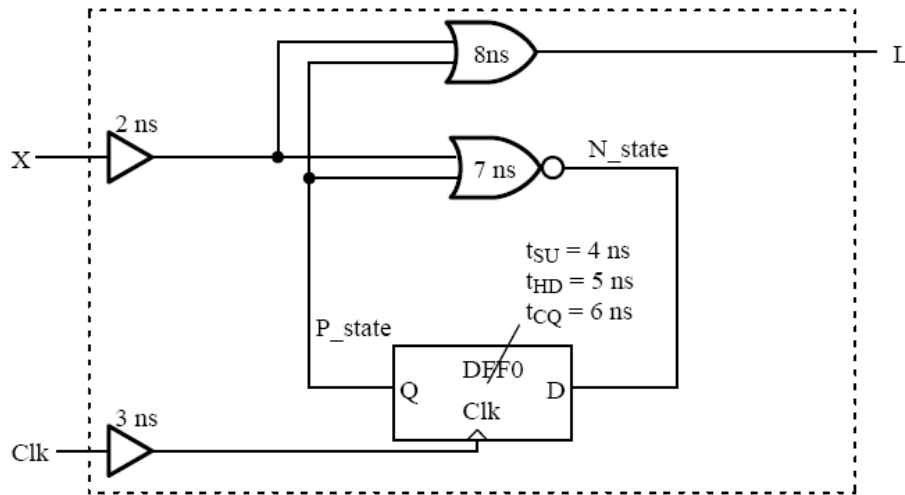
- A. Only example 1) is the correct one
- B. Only example 2) is the correct one
- C. Only example 3) is the correct one
- D. All examples are incorrect
- E. All examples are correct.

**b) Problems**

**1b1.**

For the presented circuit:

- What are the external setup and hold times for input X?
- What is the delay from the clock to output L?
- What is the clock cycle time based on register-to-register delays (the flip-flop is drawn “backwards”)?



**1b2.**

Consider the following VHDL code.

```

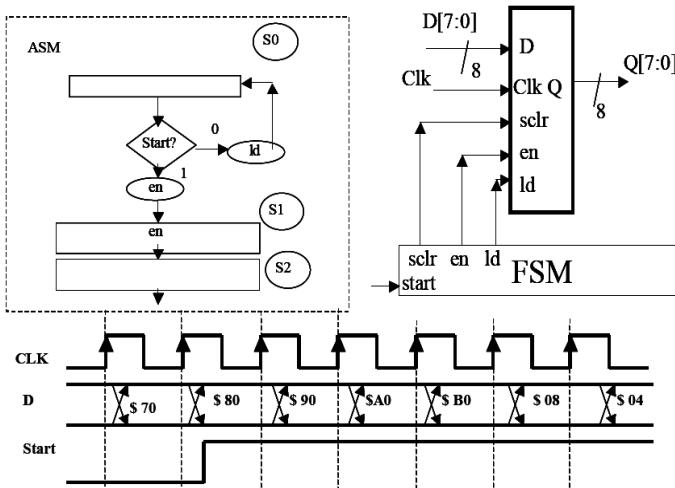
library ieee;
use ieee.std_logic_1164.all;
entity pulsedet is port(
signal clk,reset,pulse_in: in std_logic;
signal pulse_out: out std_logic
);
end pulsedet;
architecture behavior of pulsedet is
signal dffout : std_logic_vector(2 downto 0);
begin
dffs: process(clk,reset)
begin
if (reset = '1') then
dffout <= "000";
elsif (clk'event and clk='1') then
dffout(2) <= dffout(1);
dffout(1) <= dffout(0);
dffout(0) <= pulse_in;
end if;
end process;
pulse_out <= dffout(2) and not dffout(1);
end behavior;

```

Draw a diagram of logic that implements the VHDL code. (Show logic gates and D flip-flops.)

**1b3.**

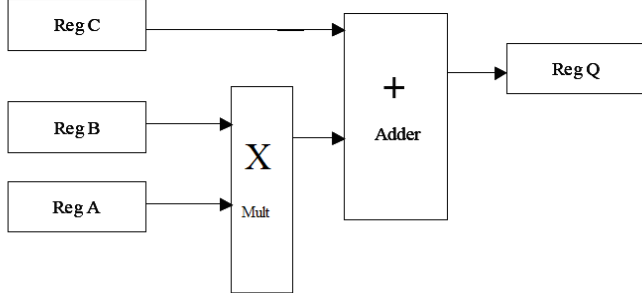
On the waveforms below, complete the waveforms for state, ld, en, and Q. The FSM is controlling the UP counter. Assume the initial state is S0.



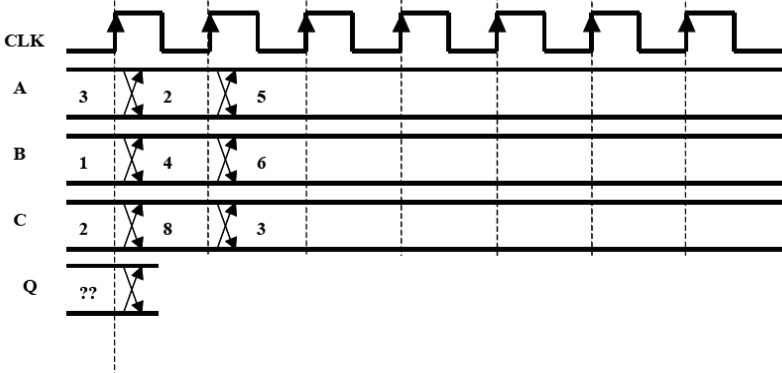
**1b4.**

For the figure below:

- Give the maximum register-to-register delay.
- Modify the diagram to add one level of pipelining but still maintain the same functionality. Add the pipeline stage in the place that will improve the register-to-register delay the most. Compute the new maximum register-to-register.
- With the pipeline stage added, complete the 'Q' waveform shown below. Input registers change values as shown; assume Reg Q is loaded every clock cycle. All waveforms represent register outputs.



Mult Delay = 18 ns, Adder delay = 7 ns, Tcq = 3 ns, Setup time = 1 ns, Hold time = 2 ns



**1b5.**

Design a circuit for a CMOS cell which is realized by  $Z = \overline{!(A(B+C)+BD)}$  logic function.

**1b6.**

Design a MS D-FF based on transmission gates (TG) with SET input.

**1b7.**

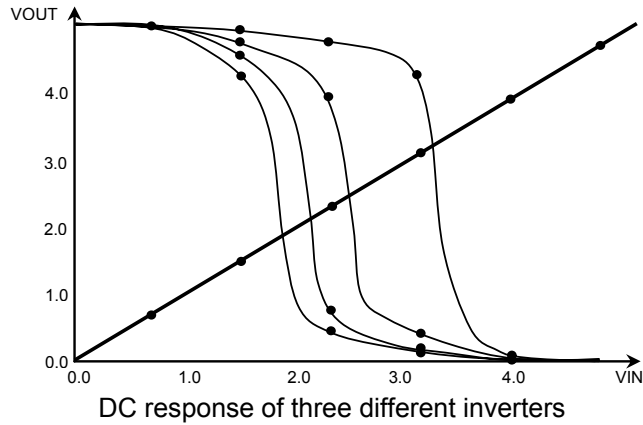
Design a MS D-FF based on switching keys with SET and RESET inputs.

**1b8.**

Design a CMOS cell circuit which is described by  $Y = \sum(1,2,6,7)$  function.

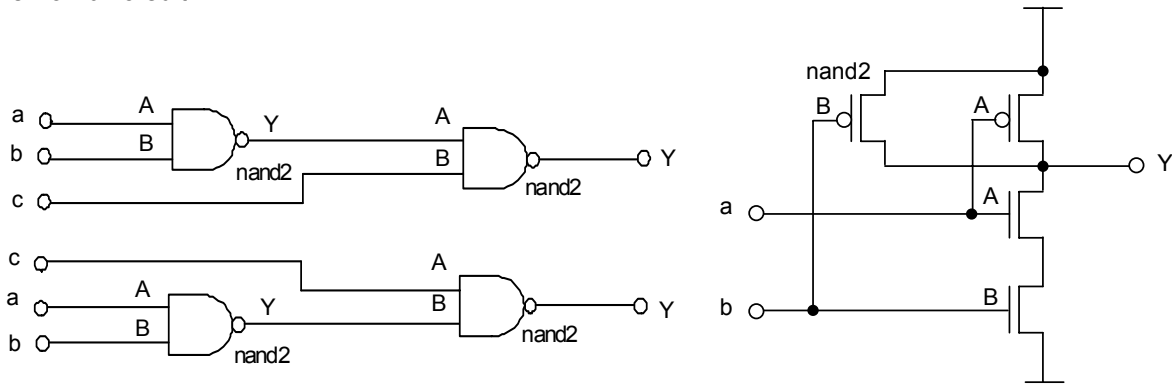
**1b9.**

Four inverters ( $V_{DD}=5V$ ) have  $W_N$ , changing from  $3\mu m$  to  $12\mu m$  while  $W_P$  is the same for each inverter ( $10\mu m$ ). Identify which  $W_P/W_N$  ratio produces the most left curve in the figure. In this process  $V_{DD}$  is  $5V$ . Which inverter has the most even noise margins ( $V_{THN}=0.6V$ ,  $V_{THP}=-0.8V$ ,  $LN=LP$ )?



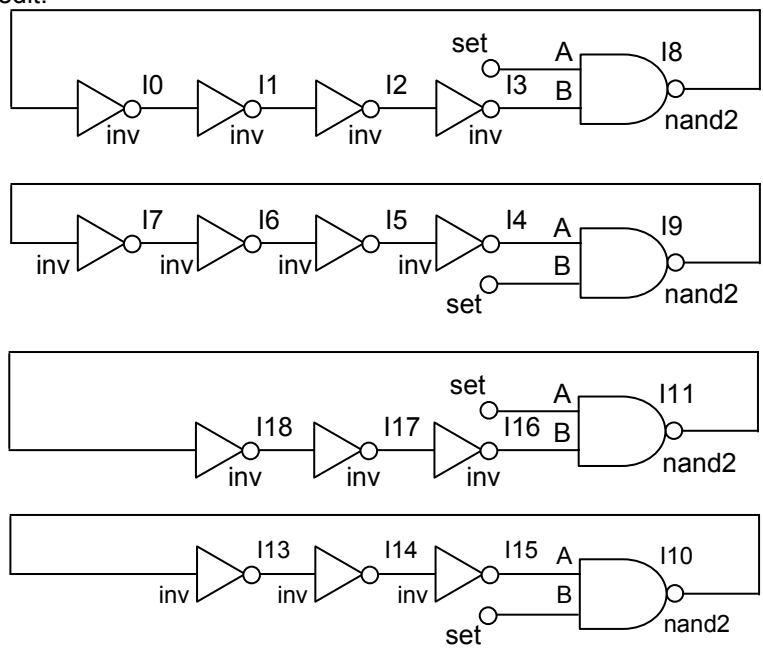
**1b10.**

Both circuits below have the same function. Which one has the smallest, best case delay? Explain the answer for full credit.

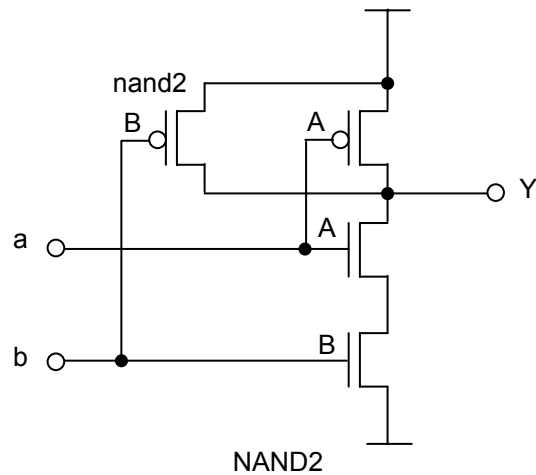


**1b11.**

It is necessary to design a ring oscillator that is to oscillate as fast as possible. Usually a set pin is needed to start the ring oscillator properly. Which circuit would be used for the fastest ring oscillator? Explain why for full credit.



Circuits of a ring oscillator (cont.)



**1b12.**

Figure shows a DFF. Using the timing data presented in the table, calculate how long CK has to remain high for Q and NQ to get the value from the output of I12 (nand2). Figure shows a DFF. Using the timing data presented in the table, calculate how long D has to remain stable before the rising clock edge so that the outputs of I12 and I13 properly get the value of D (or not(D)).

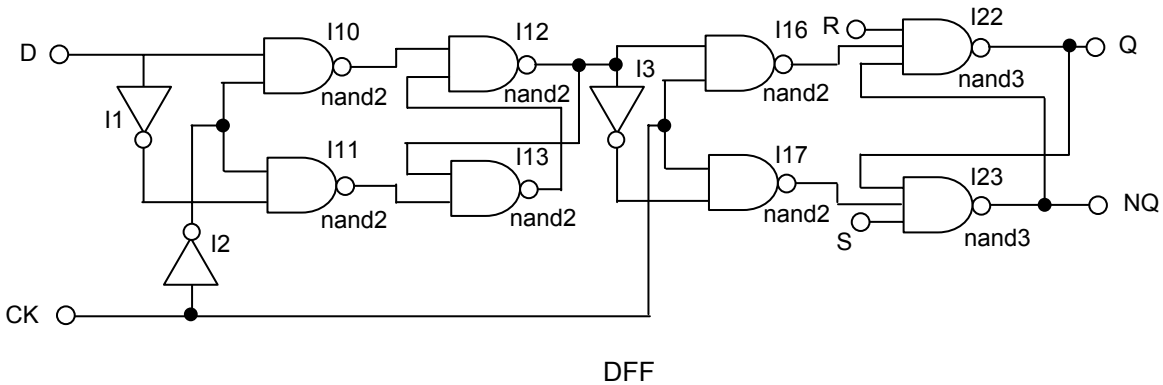


Table: Various times for various gates

|                   | INV | NAND2 | NAND3 |
|-------------------|-----|-------|-------|
| TFALL (ps)        | 200 | 500   | 600   |
| TRISE (ps)        | 300 | 500   | 650   |
| $\tau_{phl}$ (ps) | 100 | 250   | 300   |
| $\tau_{plh}$ (ps) | 150 | 250   | 325   |

**1b13.**

Using the AOI technique design a CMOS circuit to implement the following logic function:  $Z=(ABCD+EFG)H$   
Show the PNET and the NNET connected into a circuit.

**1b14.**

Using the AOI technique, design a CMOS circuit to implement the following logic function:  $Z=(AB+CD+EF)G$   
Show the PNET and the NNET connected into a circuit.

**1b15.**

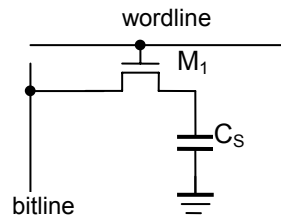
Using the AOI technique, design a CMOS circuit to implement the following logic function:  $Z=(ABC+DE+F)G$   
Show the PNET and the NNET connected into a circuit.

**1b16.**

Using the AOI technique, design a CMOS circuit to implement the following logic function: Minimize area and delay. (Show Euler path, but do not draw it).  $Z=(AB+CD+EFG)H$ . Show the PNET and the NNET connected into a circuit.

**1b17.**

For reading, the bitline is precharged to  $V_{DD}/2$ . Determine the settled voltage on the bitline when reading logic 1 and logic 0, if  $V_{tn}=0.3V$ ,  $V_{DD}=1.2V$ ,  $C_{BL}=10C_s$ . The voltage swing of the wordline control is from 0 to  $V_{DD}$ . Ignore leakages and body bias effect.

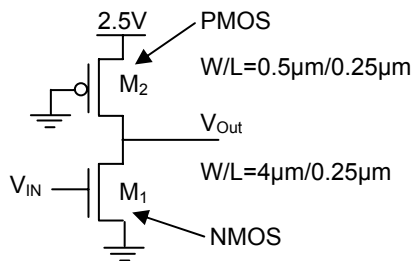


**1b18.**

Compute the following for the pseudo-NMOS inverter shown below:

$k_n = 115\mu A/V^2$ ,  $k_p = 30\mu A/V^2$ ,  $V_{tn} = 0.5V$ ,  $V_{tp} = -0.4V$

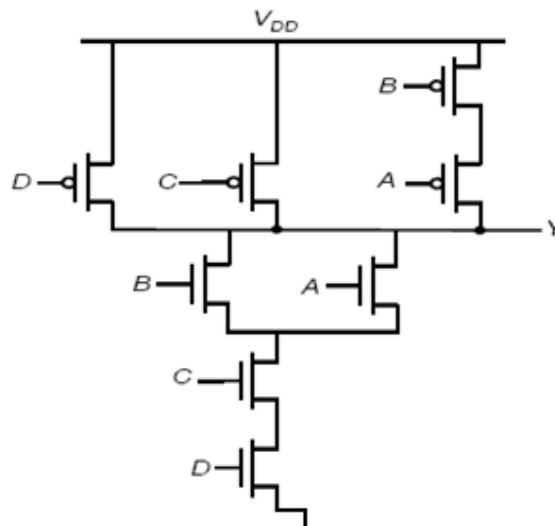
- $V_{OL}$  and  $V_{OH}$
- The static power dissipation: (1) for  $V_{in}$  low, and (2) for  $V_{in}$  high
- For an output load of 1 pF, calculate  $t_{pLH}$ ,  $t_{pHL}$  (ignore the intrinsic capacitances of transistors)



**1b19.**

Consider the circuit below.

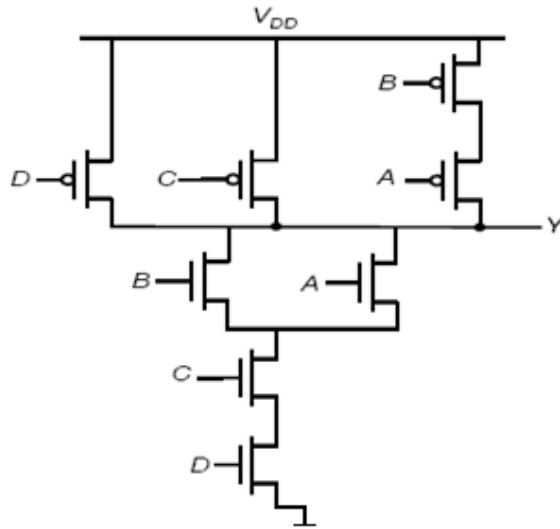
- What is the logic function implemented by CMOS transistor network? Size the NMOS and PMOS devices such that the output resistance is the same as of an inverter with an NMOS  $W/L = 4$  and PMOS  $W/L = 8$ .
- What are the input patterns that give the worst case  $t_{pHL}$  and  $t_{pLH}$ ? State the initial input patterns and tell which input(s) has to make a transition to achieve this maximum propagation delay. Consider the effect of the capacitances at the internal nodes.





**1b20.**

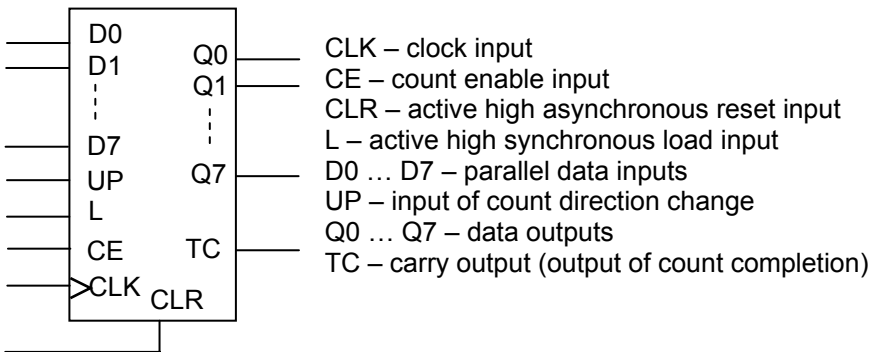
If  $P(A=1)=0.5$ ,  $P(B=1)=0.2$ ,  $P(C=1)=0.3$  and  $P(D=1)=0.8$ , determine the switching power dissipation in the logic gate. Assume  $V_{DD}=2.5V$ ,  $C_{out}=30fF$  and  $f_{clk}=250MHz$ .



**1b21.**

Design an 8-bit binary up/down counter with parallel synchronous loading, asynchronous reset and an input to enable counting.

Logic symbol for the given counter:



TC is set as soon as code  $FF_{16}$  or  $00_{16}$  appears on counter outputs.

*Truth table of the counter*

| CLR | L | CE | UP | CLK | Q7 ... Q0                           | TC              | Mode                         |
|-----|---|----|----|-----|-------------------------------------|-----------------|------------------------------|
| 1   | X | X  | X  | X   | 0 ... 0                             | 0               | Asynchronous reset           |
| 0   | 1 | X  | X  |     | D7 ... D0                           | 0               | Parallel loading (D[7:0]≠FF) |
| 0   | 1 | X  | X  |     | D7 ... D0                           | 1               | Parallel loading (D[7:0]=FF) |
| 0   | 1 | X  | X  |     | D7 ... D0                           | 1               | Parallel loading (D[7:0]≠00) |
| 0   | 1 | X  | X  |     | D7 ... D0                           | 1               | Parallel loading (D[7:0]=00) |
| 0   | 0 | 1  | 1  |     | +1                                  | 0               | Increment (Q[7:0]≠FF)        |
| 0   | 0 | 1  | 1  |     | +1                                  | 1               | Increment (Q[7:0]=FF)        |
| 0   | 0 | 1  | 0  |     | -1                                  | 0               | Decrement (Q[7:0]≠00)        |
| 0   | 0 | 1  | 0  |     | -1                                  | 1               | Decrement (Q[7:0]=00)        |
| 0   | 0 | 1  | 0  | X   | Q7 <sub>n</sub> ... Q0 <sub>n</sub> | TC <sub>n</sub> | Hold                         |

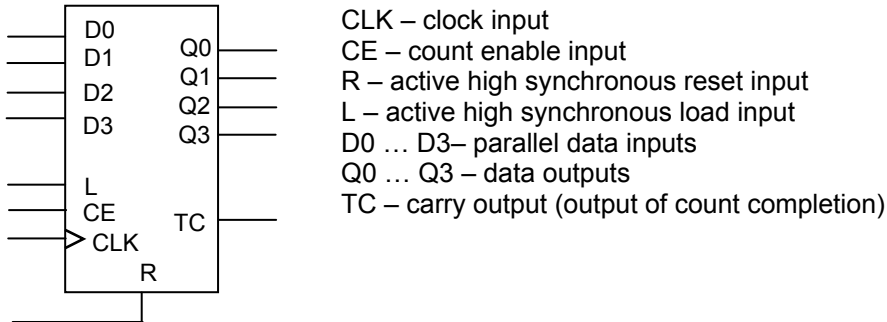
Q7<sub>n</sub> ... Q0<sub>n</sub> – previous states of the counter.

- Describe counter in Verilog and simulate by means of logic analysis tool VCS.
- Synthesize by Design Compiler tool. Obtain Verilog-out (Gate Level Netlist). Again simulate and compare the results.
- Synthesize the circuit of the given counter manually, using T flip-flops. For simplification take the number of bits equal to 4.

**1b22.**

Design a 4-bit binary-coded decimal counter with parallel synchronous loading, synchronous reset and count enable inputs.

Logic symbol for the given counter:



TC is set as soon as code  $1001_2$  appears on counter outputs.

*Truth table of the counter*

| R | L | CE | CLK | Q3 ... Q0                           | TC              | Mode                           |
|---|---|----|-----|-------------------------------------|-----------------|--------------------------------|
| 1 | X | X  | ↑   | 0 ... 0                             | 0               | Synchronous reset              |
| 0 | 1 | X  | ↑   | D3 ... D0                           | 0               | Parallel loading (D[3:0]≠1001) |
| 0 | 1 | X  | ↑   | D3 ... D0                           | 1               | Parallel loading (D[3:0]=1001) |
| 0 | 0 | 1  | ↑   | +1                                  | 0               | Increment (Q[3:0]≠FF)          |
| 0 | 0 | 1  | ↑   | +1                                  | 1               | Increment (Q[3:0]=FF)          |
| 0 | 0 | 0  | X   | Q <sub>3n</sub> ... Q <sub>0n</sub> | TC <sub>n</sub> | Hold                           |

Q<sub>3n</sub>...Q<sub>0n</sub> – previous states of the counter.

- Describe the counter in Verilog and simulate by means of logic analysis tool VCS.
- Synthesize by Design Compiler tool. Obtain Verilog-out (Gate Level Netlist). Again simulate and compare the results.
- Synthesize the circuit of the given counter manually, using T flip-flops.

**1b23.**

Design a clocked synchronous Moore FSM producing a remainder of division of a decimal number by 3 on its outputs. FSM receives the digits of a decimal number sequentially on its inputs.

- Describe counter in Verilog and Simulate using logic analysis tool VCS.
- Synthesize using Design Compiler tool. Obtain Verilog-out (Gate Level Netlist). Again simulate and compare the results.
- Synthesize the circuit of the given counter manually.

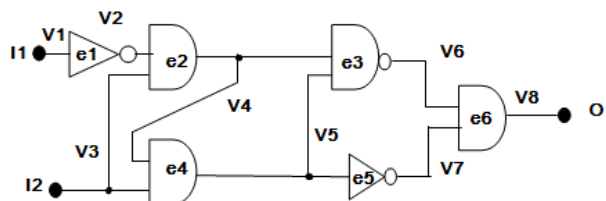
**1b24.**

Design a clocked synchronous FSM with two inputs, X and Y, and one output, Z. The output should be 1 if the number of 1 inputs is the multiple of 5 on X and Y since reset, and 0 otherwise.

- Describe counter in Verilog and Simulate using logic analysis tool VCS.
- Synthesize using Design Compiler tool. Obtain Verilog-out (Gate Level Netlist). Again simulate and compare the results.
- Synthesize the circuit of the given counter manually.

**1b25.**

Calculate the minimum and maximum signal formation time of the shown circuit in V<sub>1</sub> – V<sub>8</sub> nets by conventional units, if the cell delays are given by conventional units  $\tau_{e1}=\tau_{e5}=10$ ,  $\tau_{e3}=15$ ,  $\tau_{e2}=\tau_{e4}=\tau_{e6}=20$ : On the circuit mark I/O critical path and calculate the total delay of that path by conventional units.



**1b26.**

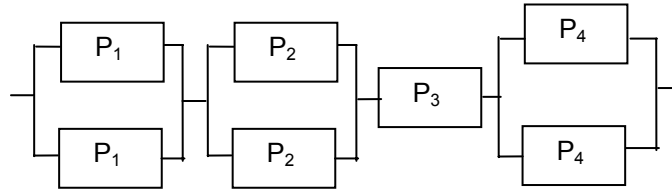
An interconnect of 300  $\mu\text{m}$  length and 0,2  $\mu\text{m}$  width is given, the sheet resistance and capacitance of which correspondingly equal:

$R_{\square}=0,2 \text{ Ohm}/\square$   $C=0,1 \text{ fF}/\mu\text{m}$ .

Construct the equivalent circuit of interconnect's 3-segment, R,C distributed parameters and calculate the delay in it.

**1b27.**

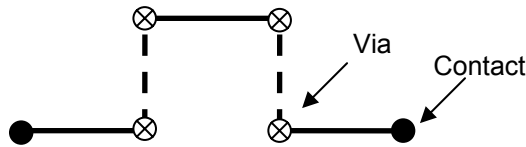
Calculate faultiness probability of IC consisting of 7 blocks if their connection, according to reliability, has the following view.



Given  $P_1=0.5$ ;  $P_2=0.6$ ;  $P_3=0.8$ ;  $P_4=0.4$ .

**1b28.**

Two contacts are connected by an interconnect containing 4 vias, as illustrated in the figure.



Construct interconnect's R, C equivalent circuit and calculate the delay in the transmission line connecting two contacts, if given:

- Each transmission line's capacitance of interconnect equals 100 fF.
- Each programmable contact resistance equals 1 Ohm.

Ignore ohmic resistances of transmission lines and contacts as well as the capacitances of vias.

**1b29.**

Based on the logic of provided VHDL code, develop a digital circuit consisting of logic gates.

```

library ieee;
use ieee.std_logic_1164.all;
use ieee.std_logic_unsigned.all;
entity adder2 is port(
 signal
 operand1,
 operand2: in std_logic_vector(1 downto 0);
 signal
 sum: out std_logic_vector(2 downto 0);
);
end adder2;
architecture behavior of adder2 is
begin
 sum <= operand1 + operand2;
end behavior;

```

Determine whether the provided logic is a combinational logic or not. Argue the answer.

**1b30.**

Based on the logic of provided VHDL code, develop a digital circuit consisting of logic gates.

```

library ieee;
use ieee.std_logic_1164.all;
use ieee.std_logic_unsigned.all;
entity incr1 is port(
 signal
 operand: in std_logic_vector(2 downto 0);
 signal
 incr_result: out std_logic_vector(3 downto 0);
);
end incr1;
architecture behavior of incr1 is
begin
 incr_result <= operand + 1;
end behavior;

```

Determine whether the provided logic is a combinational logic or not. Argue the answer.

**1b31.**

Based on the logic of provided VHDL code, develop a digital circuit consisting of logic gates.

```

library ieee;
use ieee.std_logic_1164.all;
library types;
use types.conversions.all;
entity shift_ll is port(
 signal
 operand: in std_logic_vector(3 downto 0);
 shift_size: in std_logic_vector(1 downto 0);
 signal
 shift_result: out std_logic_vector(3 downto 0);
);
end shift_ll;
architecture behavior of shift_ll is
begin
 process (operand, shift_size)
 begin
 --left logical shift by shift_size
 shift_result <= operand sll to_uint(shift_size);
 end process;
end behavior;

```

Determine whether the provided logic is a combinational logic or not. Argue the answer.

**1b32.**

Based on the logic of provided VHDL code, develop a digital circuit consisting of logic gates.

```

library ieee;
use ieee.std_logic_1164.all;
library types;
use types.conversions.all;
entity shift_rl is port(
 signal
 operand: in std_logic_vector(3 downto 0);
 shift_size: in std_logic_vector(1 downto 0);
 signal
 shift_result: out std_logic_vector(3 downto 0);
);
end shift_rl;
architecture behavior of shift_rl is
begin
 process (operand, shift_size)
 begin
 --right logical shift by shift_size
 shift_result <= operand srl to_uint(shift_size);
 end process;
end behavior;

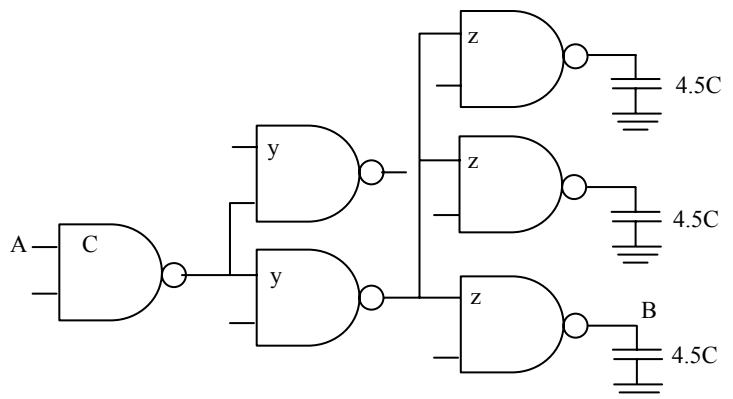
```

Determine whether the provided logic is a combinational logic or not. Argue the answer.

**1b33.**

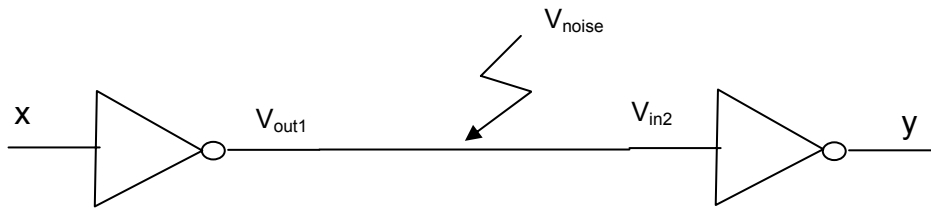
The sizes of the first cell in the shown logic circuit are selected such that it has a driving strength of a minimal size inverter having input capacitance C, i.e. NMOS transistor sizes have been increased to compensate the consequence of serial connection.

Define the second and third stage scaling ratios y and z, from the condition to get minimum delay in the A-to-B path. Ignore intrinsic output and interconnects capacitances of cells.



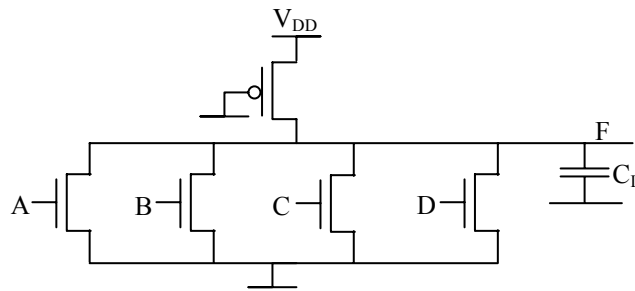
**1b34.**

For the following circuit define the maximum permissible noise magnitude  $V_{noise}$ , if the inverter's VTC parameters are  $V_{OHnom}=1V$ ,  $V_{OHmin}=0.9V$ ,  $V_{OLnom}=0V$ ,  $V_{OLmax}=0.15V$ ,  $V_{SP}=0.48V$ ,  $V_{IHmin}=0.58V$ ,  $V_{ILmax}=0.44V$ .



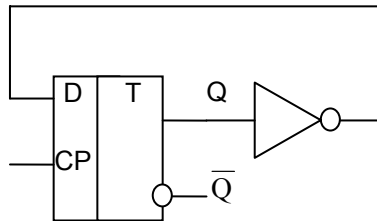
**1b35.**

Define four input NOR cell's output low and high levels (see the figure) (a) when only one input switches (b) when all the inputs switch simultaneously if  $C_L=0.05$  pF. Use the following technological parameters  $V_{DD}=1.8$  V,  $T_{ox}=10^{-8}$  m,  $\mu_n=270$  cm<sup>2</sup>/Vv,  $V_{in}=0.5V$ ,  $\mu_p=70$  cm<sup>2</sup>/Vv,  $V_{tp}=-0.5V$ ,  $L_n=0.18$  um,  $W_n=20$  um,  $W_p=5$  um.



**1b36.**

A circuit of a frequency divider is presented.

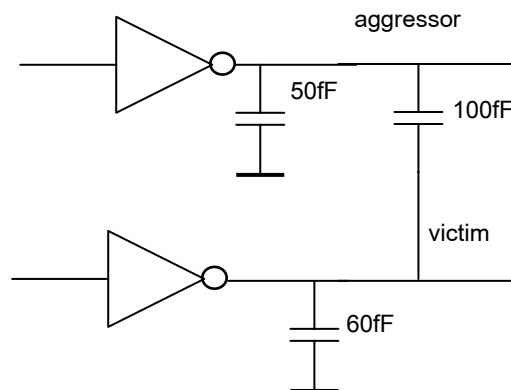


Define the minimum period of clock pulses if  $t_{su}=20ps$ ,  $t_{hd}=-15ps$ ,  $t_{c2q}=100ps$ ,  $t_{pinv}=30ps$ .

**1b37.**

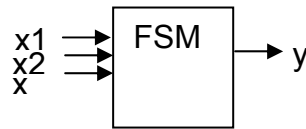
For the given circuit define:

- the maximum value of the noise in victim line that occurs when the signal in the aggressor line switches from 1.0 V to 0V.
- the effective capacitance in victim line for delay calculation if the signals in aggressor and victim lines switch in opposite directions.



**1b38.**

Design Moore FSM which has 3 inputs  $x_1$ ,  $x_2$ ,  $x_3$  and 1 output. The output of FSM equals 1 when total number of ones, which are given to FSM inputs, is divided by 7. Describe by Verilog. Create a testbench and simulate by VCS.



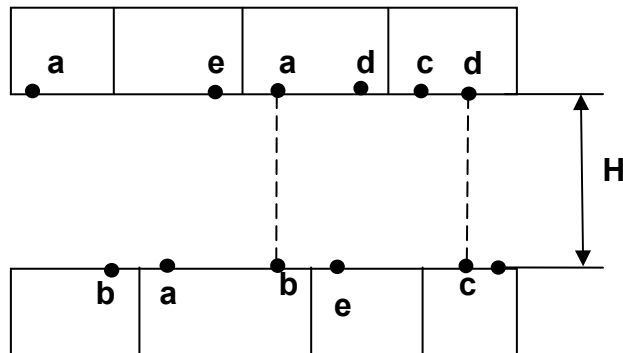
**1b39.**

Describe by Verilog 5-bit polynomial counter (LFSR). In feedback circuit  $d(x) = 1 + x^3 + x^5$  is polynomial. The counter has 10000 initial state asynchronous preset input. The shift of information is realized by positive edge of clock signal.

Describe by Verilog. Create a testbench and simulate using VCS.

**1b40.**

Define the minimum H distance between two rows of cells which is necessary for routing of two-layer co-perpendicular routing of a, b, c, d, e nets if the minimum permissible size between interconnects width and space is 0,1 um, and the minimum distance between interconnects and 0,2 um.

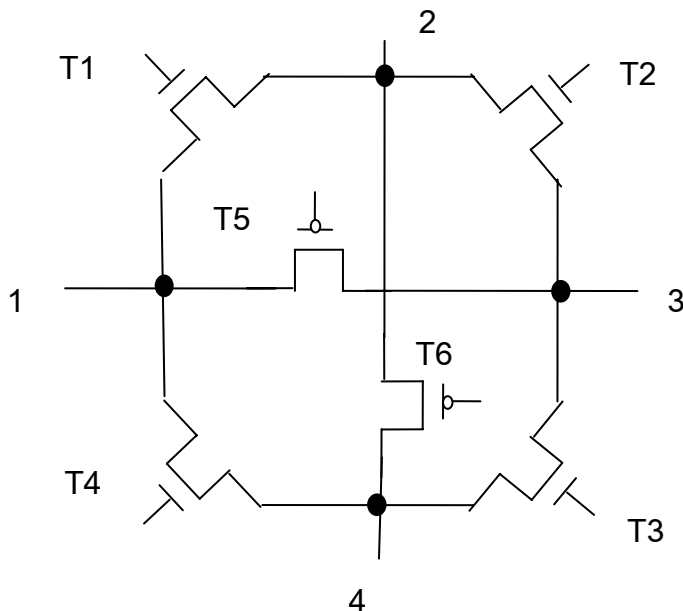


**1b41.**

Switching block with 4 pins (1, 2, 3, 4) is shown in the figure. It consists of 4 NMOS and 2 PMOS transistors.

What kind of logic level signals (0 or 1) should be given to each gate of T1-T6 transistors to provide:

- a) Simultaneous signal transfer from pin 1 to 3 and from pin 2 to 4;
- b) Simultaneous signal transfer from pin 1 to 4 and from pin 2 to 3;



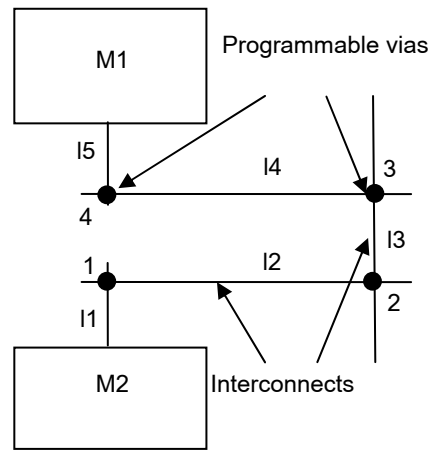
**1b42.**

Two modules (M1 and M2) are connected with 5 interconnects (I1-I5) and 4 programmable vias (1-4). Construct interconnect's R, C equivalent circuit and calculate the delay in the transmission line connecting two modules, if given:

Each transmission line capacitance of interconnect equals 100 fF.

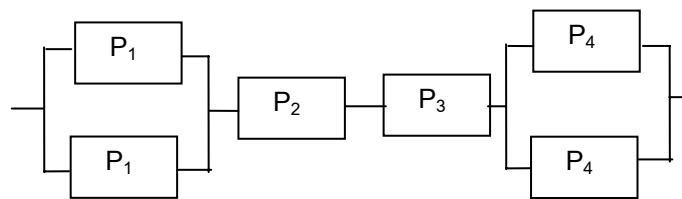
Each programmable contact resistance equals 1 Ohm.

Ignore ohmic resistances of interconnects and contacts as well as the capacitances of programmable vias.



**1b43.**

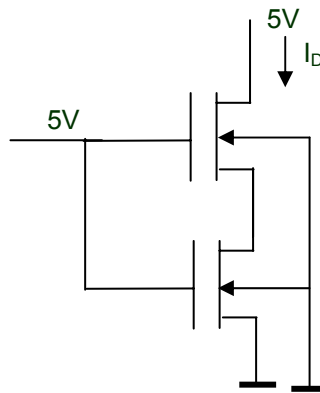
Calculate faultness probability of IC consisting of 7 blocks if their connection, according to reliability, has the following view.



Given  $P_1=0,5$ ;  $P_2=0,6$ ;  $P_3=0,8$ ;  $P_4=0,4$ .

**1b44.**

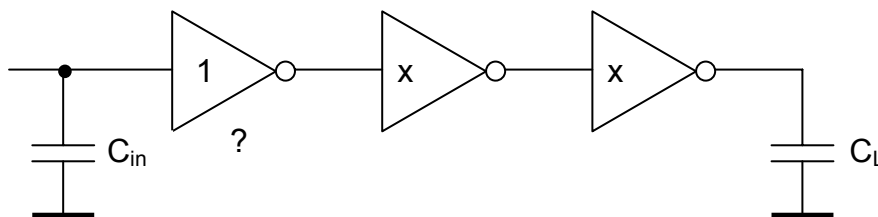
Using the following parameters, define the current through series connected transistors.  $K_p'=25 \text{ mA/V}^2$ ,  $V_{T0}=1.0\text{V}$ ,  $\gamma=0.39\text{V}^{1/2}$ ,  $2|\Phi_F|=0.6\text{V}$ ,  $W/L=1$ . Consider the body bias effect. Several iterations will be needed for the solution.



**1b45.**

The first inverter of the presented buffer is of minimal size, input capacitance is  $C_{in}=10 \text{ fF}$ , delay 70 ps. The load capacitance of the buffer is  $C_L=20 \text{ pf}$ .

- Define the sizes of the other two inverters with respect to the minimal one. Use the condition to get minimum delay, consider that input capacitances are proportional to sizes.
- Add any number of inverters to get minimum delay. Define the total delay.
- Define the power consumption of the circuit if the supply voltage is 2.5V, operating frequency 200 MHz.



**1b46.**

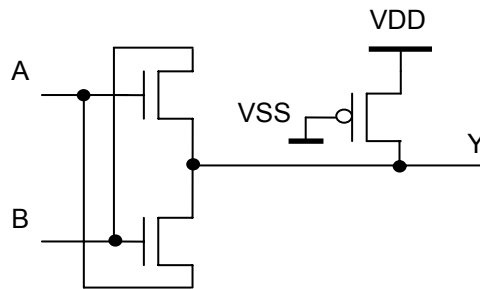
Design a CMOS cell, implementing  $F = AB + AC + BC$  function.

Choose a transistor sizes such that NMOS and PMOS net resistances are equal to the minimal size inverter's resistances. Consider  $2k'_p = k'_n$ . For what input combinations will the best and the worst resistances be obtained?

**1b47.**

A cell with transistor switches is presented in the figure.

- What function does the cell implement? Show the truth table.
- Considering inputs 0 and 2.5V, choose PMOS transistor's sizes such that  $V_{OL} = 0.3V$ . Consider  $2k'_p = k'_n$  and  $|V_{tp}| = V_{tn} = 0.5V$ , for NMOS transistors  $W/L = 1.5\mu m / 0.25\mu m$ .
- Will the cell continue to operate correctly if a PMOS transistor is removed? Does the PMOS transistor implement any helpful function?



**1b48.**

In the presented figure  $C_x = 50$  fF, M1 transistor's  $W/L = 0.375\mu m / 0.375\mu m$ , M2 transistor's  $W/L = 0.375\mu m / 0.25\mu m$ . Note that the inverter does not switch until the input  $V_x$  voltage reaches  $V_{DD}/2$ .  $V_{DD} = 2.5V$

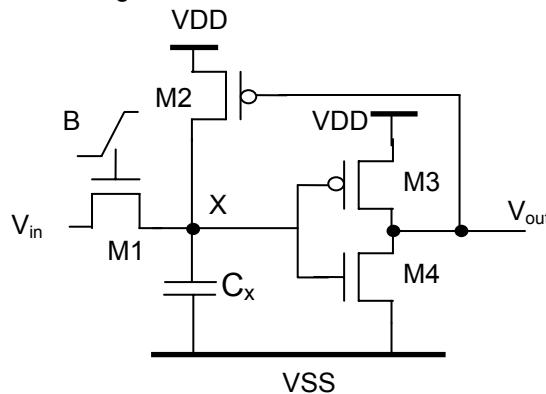
$(W/L)_3 = 1.5\mu m / 0.25\mu m$

$(W/L)_4 = 0.5\mu m / 0.25\mu m$

$k_{n'} = 115\text{mkA/V}^2$ ,  $k_{p'} = -30\text{mkA/V}^2$

$V_{tn} = 0.43V$ ,  $V_{tp} = -0.4V$

- How long will it take for M1 transistor to move x node from 2.5V to 1.25V if  $V_{in} = 0V$ ,  $V_B = 2.5V$ ?
- How long will it take for M2 transistor to move x node from 0V to 1.25V if  $V_{in} = 2.5V$ ,  $V_B = 2.5V$ ?
- What is the minimum value of  $V_B$  voltage for  $V_x = 1.25V$  when  $V_{in} = 0V$ ?

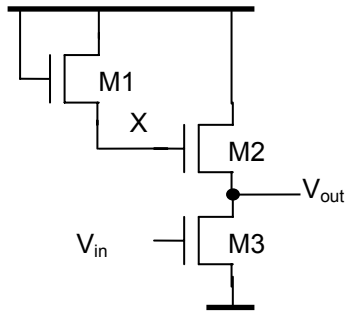


**1b49.**

For the figure shown below in a MOS inverter, consider that all transistor bulks are connected to the ground and the IN input changes from 0V to 2.5V.

- Get the expression to compute x node's voltage considering  $\gamma = 0.5$ ,  $2|\Phi_f| = 0.6V$ ,  $V_{t0} = 0.5V$ ,  $V_{DD} = 2.5V$ .
- Considering  $\gamma = 0$ , define the operating modes of M2.
- Considering  $\gamma = 0$ , define the output value when  $V_{in} = 0V$ .
- Considering  $\gamma = 0$ , get the expression of inverter's switching point voltage ( $V_{SP}$ ) (in the switching point  $V_{IN} = V_{OUT}$ ). Consider M1, M2 and M3 transistor sizes  $(W/L)_1$ ,  $(W/L)_2$  and  $(W/L)_3$  respectively. Define changing limits of  $V_{SP}$  when (1)  $(W/L)_1 \gg (W/L)_2$ , (2)  $(W/L)_2 \gg (W/L)_1$ .

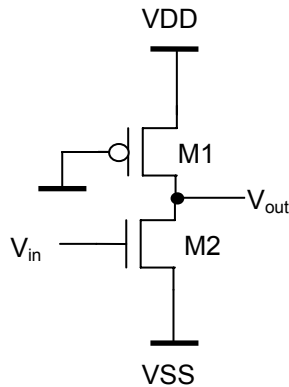




**1b50.**

For the given inverter  $(W/L)_1=5\mu\text{m}/0.25\mu\text{m}$ ,  $(W/L)_2=4\mu\text{m}/0.25\mu\text{m}$ ,  $k_n = 120\mu\text{A}/\text{V}^2$ ,  $k_p = 30\mu\text{A}/\text{V}^2$ ,  $V_{tN} = 0.5\text{V}$ ,  $V_{tP} = -0.4\text{V}$ ,  $V_{DD}=2.5\text{V}$ .

- a)  $V_{OL}$  and  $V_{OH}$
- b)  $V_{SP}$



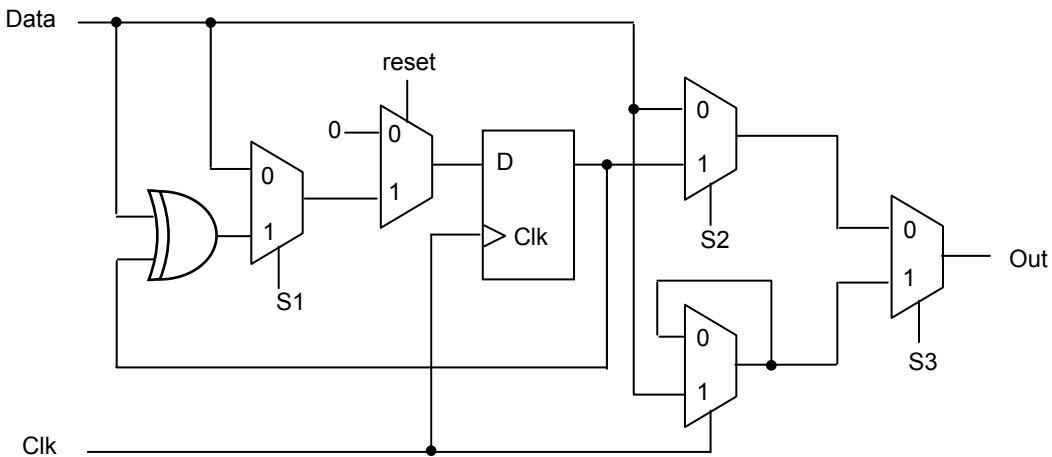
**1b51.**

Design a cascade differential voltage switching logic cell which is described by the equation  $F = ABC\bar{D} + A\bar{B}CD$ . Use minimum number of transistors. Consider that normal and inverted values of inputs are available.

**1b52.**

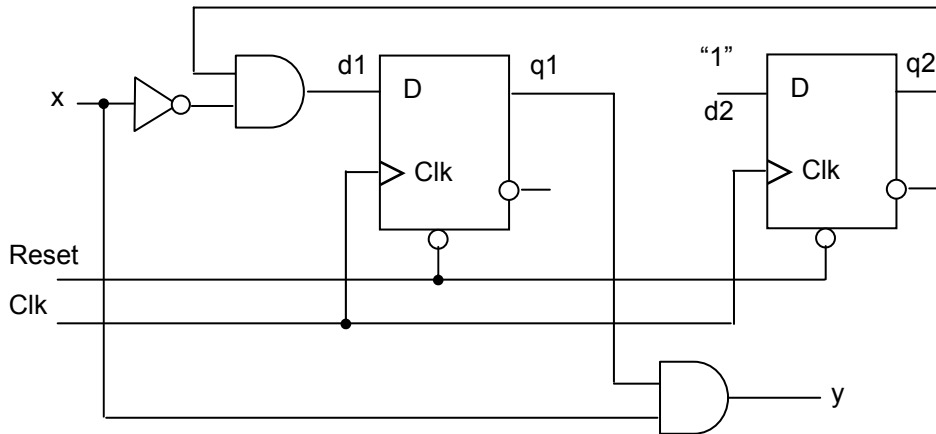
A circuit of a flip-flop with combined logic is shown. Define for which values of S1, S2, S3 signals this circuit will operate as:

- Positive edge-triggered D-flip-flop with synchronous reset
- Positive edge-triggered T- flip-flop with synchronous reset
- D-latch
- Transfer of input data to output



**1b53.**

Define what input binary sequence detector the given circuit is. The beginning of the following sequence can be the end of the previous one.



**1b54.**

Using three- and four-bit Johnson counters (one of them having odd number of states), construct a counter with 42 states. Construct a decoder that will decode every 6<sup>th</sup> state starting from 0.

**1b55.**

It is required to design an arbiter circuit that performs the following function:

allows availability to four devices into the general resource of the given system. At any time only one device can use the resource. Each device creates a request which is given to the arbiter input by clock: The FSM for each device generates an access to permission signal – grant. The requests are processed as per priority. After processing the request from some device, the device removes the request and the FSM comes to its initial state. The system also has an additional input that defines the priorities of requests. If direction=0, the highest priority is r0 request, then r1, r2 and r3. And if direction=1, the priorities have opposite order r3,r2,r1,r0.

1. Devise transition graph of arbiter states
2. Describe the FSM in Verilog.

**1b56.**

Suppose there is a given "data\_in" signal with one hot logic vector of 8-bit of width. And the goal is to calculate the encoded decimal value of the bit index having value of 1'b1 (the following 'case' statements show the required logic).

```

input [7:0] data_in;
output [3:0] data_out;
 casez(data_in) //data_in can have only one 1 at a time
 8'b??????1: data_out = 3'd0;
 8'b?????10: data_out = 3'd1;
 8'b?????100: data_out = 3'd2;
 8'b????1000: data_out = 3'd3;
 8'b????10000: data_out = 3'd4;
 8'b???100000: data_out = 3'd5;
 8'b?1000000: data_out = 3'd6;
 8'b10000000: data_out = 3'd7;
 default: data_out = 3'd0;
 endcase

```

Note, 'one hot logic vector' means that this vector will have only one.

1'b1 value in its bits. E.g.:

- 0001
- 0010
- 0100
- 1000

Provide an optimal gate-level logic equivalent to this case statement (using only logical AND, OR and NOT gates). The answer may be provided in either VerilogHDL or schematic drawing interpretation.

- a) Provide the gate level derivation logic for data\_out[0].
- b) Provide the gate level derivation logic for data\_out[1].
- c) Provide the gate level derivation logic for data\_out[2].

## 2. ANALOG INTEGRATED CIRCUITS

### a) Test questions

- 2a1.** How is the balancing of the differential amplifier executed?  
A. *By applying additional biasing to one of inputs*  
B. *Through external potentiometer connected between load resistors of two branches*  
C. *By changing supply voltage value*  
D. *A. and B. answers are correct*  
E. *All the answers are wrong*
- 2a2.** What differences are between inverting and non-inverting adders based on operational amplifier?  
A. *Output signal's phase*  
B. *There is interaction of signal sources in inverting adder*  
C. *In inverting adder inputs are limited*  
D. *Interaction of signal sources is absent in inverting adder*  
E. *B. and C. answers are correct*
- 2a3.** Input resistance for differential signal of differential amplifier can be increased by:  
A. *The increase of resistors in emitter circuits*  
B. *The increase of transistors'  $\beta$ s*  
C. *The application of Darlington's transistors*  
D. *The application of field transistors*  
E. *All the answers are correct*
- 2a4.** What is the advantage of "R - 2R" matrix towards "R - 2R - 4R - 8R matrix in digital-analog converters?  
A. *More precise*  
B. *Can be multibit*  
C. *The current of reference voltage is constant*  
D. *Is being heated in a more uniform way*  
E. *All the answers are correct*
- 2a5.** Which analog-digital converter (ADC) is the fastest?  
A. *Sequential ADC*  
B. *Parallel ADC*  
C. *Double integration ADC*  
D. *The speed depends on the applied cells*  
E. *The most high-speed is not mentioned among the answers*
- 2a6.** Why does the increase of collector resistor value lead to transistor's saturation mode in a common emitter circuit?  
A. *Because it leads to collector voltage increase*  
B. *Because it leads to base current increase*  
C. *Because it leads to collector current increase and contributes to transistor opening*  
D. *All the answers are correct*  
E. *All the answers are wrong.*
- 2a7.** What is the differential amplifier's application limited by?  
A. *Large input resistance*  
B. *Large output resistance*  
C. *The difference of input resistances for common mode and differential signals*  
D. *Large amplifier coefficient*  
E. *All the answers are correct*
- 2a8.** What is the reason of occurrence of disbalance of differential amplifier?  
A. *Transistors of two branches are not similar*  
B. *Difference of resistance values between two branches*  
C. *The summary difference between both transistors and resistors of two branches*  
D. *Non ideal nature of the power source*  
E. *All the answers are correct*
- 2a9.** What properties are demonstrated by active integrator from the perspective of frequency?  
A. *High pass filter*  
B. *Low pass filter*  
C. *Band-pass filter*  
D. *Rejecter filter*  
E. *Has no filtering properties*

- 2a10.** The random variable signal conversion accuracy of ADC depends on:  
 A. *Comparator accuracy*  
 B. *Resistors accuracy in R-2R matrix of internal DAC*  
 C. *Bit count of ADC*  
 D. *Performance of ADC elements*  
 E. *All the answers are correct*
- 2a11.** The quality of current source in differential amplifier depends on  
 A. *High internal resistance*  
 B. *Thermostability*  
 C. *Current value*  
 D. *A, B, C answers are correct*  
 E. *A, B answers are correct*
- 2a12.** Analog IC production group method is based on the following factors:  
 A. *Parameter similarity of elements produced during the same technological process*  
 B. *Elements on crystal, which are placed close to each other, heat evenly*  
 C. *Elements thermal coefficient similarity*  
 D. *A, B, C answers are correct*  
 E. *Technological restrictions on element implementation*
- 2a13.** What is the minimum value of the resistance of OpAmp negative feedback limited by?  
 A.  *$K_u=1$  request*  
 B. *Thermal instability of input current*  
 C. *Permissible minimum value of resistance of OpAmp output load*  
 D. *No limitation*  
 E. *All the answers are wrong*
- 2a14.** In voltage stabilizer by OpAmp application why is the feedback given to the inverse input not from OpAmp output, but from the output of output emitter repeater  
 A. *To fade the loading of OpAmp*  
 B. *To fade stabilitrion current given to the direct input*  
 C. *To compensate thermal instability of the output emitter repeater*  
 D. *To increase amplifier's coefficient of OpAmp*  
 E. *All the answers are correct*
- 2a15.** What factors is the redundancy principle of analog microcircuitry based on?  
 A. *Technological restrictions of element preparation*  
 B. *On those elements of the circuit which are not possible to carry out in crystal or occupy too much area, are substituted by multielement node which implements the same function.*  
 C. *A. and B. answers are correct*  
 D. *On placing redundant elements on crystal area*  
 E. *Minimization of the number of circuit elements*
- 2a16.** Why does not the emitter oscillate the voltage?  
 A. *Because the output voltage must always be smaller than the one of the input for open state of the transistor*  
 B. *Because it is not possible to increase large nominal resistance in emitter circuit*  
 C. *Because the output signal is taken from transistor's emitter*  
 D. *Because the emitter repeater cannot provide large output resistance*  
 E. *All the answers are correct*
- 2a17.** What is the role of additional emitter repeater in current mirror?  
 A. *Has no influence*  
 B. *Increases the output current of current mirror*  
 C. *Balances current mirror*  
 D. *Increases the output resistance of current mirror*  
 E. *All the answers are wrong*
- 2a18.** What is the comparator's sensibility, built on OpAmp, conditioned by?  
 A. *Input resistances of OpAmp*  
 B. *Value of supply voltage*  
 C. *Own amplifier's coefficient of OpAmp*  
 D. *Debalance of OpAmp*  
 E. *All the answers are wrong*

- 2a19.** What is the advantage of double integration of ADC?  
 A. *Increases the conversion accuracy*  
 B. *Thermal stability*  
 C. *Compensates the thermal instability of integrator's capacitor*  
 D. *A., B. answers are correct*  
 E. *A., B., C. answers are correct*
- 2a20.** Between what points is the input resistance for differential signal of differential amplifier distributed?  
 A. *Between inputs of differential amplifier*  
 B. *Between inputs of differential amplifier and ground*  
 C. *Between one input of differential amplifier and ground*  
 D. *Between one input of differential amplifier and negative power source*  
 E. *All the answers are wrong*
- 2a21.** In what state are bipolar transistor junctions in saturation mode?  
 A. *Emitter junction is close, collector - open*  
 B. *Emitter junction is open, collector - close*  
 C. *Both junctions are open*  
 D. *Both junctions are close*  
 E. *Saturation mode has no connection with the states of junctions*
- 2a22.** Why is not resistance applied as a stable current source in a differential amplifier?  
 A. *Large resistances, characteristic of current source, are not possible to realize in semiconductor ICs*  
 B. *Large voltage drop is obtained on the resistance of large nominal which leads to the increase of power supply voltage value*  
 C. *The resistance cannot be current source at all*  
 D. *A., B. answers are wrong*  
 E. *All the answers are wrong*
- 2a23.** What is the function of the output cascade of OpAmp?  
 A. *Current amplifier*  
 B. *Power amplifier*  
 C. *Provides small output resistance*  
 D. *B., C. answers are correct*  
 E. *All the answers are wrong*
- 2a24.** Why are the minimum values of output signals of differential amplifier for small signal application limited by approximately -0.7V level with bipolar transistors?  
 A. *Due to one of transistors falling into saturation mode*  
 B. *Due to one of transistors collector opening*  
 C. *Due to closing of one of transistors*  
 D. *A., B. answers are correct*  
 E. *Due to value limitation of collector resistance*
- 2a25.** What should the structure of a MOS transistor look like to reduce body effect?  
 A. *Square of the bulk diffusion must be large*  
 B. *Bulk diffusion must be rounded over transistor*  
 C. *Bulk diffusion must be as close to the transistor as possible*  
 D. *A and C answers are correct*  
 E. *B and C answers are correct*
- 2a26.** What signals are the switching capacitances controlled by?  
 A. *Overlap clock signals*  
 B. *Clock signals*  
 C. *Multi-level clock signals*  
 D. *Non overlap clock signals*  
 E. *Non clock signals*
- 2a27.** Which ADC is basically the fastest?  
 A. *Sigma-delta*  
 B. *Dual slope integrating AC*  
 C. *SAR*  
 D. *Integrating*  
 E. *Pipeline*

- 2a28.** How can the channel modulation effect of a MOS transistor be reduced?  
 A. *By decreasing bulk potential*  
 B. *By decreasing transistor's L*  
 C. *By decreasing transistor's W*  
 D. *By increasing transistor's W*  
 E. *By increasing transistor's L*
- 2a29.** How much is the NMOS source follower output voltage when the input transistor is in saturation mode?  
 A. *Equal to supply voltage*  
 B. *Equal to zero*  
 C. *Larger than input voltage by threshold voltage of an input transistor*  
 D. *Smaller than input voltage by threshold voltage of an input transistor*  
 E. *Equal to input voltage*
- 2a30.** What is the advantage of using a field transistor in the input of DiffAmp?  
 A. *Input capacitance decreases*  
 B. *Input resistance increases*  
 C. *Input offset error decreases*  
 D. *Has no advantage*  
 E. *A, B and C answers are correct*
- 2a31.** Which of the listed ADC contains DAC?  
 A. *SAR*  
 B. *Flash*  
 C. *Integrating*  
 D. *Dual slope integrating*  
 E. *All the answers are correct*
- 2a32.** By what element is channel modulation of a MOS transistor presented in a small signal model?  
 A. *Controlled voltage source*  
 B. *Resistance*  
 C. *Controlled current source*  
 D. *Capacitor*  
 E. *RC circuit*
- 2a33.** The effect of what errors reduces ADC's digital correction application in a pipeline?  
 A. *Errors depending on gain of OpAmp*  
 B. *Errors depending on capacitor values scaling*  
 C. *Comparators offset error*  
 D. *Comparators sensitivity*  
 E. *C and D answers are correct*
- 2a34.** Which of the mentioned DAC is not used in ICs?  
 A. *R-string DAC*  
 B. *R-2R DAC*  
 C. *Charge scaling DAC*  
 D. *Current DAC*  
 E. *All are used*
- 2a35.** What is the IC lifetime conditioned by?  
 A. *Supply voltage value*  
 B. *Migration*  
 C. *Leakage power*  
 D. *Maximum clock frequency of IC*  
 E. *All the answers are correct*
- 2a36.** By what element is the body effect of a MOS transistor presented in a small signal model?  
 A. *Controlled voltage source*  
 B. *Resistance*  
 C. *Controlled current source*  
 D. *Capacitor*  
 E. *RC circuit*

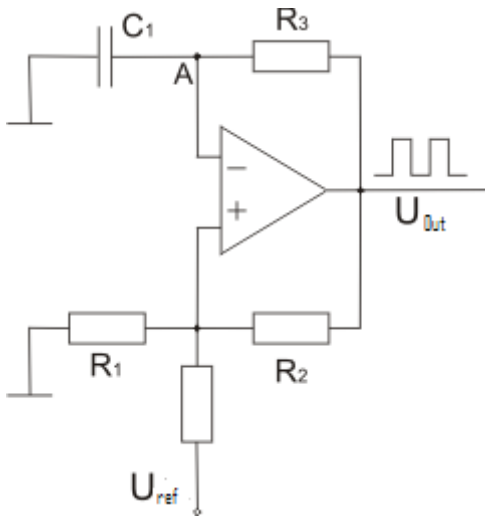
- 2a37.** What is the minimum value of OpAmp's negative feedback limited by?  
 A. *Input resistance of OpAmp*  
 B. *Amplifying coefficient of OpAmp*  
 C. *Minimum value of OpAmp's output load resistance*  
 D. *A=1 value*  
 E. *Minimum value of OpAmp's input current*
- 2a38.** What is the reason of differential amplifier's debalance?  
 A. *Non similarity of diffpair (transistor)*  
 B. *Accuracy of current source*  
 C. *Loads non matching*  
 D. *A and B answers are correct*  
 E. *A and C answers are correct*
- 2a39.** How many comparators does an 8 bit two-stage Flash ADC contain?  
 A. *15*  
 B. *30*  
 C. *255*  
 D. *31*  
 E. *63*
- 2a40.** How many comparators does a single-stage 8-bit Flash analog to digital converter have?  
 A. *31*  
 B. *7*  
 C. *255*  
 D. *63*  
 E. *127*
- 2a41.** What is the main disadvantage of a SAR analog to digital converter?  
 A. *Limited accuracy*  
 B. *low performance*  
 C. *A and B answers are correct*  
 D. *A, B and E answers are correct*  
 E. *Thermal instability*
- 2a42.** What is the high performance of an ADC conditioned by?  
 A. *High performance of input switches*  
 B. *Output change rate of output operational amplifier*  
 C. *Converter's capacity*  
 D. *A, B and C answers are correct*  
 E. *A and B answers are correct*
- 2a43.** Why are differential amplifiers applied only in case of high ohmic loads?  
 A. *Because they have two outputs*  
 B. *Because the reduction of load resistance reduces the amplification coefficient of differential signal*  
 C. *Because the reduction of load resistance leads to amplification of common-mode signal*  
 D. *B, C and D answers are correct*  
 E. *B and C answers are correct*
- 2a44.** Why are not differential amplifiers used as a standalone amplifiers?  
 A. *Because of too high output resistance*  
 B. *Because they cannot work with low ohmic loads*  
 C. *Because amplifying of differential signal changes with load resistance*  
 D. *A, B and C answers are correct*  
 E. *A and B answers are correct*
- 2a45.** What is the body effect in a small signal model of a MOS transistor presented by?  
 A. *Current controlled current source*  
 B. *Voltage controlled voltage source*  
 C. *Voltage controlled current source*  
 D. *Resistance*  
 E. *Current controlled voltage source*

**2a46.** By what is channel length modulation in a small signal model of a MOS transistor presented?  
 A. Current controlled current source  
 B. Voltage source  
 C. Capacitor  
 D. Resistor  
 E. Current controlled voltage source

**2a47.** What is the body effect of a MOS transistor conditioned by?  
 A. Potentials' difference of source and drain  
 B. Potentials' difference of source and gate  
 C. Potentials' difference of drain and substrate  
 D. Potentials' difference of gate and substrate  
 E. Potentials' difference of source and substrate

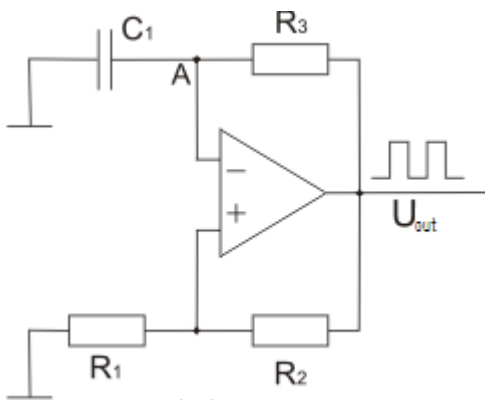
**2a48.** What is the channel length modulation of a MOS transistor conditioned by?  
 A. Potentials' difference of source and drain  
 B. Potentials' difference of source and gate  
 C. Potentials' difference of drain and substrate  
 D. Potentials' difference of gate and substrate  
 E. Potentials' difference of source and substrate

**2a49.** How will the change of R1 affect this generator?



- A. Output amplitude will change
- B. Output frequency will change
- C. Amplitude at point A will change
- D. Frequency at point A will change
- E. All the answers are wrong

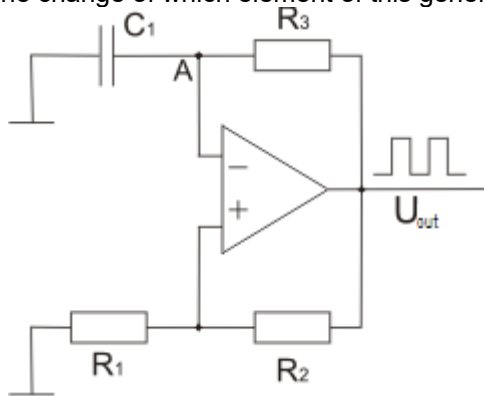
**2a50.** How will R1 change affect this generator?



- A. The amplitude of output pulses will change
- B. The frequency of output pulses will change
- C. The amplitude of A point pulses will change
- D. The frequency of A point pulses will change
- E. B, C and D answers are correct



2a51. The change of which element of this generator will lead to the change of frequency of output signal?



- A.  $R_2$
- B.  $R_3$
- C.  $C_1$
- D. B and C answers are correct
- E. A, B and C answers are correct

2a52. Between which pins of a MOS transistor there is no direct capacitance?

- A. Source and gate
- B. Drain and gate
- C. Gate and substrate
- D. Source and substrate
- E. Source and drain

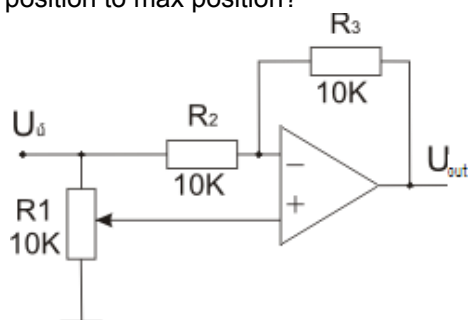
2a53. How can the conductance of a MOS transistor change in case of the given technology and gate source voltage (ignore secondary effects)?

- A. Change in gate drain voltage of a transistor
- B. Change in drain source voltage of a transistor
- C. Change in the sizes of channel
- D. Not possible to change
- E. Change in the sizes of source and drain

2a54. What does the FET transistor in saturation mode represent?

- A. Current source
- B. Voltage source
- C. Linear resistance
- D. Resistance
- E. Infinite small resistance

2a55. Within what limitations will  $U_{\text{output}}$  voltage change when moving control of  $R_1$  potentiometer from min position to max position?



- A. From  $U_m$  to  $2U_m$
- B. From  $-U_m$  to  $-2U_m$
- C. From  $-2U_m$  to  $U_m$
- D. From  $-U_m$  to  $U_m$
- E. From  $-U_m$  to  $2U_m$

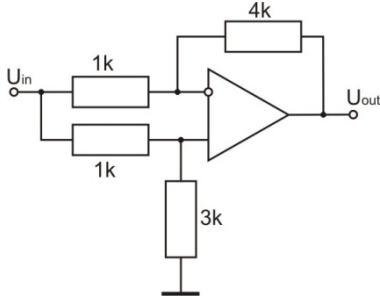
2a56. What is the accuracy of a digital-analog converter with R-2R matrix conditioned by?

- A. Accuracy of making matrix resistors
- B. Value of voltage offset error of output OpAmp
- C. Gain of output OpAmp
- D. A and B answers are correct
- E. A, B and C answers are correct

2a57. How can the amplification coefficient of a single stage common source resistive load amplifier increase?

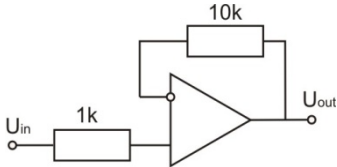
- A. By increasing resistance value
- B. By increasing channel width
- C. By increasing channel length
- D. A and B answers are correct
- E. A and C answers are correct

2a58. What is the value of amplification coefficient of this amplifier?



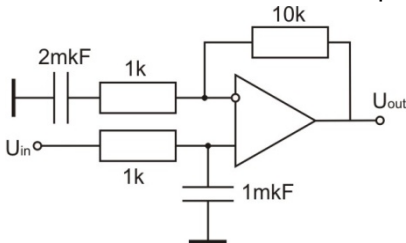
- A.  $K_U=10$
- B.  $K_U=1$
- C.  $K_U=-1$
- D.  $K_U=11$
- E.  $K_U=2$

2a59. What is the value of amplification coefficient of this amplifier?



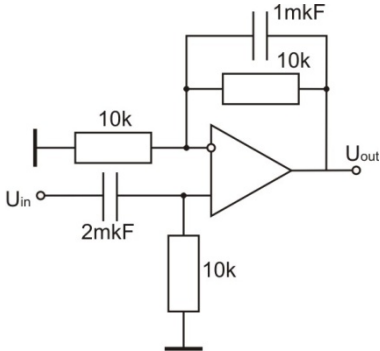
- A.  $K_U=0$
- B.  $K_U=1$
- C.  $K_U=11$
- D.  $K_U=10$
- E.  $K_U=2$

2a60. What function does this circuit implement?



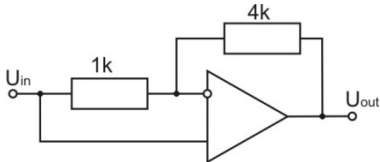
- A. *Low pass filter*
- B. *Band-pass filter*
- C. *Band-stop filter*
- D. *Only an amplifier*
- E. *High pass filter*

2a61. What function does this circuit implement?



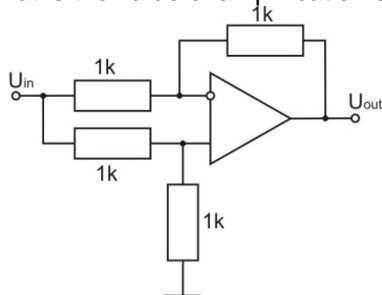
- A. *Band-pass filter*
- B. *Low pass filter*
- C. *Band-stop filter*
- D. *Only an amplifier*
- E. *High pass filter*

2a62. What is the value of amplification coefficient of this amplifier?



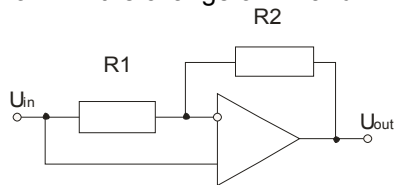
- A.  $K_U=-4$
- B.  $K_U=5$
- C.  $K_U=-1$
- D.  $K_U=1$
- E.  $K_U=0$

2a63. What is the value of amplification coefficient of this amplifier?



- A.  $K_U=0$
- B.  $K_U=1$
- C.  $K_U=-1$
- D.  $K_U=-2$
- E.  $K_U=2$

2a64. How will the change of R1 and R2 affect an output voltage value?



- A.  $R_1$  decrease will lead to  $U_{out}$  decrease
- B.  $R_1$  decrease will lead to  $U_{out}$  increase
- C.  $R_2$  increase will lead to  $U_{out}$  increase
- D. The change of  $R_1$  and  $R_2$  will not affect on  $U_{out}$
- E. A. B. answers are correct

2a65. What is the reason of differential amplifier's offset?

- A. Non similarity of diffpair (transistor)
- B. Accuracy of current source
- C. Low gain
- D. High gain
- E. Value of phase shift

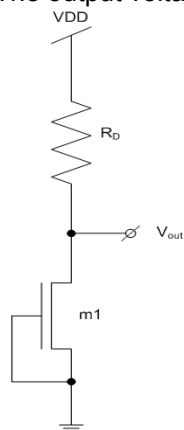
2a66. Which of the mentioned single stage amplifier has the highest amplification coefficient?

- A. Common source with the resistive load
- B. Common source with the current source load
- C. Source follower
- D. Cascode stage
- E. Common gate

2a67. Which OpAmp parameter is measured with AC analysis?

- A. Slew rate
- B. Settling time
- C. Gain margin
- D. Power consumption
- E. The correct answer is missing

2a68. The output voltage of the presented circuit equals:



- A. 0
- B.  $v_{dd}/2$
- C.  $v_{in}$
- D.  $v_{dd}$
- E.  $v_{in} - v_{th}$

2a69. One of the advantages of differential operation vs single-stage operation is:

- A. High immunity to noise
- B. Low immunity to noise
- C. Possible reduction of power consumption
- D. Possible reduction of area
- E. Low voltage operation range

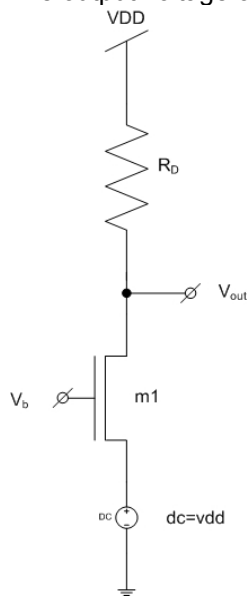
2a70. Which of the mentioned OpAmps is the fastest one (has the largest gain bandwidth)?

- A. Folded cascode
- B. Telescopic
- C. Two-stage
- D. Gain-boosted
- E. All

2a71. Which OpAmp parameter is not measured with AC analysis?

- A. Slew rate
- B. Phase margin
- C. DC gain
- D. Unity gain bandwidth
- E. Power supply rejection ratio

2a72. The output voltage of the presented circuit equals:



- A.  $v_{in}$
- B.  $v_{dd}$
- C. 0
- D.  $v_{dd}/2$
- E.  $V_{th}$

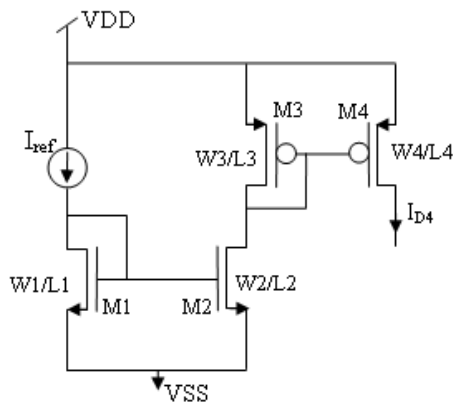
2a73. Cascode stage is the cascade connection of

- A. Source follower and common gate
- B. Source follower and common source
- C. Common source and common gate
- D. Cascade stage is not a cascade connection
- E. The correct answer is missing

**b) Problems**

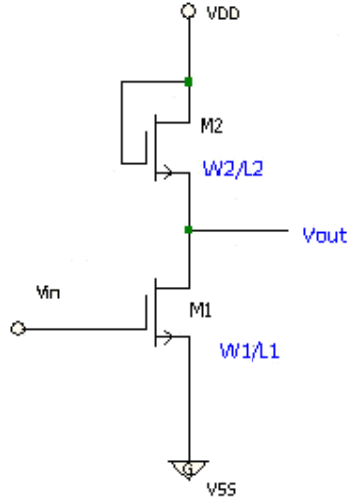
2b1.

For the following circuit find  $I_{D4}=f(I_{ref})$  dependence if  $\lambda=0$ ,  $L_1=L_2=L_3=L_4$ ,  $W_1=W_2$ ,  $W_3=W_4$ . Note that all the transistors are in saturation mode.



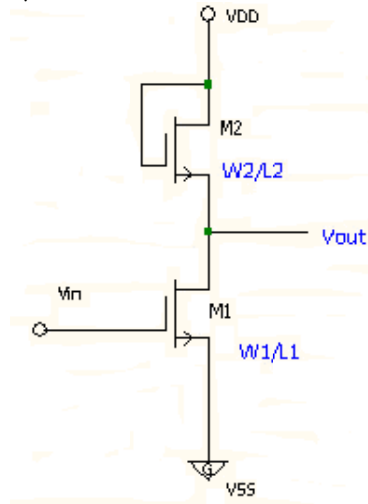
**2b2.**

For the following circuit define small signal gain constant (assume M1 is saturated and  $\lambda=0, \gamma=0$ ).



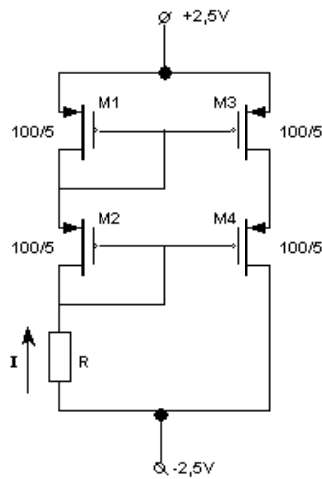
**2b3.**

For the following circuit define the value of input voltage in case of which M1 is out of saturation mode if  $(W/L)_1=49 (W/L)_2=9 V_{TH}=0.7 V_{DD}=3, \lambda=\gamma=0$ .



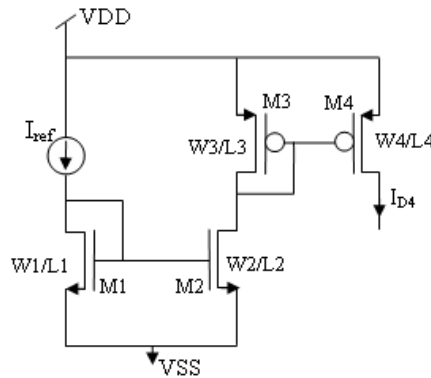
**2b4.**

A current mirror is given by transistors' geometrical sizes. Find the gate voltage of M1 transistor.



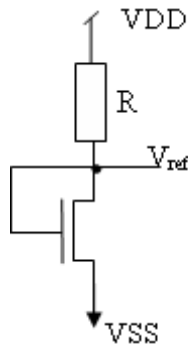
**2b5.**

For the following circuit find the drain current of M4 transistor if all the transistors are in saturation. Ignore channel length modulation ( $\lambda=0$ ).



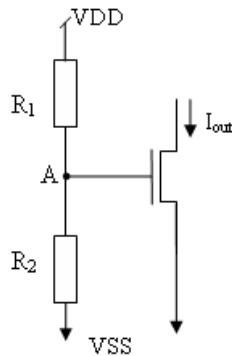
**2b6.**

For the following circuit find  $V_{ref}$ , if  $\mu_n=550 \text{ cm}^2/\text{Vs}$ ,  $\epsilon_{SiO_2}=3.9$ ,  $\epsilon_0=8.85 \cdot 10^{-14} \text{ F/cm}$ ,  $t_{ox}=0.16 \text{ nm}$ ,  $V_{thn}=0.8\text{V}$ ,  $V_{DD}=2\text{V}$ ,  $V_{SS}=0\text{V}$ ,  $W=L=10 \text{ }\mu\text{m}$ ,  $\lambda=0$ ,  $R=10 \text{ k}\Omega$ .



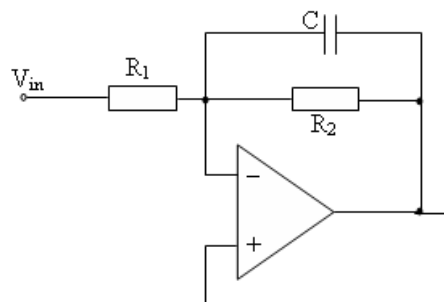
**2b7.**

For the following circuit define how much  $I_{out}$  will change if  $V_{DD}$  changes by  $\pm 10\%$  and if the transistor is in saturation and  $W=50\mu\text{m}$ ,  $L=0.5\mu\text{m}$ ,  $I_{out}=0.5\text{mA}$ ,  $K_n=120 \text{ }\mu\text{A/V}^2$ ,  $V_{TH}=0.5\text{V}$ ,  $V_{DD}=3\text{V}$ , (nominal value),  $R_2/R_1=0.35$ ,  $\lambda=0$ .



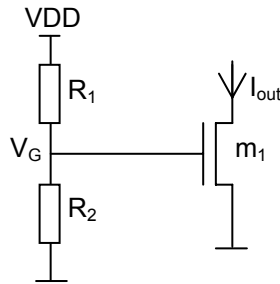
**2b8.**

Find the cutoff frequency of the following circuit and build amplitude-frequency characteristics, if  $R_1$ ,  $R_2$  and  $C$  are known.



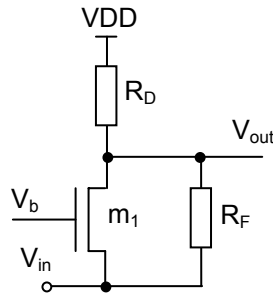
**2b9.**

What does  $I_{out}$  equal to when the transistor is in saturation (express transistor's conductivity by  $g_m$ ). Ignore body effect and channel modulation.



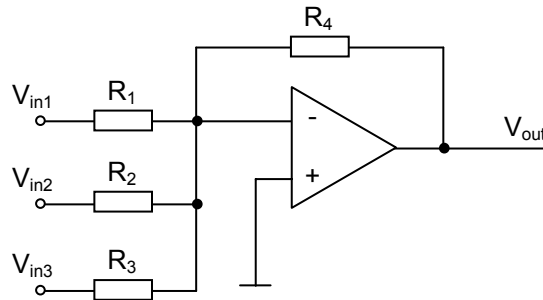
**2b10.**

Find  $V_{out}$  depending on  $V_{in}$ , if the transistor is in saturation. Ignore body effect and channel modulation.



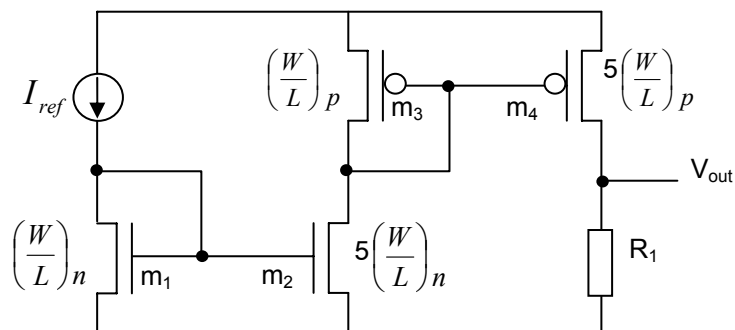
**2b11.**

$R_1, R_2, R_3, R_4$  resistances are given. Find  $V_{out} = f(V_{in1}, V_{in2}, V_{in3})$ , considering the real  $K_A$  amplifying coefficient.



**2b12.**

$\left(\frac{W}{L}\right)_n, \left(\frac{W}{L}\right)_p, I_{ref}, R_1$  values are given. Find  $V_{out}$ , ignore channel modulation.

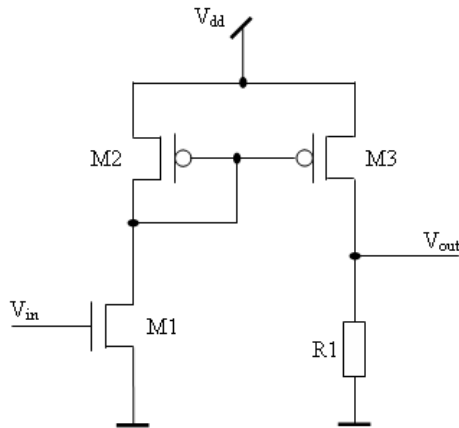






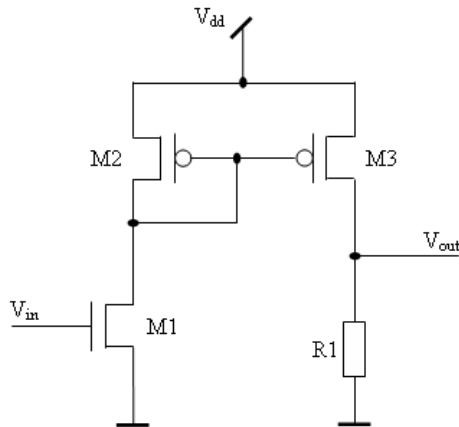
**2b16.**

Find  $k = \frac{dV_{out}}{dV_{in}}$  coefficient (by variable component). Given  $g_{m1}, g_{m2}, g_{m3}, R$ .



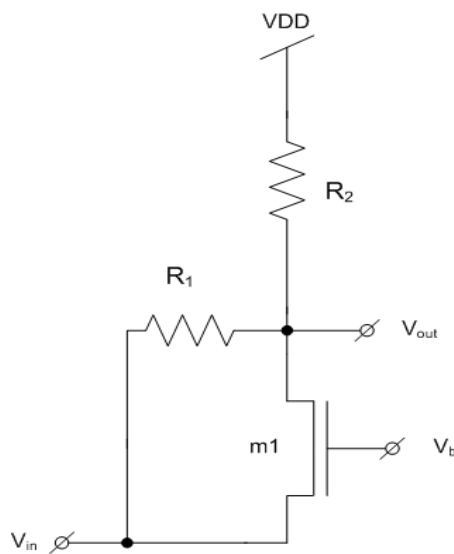
**2b17.**

Find  $A_v = dV_{out}/dV_{in}$  small signal gain:  $g_{m1}, g_{m2}, g_{m3}, R_1$  values are known. Ignore secondary effects.



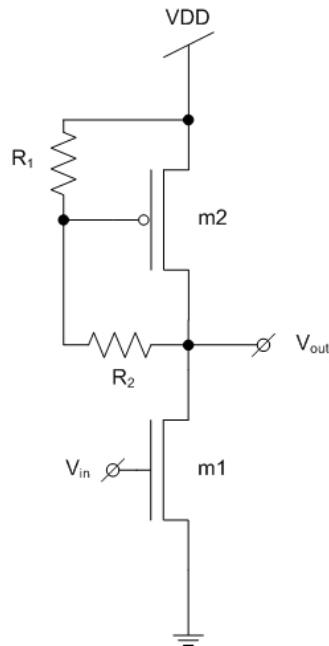
**2b18.**

Find  $A_v = dV_{out}/dV_{in}$  small signal gain:  $g_{m1}, g_{mb1}, R_1, R_2$  values are known. Ignore channel length modulation.

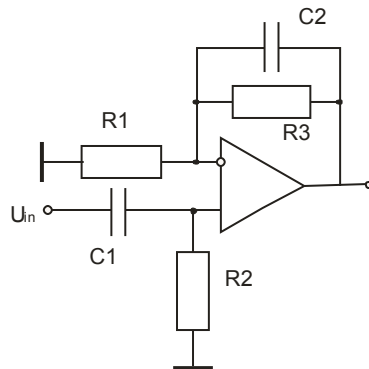


**2b19.**

Sketch  $V_{out}$  versus  $V_{in}$  as  $V_{in}$  varies from 0 to  $V_{DD}$ . Identify the important transition points.

**2b20.**

Calculate the maximum gain and central frequency of the filter based on OpAmp if  $R1=1\text{k}\Omega$ ,  $R2=2\text{k}\Omega$ ,  $R3=10\text{k}\Omega$ ,  $C1=10\text{m}\mu\text{F}$ ,  $C2=1\text{m}\mu\text{F}$ .

**2b21.**

The sequence  $x(n) = \cos(\pi n / 4)$  is a result of  $u(t) = \cos(2\pi f_0 t)$  analog signal sampling with sampling frequency  $f_s=1000\text{ Hz}$ :  $x(n) = u(n\Delta t)$  where  $\Delta t=1/f_s$ . Find two minimum  $f_0$  values for which it takes place.

**2b22.**

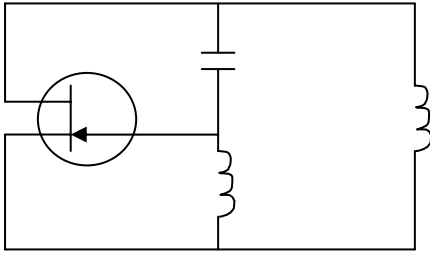
The sequence  $x(n) = \cos(\pi n / 4)$  is a result of  $u(t) = \cos(2\pi f_0 t)$  analog signal sampling with sampling frequency  $f_s=800\text{ Hz}$ :  $x(n) = u(n\Delta t)$ , where  $\Delta t=1/f_s$ . Find two minimum  $f_0 > 200\text{ Hz}$  values for which it takes place.

### 3. RF CIRCUITS

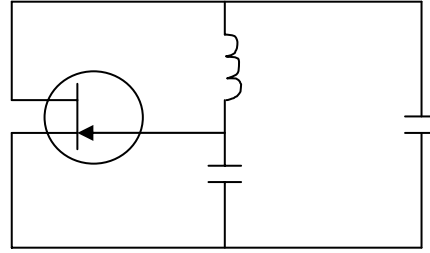
#### a) Test questions

- 3a1.** Define super-heterodyne receiver's intermediate frequency if the input signal frequency is  $f_s = 900$  MHz, and heterodyne frequency is  $f_h = 700$  MHz.
- A. 200 MHz
  - B. 600 MHz
  - C. 1400 MHz
  - D. 1600 MHz
  - E. 1800 MHz

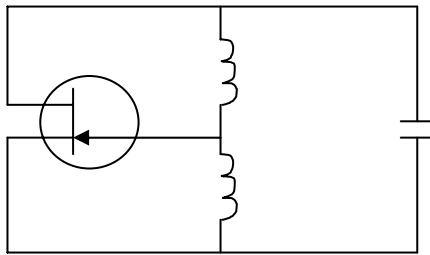
- 3a2.** Which of the shown circuits is called "Inductive triple point"?



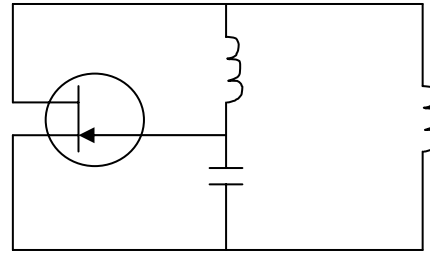
A.



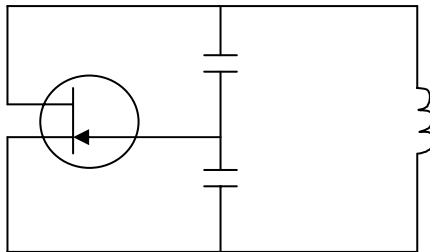
B.



C.



D.



E.

- 3a3.** Find the initial phase of second harmonic of the following signal:

$$s(t) = \sum_{n=1}^2 \frac{2}{n} \cos \left[ 2\pi n \cdot 10^6 t + \frac{\pi}{n} (-1)^{n+1} \right]$$

- A.  $-\pi$
- B.  $-\frac{\pi}{2}$
- C. 0
- D.  $\frac{\pi}{2}$
- E.  $\pi$

## 4. SEMICONDUCTOR PHYSICS AND ELECTRONIC DEVICES

### a) Test questions

- 4a1. Which circuit based on operational amplifier has transfer function with hysteresis?  
A. *Non-inverting amplifier*  
B. *Inverting adder*  
C. *Comparator without feedback*  
D. *Comparator with positive feedback*  
E. *Comparator with negative feedback*
- 4a2. Does semiconductor diode's I/V characteristics differ from I/V characteristics of ohmic resistance?  
A. *Yes, it depends on the applied voltage direction and is nonlinear*  
B. *Yes, it depends on the applied voltage direction and is linear*  
C. *No, as the more direct voltage, the more is the current*  
D. *Partially, as current exists irrespective of voltage direction*  
E. *The correct answer is missing*
- 4a3. What parameters of semiconductor material are needed for transistor fabrication?  
A. *Charge carriers' mobility and concentration*  
B. *Charge carriers' concentration, minority charge carriers' life time, mobility*  
C. *Charge carriers' concentration and diffusion coefficient*  
D. *Charge carriers' concentration, diffusion coefficient, mobility, band gap*  
E. *The correct answer is missing*
- 4a4. Which regions does the graph of drain current dependence on source-drain voltage for p-n junction field effect transistor consist of?  
A. *Linear dependence, saturation*  
B. *Linear dependence, saturation, breakdown*  
C. *Linear dependence, transition, saturation, breakdown*  
D. *Linear dependence, transition, breakdown*  
E. *The correct answer is missing.*
- 4a5. How are the oxide layer capacitor  $C_0$ , the surface state capacitor  $C_{ss}$  and the differential capacitor of the surface charge layer connected between one another in a MOS structure?  
A.  *$C_{ss}$  and  $C_0$  parallel, and with  $C_{sc}$  sequentially*  
B.  *$C_{ss}$  and  $C_{sc}$  parallel, and with  $C_0$  sequentially*  
C.  *$C_0$  and  $C_{sc}$  sequentially, and with  $C_{sc}$  parallel*  
D. *All capacitors are connected parallel to one another*  
E. *The correct answer is missing.*
- 4a6. How does differential resistance of p-n junction change parallel to direct current increase?  
A. *Does not change*  
B. *Decreases*  
C. *Increases*  
D. *Increases, then decreases*  
E. *The correct answer is missing*
- 4a7. How many times will diffusion capacitance of a bipolar transistor increase if its base length increases twice?  
A. *Will not change*  
B. *Will increase  $\sqrt{2}$  times*  
C. *Will increase 4 times*  
D. *Will decrease twice*  
E. *The correct answer is missing*
- 4a8. How does the p-n-p bipolar transistor's transfer factor depend on diffusion length of holes  $L_p$  ?  
A. *No dependence*  
B. *Increase with the increase of  $L_p$  by  $\beta = 1 - \frac{W}{2L_p}$  law*  
C. *Increase with the increase of  $L_p$  by  $\beta = 1 - \frac{1}{2} \left( \frac{W}{L_p} \right)^2$  law*

- D. *Decreases*
- E. *The correct answer is missing*

**4a9.** The light of what maximum wavelength can affect the current of silicon ( $E_g = 1.1$  eV) photodiode?

- A.  $\lambda_{max} = 1130nm$
- B.  $\lambda_{max} = 550nm$
- C.  $\lambda_{max} = 1240nm$
- D.  $\lambda_{max} = 335 nm$
- E. *The correct answer is missing*

**4a10.** How is bipolar transistor's current gain expressed in common base circuit with the help of emitter effectiveness  $\gamma$ , transition coefficient  $\beta$  and collector's avalanche multiplication factor  $M$ ?

- A.  $\alpha = \frac{\gamma\beta}{M}$
- B.  $\alpha = \gamma\beta M$
- C.  $\alpha = \frac{M}{\gamma} \beta$
- D.  $\alpha = \frac{\beta}{\gamma M}$
- E. *The correct answer is missing*

**4a11.** Is operating temperature range of ICs, computers and other semiconductor devices conditioned by the used semiconductor material's band gap?

- A. *Yes, the more the band gap, the more temperature range*
- B. *No, as concentration of minority charge carriers is independent from temperature*
- C. *Conditioned partially as by increasing the temperature, carriers' mobility decreases*
- D. *No, as band gap does not depend on temperature*
- E. *The correct answer is missing*

**4a12.** Which expression is wrong?

- A. *Diode subtypes are: point-junction diodes, stabilitrans, varicaps and tunnel diodes*
- B. *In tunnel diodes reverse current for the same voltage is higher than direct current value*
- C. *In varicaps with increase of voltage barrier capacitance increases*
- D. *Schottky diodes operation is based on processes which take place in semiconductor-metal contact*
- E. *The response time of Schottky diodes, therefore, frequency properties are conditioned by barrier capacitance*

**4a13.** Field effect transistors, compared with bipolar transistors

- A. *Have small input resistance*
- B. *Have small noise coefficient*
- C. *The current is at the same time conditioned by electrons and holes*
- D. *Provide current amplification*
- E. *The performance is mainly conditioned by injection of minority carriers*

**4a14.** Through which device is the electrical signal amplification implemented?

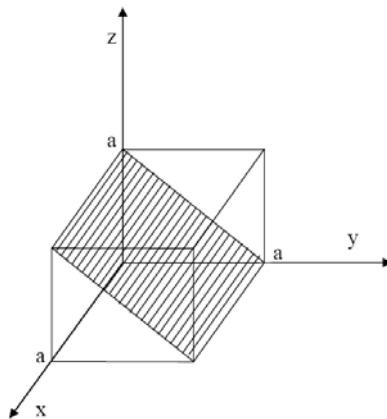
- A. *Resistor*
- B. *Capacitor and inductor*
- C. *Diode*
- D. *Transistor*
- E. *Photodiode*

- 4a15.** How many pins does the field effect transistor have?  
 A. 1 gate  
 B. 2 sources and 1 drain  
 C. 3 sources, 1 gates and 1 drain  
 D. 2 bases and 1 collector  
 E. 1 source, 1 gate and 1 drain
- 4a16.** Is the gate of a field effect transistor isolated from its channel?  
 A. Yes  
 B. No  
 C. Partially and there is weak tunnel coupling  
 D. In saturation mode of field effect transistor most part of channel current flows through gate  
 E. The correct answer is missing
- 4a17.** n – type Ge sample, which is anticipated for making a transistor, has 1.5 Ohm·cm specific resistance and  $5.4 \cdot 10^3 \text{ cm}^3/\text{KI}$  Holy coefficient. What does the charge carriers' concentration and their mobility equal?  
 A.  $1.6 \cdot 10^{21} \text{ m}^{-3}$ ,  $5 \text{ m}^2/\text{V}\cdot\text{s}$   
 B.  $1.6 \cdot 10^{21} \text{ m}^{-3}$ ,  $0.1 \text{ m}^2/\text{V}\cdot\text{s}$   
 C.  $1.16 \cdot 10^{21} \text{ m}^{-3}$ ,  $0.36 \text{ m}^2/\text{V}\cdot\text{s}$   
 D.  $2 \cdot 10^{20} \text{ m}^{-3}$ ,  $0.36 \text{ m}^2/\text{V}\cdot\text{s}$   
 E. The correct answer is missing.
- 4a18.** By means of what semiconductor device can light influence be detected?  
 A. Posistor  
 B. Resistor  
 C. Photodiode  
 D. Capacitor  
 E. Inductor
- 4a19.** Which materials' conductivity is higher?  
 A. Dielectrics  
 B. Semiconductors  
 C. Metals  
 D. All have low conductivity  
 E. All have high conductivity
- 4a20.** How does the negative differential resistance current range change depending on the density of lightly degenerated n-region impurities in tunnel diode?  
 A. Interval decreases when increasing density  
 B. Interval increases when increasing density  
 C. It is not conditioned by density of impurity  
 D. Interval increases when reducing density  
 E. All the conditions are true
- 4a21.** Generally, what is the response time of photodiode conditioned by?  
 A. The diffusion time of equilibrium carriers in the base  
 B. Their transit time through the layer of p-n junction  
 C. RC constant of diode structure  
 D. Only A and C  
 E. Conditions A, B, C
- 4a22.** Which statement mentioned below is not true for ohmic contact?  
 A. Electrical resistance of ohmic contact is small  
 B. Electrical resistance of ohmic contact does not depend on the current direction if the current value does not exceed the given value  
 C. Electrical resistance of ohmic contact does not depend on the current direction in case of any current value flowing through it  
 D. Most part of ohmic contacts is formed on the basis of n-n+ or p-p+ type contacts  
 E. All the answers are correct

- 4a23.** By increasing the lifetime of electrons 4 times, their diffusion length
- Increases 4 times*
  - Increases twice*
  - Does not increase*
  - Reduces twice*
  - The correct answer is missing*
- 4a24.** What does the generation frequency depend on in Gunn diode?
- Mobility speed of field domain*
  - Impurity density in semiconductor*
  - Sample length*
  - Dielectric permeability of material*
  - All the answers are correct*
- 4a25.** How can the cutoff voltage of MOS transistor change?
- By opposite voltage of substrate-channel junction, when substrate is higher ohmic than the channel*
  - By opposite voltage of substrate-channel junction when substrate resistance is equal or smaller than the channel resistance*
  - By voltage applied to the gate*
  - By A and C*
  - By B and C*
- 4a26.** Which statement is wrong for unipolar transistors?
- In unipolar transistors, physical processes of current transport are conditioned by one sign carriers-electrons or holes*
  - In unipolar transistors, physical processes of current transport are conditioned by the injection of minority carriers.*
  - In unipolar transistors current control is carried out by the vertical electrical field*
  - The surface channel unipolar transistor includes metal-dielectric-semiconductor structure*
  - The correct answer is missing*
- 4a27.** What is the high frequency property of Schottky diode conditioned by?
- Moving the majority carriers through diode*
  - Excluding minority carriers' accumulation in diode*
  - Value of Schottky barrier*
  - Impurity density in a semiconductor*
  - Only C and D*
- 4a28.** Which of the below written statements is wrong for an integrated capacitor?
- An integrated capacitor represents IC element consisting of conductive electrodes (plates), divided by isolation layer*
  - In ICs the role of an integrated capacitor is often performed by reverse-biased p-n junctions of a transistor structure*
  - The quality factor of an integrated capacitor is defined by the following:  $Q = 2 \pi f R C$  where  $f$  – operating frequency,  $C$  – capacitance of a capacitor,  $R$  – resistance of a resistor sequentially connected with the transistor*
  - The quality factor of an integrated capacitor characterizes loss of power at capacitive current junction*
  - All the answers are correct*
- 4a29.** Which of the below mentioned statements is wrong for electronic lithography?
- In this method the electron beams are used as a source of radiation*
  - The method of electron beam lithography is based on non-thermal influence left by electron beam on resist*
  - The ultraviolet beams fall on resist surface at electron beam lithography*
  - It is possible to reduce diffraction effects by increasing the electron accelerating voltage in electron beam lithography*
  - The correct answer is missing*

- 4a30. Which of the below mentioned statements is correct for a bipolar transistor in saturation mode?
- Emitter and collector junctions are forward-biased
  - Emitter junction is forward-biased, and collector junction –reverse-biased
  - Transistor base resistance in this mode is maximum, as emitter and collector junctions inject large number of free particles to base region
  - Free carriers' extraction takes place from transistor base being in this mode
  - The correct answer is missing

- 4a31. What crystallographic plane is underlined in cubic lattice?



- (100)
- (101)
- (001)
- (011)
- (110)

- 4a32. Due to what properties is silicon considered the main material of microelectronics?
- Exclusive combination of its band gap and electro-physical parameters
  - Its oxide stability and isolating properties
  - High development of technological methods related to it in different physical-chemical processes
  - The value of its natural resources
  - All the answers are correct
- 4a33. What statement is wrong for metal-nitride-oxide semiconductor (NMOS) field effect transistor?
- In a such transistor, silicon nitride and silicon dioxide double structure serves as a subgate isolator
  - There are many deep levels (traps) for electrons in silicon nitride layer
  - The layer's thickness of silicon dioxide is selected such that it is not tunnel transparent
  - After removing the voltage on MNOS-field transistor's gate, the injected charge remains long trapped which corresponds to the existence of induced inversion layer.
  - The correct answer is missing
- 4a34. If the density of semiconductor's defects is large,
- The lifetime of carriers will be large
  - The conductivity of semiconductor will decrease
  - The recombination speed will increase
  - The current will remain constant
  - The generation speed will increase
- 4a35. The performance of tunnel diodes is larger than the one of p – n junction as
- Injection level is large
  - Injection mechanism is different
  - Junction capacitance is small
  - Current is formed by electrons and holes
  - Potential barrier's height is small
- 4a36. What diode is called stabilatron?
- Direct current of which exponentially increases with the voltage
  - Reverse current of which is saturated
  - Reverse branch of its the volt-ampere characteristic has a region very strict dependence of current on voltage
  - The volt-ampere characteristic of which has an N- type region
  - The correct answer is missing
- 4a37. Threshold voltage of a short channel MOS transistor is smaller than the one of a long channel MOS as
- It occupies smaller area
  - The number of technological processes is small



- C. *Gate controls smaller number of charge*  
 D. *There are piled charges on the boundary of oxide-semiconductor junction*  
 E. *Intrinsic capacitance is large*
- 4a38.** In Gann diode, negative conductivity occurs  
 A. *Due to ohmic property of contacts*  
 B. *When p-n junction is controlled by larger voltage*  
 C. *Due to field domain*  
 D. *Carriers' flight time is small*  
 E. *No answer is correct*
- 4a39.** To increase MOS transistor's drain conductance which statement below is wrong?  
 A. *It is necessary to reduce channel length and increase its width*  
 B. *It is necessary to increase the thickness of subgate isolator*  
 C. *It is necessary to use dielectric with more dielectric permittivity*  
 D. *It is necessary to use as a transistor's substrate a semiconductor with more carrier's mobility*  
 E. *The correct answer is missing*
- 4a40.** How does base volume resistance affect on semiconductor's diode characteristic?  
 A. *It leads to the sharp increase of the current*  
 B. *The direct current, depending on the voltage, increases slower than exponential law*  
 C. *Volt - ampere characteristic, starting with the smallest value of voltage, becomes ohmic*  
 D. *Negative differential conductance region appears on volt-ampere characteristic of the diode*  
 E. *The correct answer is missing*
- 4a41.** Carriers' mobility, depending on temperature, changes:  
 A. *Increases linearly*  
 B. *Decreases exponentially*  
 C. *Remains constant*  
 D. *Changes nonlinearly*  
 E. *No answer is correct*
- 4a42.** The reduction of oxide layer's thickness in a MOS transistor leads to:  
 A. *Increase of transistor's performance*  
 B. *Decrease of transistor's performance*  
 C. *Increase of intrinsic capacitance*  
 D. *Increase of leakage currents*  
 E. *Decrease of occupied area*
- 4a43.** Numbers of basic equivalent minimums of silicon and germanium conduction bands correspondingly are:  
 A. *6 and 6*  
 B. *6 and 4*  
 C. *6 and 8*  
 D. *4 and 6*  
 E. *2 and 4*
- 4a44.** The band gap of a semiconductor with the increase of magnetic field  
 A. *Increases linearly*  
 B. *Increases exponentially*  
 C. *Decreases linearly*  
 D. *Decreases exponentially*  
 E. *Remains unchanged*
- 4a45.** Fermi level of full compensated semiconductor at 0K temperature is located on:  
 A. *Middle of conduction band bottom and donor level*  
 B. *Middle of donor and acceptor levels*  
 C. *Middle of band gap*  
 D. *Donor level*  
 E. *Acceptor level*
- 4a46.** Two contacting semiconductors are in equilibrium if  
 A. *Forbidden band gaps are equal*  
 B. *Fermi levels are equal*  
 C. *Free carrier concentrations are equal*  
 D. *Current carrier lifetimes are equal*  
 E. *Current carrier diffusion coefficients are equal*

- 4a47.** If Fermi and donor levels are equal, the probability of electron occupancy in donor level is  
 A. 1/2  
 B. 1  
 C. 2/3  
 D. 1/3  
 E. 0
- 4a48.** Fundamental parameters of a semiconductor are  
 A. *Electron effective mass*  
 B. *Electron and hole lifetimes*  
 C. *Electron and hole mobility*  
 D. *Resistivity*  
 E. *Coefficient of lattice thermal conductivity*
- 4a49.** p-n junction potential barrier under influence of absorbing light  
 A. *Increases*  
 B. *Increases, then decreases*  
 C. *Decreases, then increases*  
 D. *Decreases*  
 E. *Remains unchanged*
- 4a50.** When p-n junction occurs, which part of it acquires i-type conductivity?  
 A. *High ohmic part outside p-n junction*  
 B. *Contact domain of p- and n- parts*  
 C. *Deep domains of p- and n- parts*  
 D. *The layer of spatial charges the layer of p-n junction*  
 E. *Possible high ohmic part, adjacent to ohmic contact*
- 4a51.** When p-n junction occurs, which flow of carriers gives rise to potential barrier?  
 A. *Diffusion of majority carriers of p- and n- domains*  
 B. *Drift of minority carriers of p- and n- domains*  
 C. *Two flows together*  
 D. *Only diffusion of electrons*  
 E. *Only drift of holes*
- 4a52.** In case of being connected by general base of a bipolar transistor, when injection is missing from the emitter, on what part does the voltage, supplying collector p-n junction, fall?  
 A. *The part of load resistance, connected to collector.*  
 B. *The layer of drift charges of collector p-n junction.*  
 C. *Both A and B.*  
 D. *High ohmic part of collector contact.*  
 E. *The part outside spatial charges of a collector.*
- 4a53.** What is comparably high temperature stability of a field transistor conditioned by?  
 A. *Reverse bias voltage of gate*  
 B. *Possibility of modulation of channel resistance*  
 C. *Output current conditioned by majority carriers*  
 D. *Both B and C*  
 E. *Quality of ohmic contacts*
- 4a54.** How does the drain current of a filed transistor change when increasing the temperature?  
 A. *Increases on the account of getting rid of surface state electrons*  
 B. *Decreases due to decreasing carriers' mobility*  
 C. *Both A and B occur*  
 D. *Does not change*  
 E. *Increases together with output contact heating*
- 4a55.** What is the high performance of Schottky diode conditioned by?  
 A. *Output operation of a semiconductor*  
 B. *Lack of minority carriers' accumulation in a semiconductor*  
 C. *Move of majority carriers conditioned by diode operation*  
 D. *Presence of charge capacitance*  
 E. *Output operation of a metal*

- 4a56.** Why is the light diode's radiation spectrum not strictly one wave?  
 A. Because radiation recombination of carriers occurs between two levels  
 B. Because radiation recombination of carriers occurs between electrons which are on one group of levels and holes which are in another group  
 C. Because spectral distribution of radiation, coming from diode changes  
 D. Because it is conditioned by injection of carriers  
 E. Because recombination is missing in p-n junction
- 4a57.** When does a semiconductor amplifier, having positive feedback, become a generator?  
 A. When the mirrors, creating positive feedback, provide pure reflection  
 B. When amplification exceeds all the losses of radiation in the device  
 C. When radiation losses in unit length of active layer are minimum  
 D. When radiation is directed  
 E. When radiation is provided in the layer of p-n junction
- 4a58.** If p-n junction with  $C_{dif}$  diffusion and  $C_{ch\ arg}$  charge capacitances is in equilibrium state, then  
 A.  $C_{dif} = 0, C_{ch\ arg} = 0$   
 B.  $C_{dif} \neq 0, C_{ch\ arg} \neq 0$   
 C.  $C_{dif} \neq 0, C_{ch\ arg} = 0$   
 D.  $C_{dif} = 0, C_{ch\ arg} \neq 0$   
 E.  $C_{dif} = \infty, C_{ch\ arg} \neq 0$
- 4a59.** There is double heterojunction luminescent AlGaAs - GaAs - AlGaAs structure. Refraction index of GaAs is higher than refraction index of AlGaAs. What is the role of intermediate GaAs microsize layer for radiation?  
 A. Majority carriers  
 B. Ohmic contact  
 C. Waveguide  
 D. Reduction of radiation  
 E. Observation of radiation.
- 4a60.** Which is the key advantage of double heterojunction luminescent structure towards single heterojunction luminescent structure?  
 A. Operates at higher bias voltage  
 B. Easier realized technologically  
 C. Provides high intensity of radiation by double injection  
 D. Provides high radiation losses  
 E. Provides chaos observation of radiation
- 4a61.** What type of conductivity does strictly compensated semiconductor have?  
 A. Ion conductivity  
 B. Electronic conductivity  
 C. i-type conductivity  
 D. Hole conductivity  
 E. Has no conductivity
- 4a62.** What type of radiation is there in radiating semiconductor structures?  
 A. Spontaneous  
 B. Chaotic  
 C. Compulsive  
 D. Spontaneous and compulsive  
 E. Multiwave
- 4a63.** When does non-equilibrium electro conductance reach dynamic balance in a semiconductor, created by light?  
 A. When the increase of non-equilibrium carrier density, generated by light, occurs  
 B. When non-equilibrium hole density exceeds electrons  
 C. When the generation and recombination processes of non-equilibrium holes and electrons reach dynamic balance  
 D. When observing radiation energy is enough for moving valence electron to conductance region  
 E. When the semiconductor is in total vacuum

- 4a64.** What is the high photosensitivity of a field phototransistor conditioned by?  
 A. *Possibility to connect large input resistance*  
 B. *Gate photocurrent*  
 C. *Modulation of photogenerated carrier channel conductance, drain current gain*  
 D. *Environment to observe radiation*  
 E. *All the mentioned factors*
- 4a65.** What properties of p-n junction allow creating group technology for high efficiency ICs?  
 A. *The capacitive property of p-n junction*  
 B. *Signal correction property by one p-n junction*  
 C. *Signal amplification property by two p-n junction*  
 D. *The property of p-n junction resistance*  
 E. *All the mentioned properties*
- 4a66.** What does radiation's quantum energy depend on?  
 A. *Spectrum form of radiation*  
 B. *Intensity of radiation flow*  
 C. *Radiation frequency*  
 D. *Environment of observing radiation*  
 E. *All the mentioned factors*
- 4a67.** What does the abrupt heterojunction conductance band difference equal to?  
 A. *The difference of work-function*  
 B. *The difference of dipole moments*  
 C. *The difference of semiconductors affinities*  
 D. *The difference of band gap*  
 E. *The difference of Fermi energy*
- 4a68.** The charge carriers' mobility in a double gate MOS is larger as  
 A. *Threshold transconductance is large*  
 B. *Short channel effects are present*  
 C. *Oxide effective thickness is small*  
 D. *The perpendicular field coefficient is small in the channel*  
 E. *The channel is far from substrate*
- 4a69.** The electric field value in p-n junction at avalanche multiplication (impact ionization) regime is defined by:  
 A. *Avalanche breakdown current in the given point*  
 B. *Avalanche breakdown current change rate at time*  
 C. *Avalanche breakdown voltage*  
 D. *The electron and hole ionization rates*  
 E. *The phase difference between current and voltage*
- 4a70.** The domain formation time in Gann diode:  
 A. *Must be larger than Maxwell relaxation time*  
 B. *Must be defined by the value of applied field*  
 C. *Must equal to transit time*  
 D. *Must be larger than the lifetime of electrons and holes*  
 E. *Must be smaller than transit time*
- 4a71.** The charge carriers' mobility in the base of drift transistor is large  
 A. *As the concentration of impurities is small*  
 B. *The base width is small*  
 C. *The emitter impurity concentration is high*  
 D. *The diffusion coefficient is large*  
 E. *None of the answers is correct*
- 4a72.** The tunnel diode high frequency operation is conditioned by:  
 A. *The p-n doped impurity degree*  
 B. *Depletion layer width*  
 C. *p-n junction capacitance*  
 D.  *$J_{max}/J_{min}$  ratio*  
 E. *The value of excess current*

- 4a73.** What the depletion layer width near the drain in Shottky barrier controlled FETs is conditioned by?  
 A. *The source-drain impurity concentration*  
 B. *The drain applied voltage*  
 C. *The substrate legiration degree*  
 D. *The voltage difference between gate and channel drop voltage*  
 E. *The sum of gate voltage and channel voltage*
- 4a74.** What type of a semiconductor can be defined by thermal-compression methods if  $p=5n$ , and  $\mu_n = 10\mu_p$ ?  
 A. *Intrinsic type of conductance*  
 B. *p type*  
 C. *n type*  
 D. *Both p type and n type*  
 E. *None of the answers is correct*
- 4a75.** Can the conductance of a conductor be of p-type?  
 A. *It can if the conductance band is almost occupied (the  $F$ -is high from the middle of band gap)*  
 B. *It cannot*  
 C. *The conductance of a conductor will be of n-type*  
 D. *Both p type and n type*  
 E. *None of the answers is correct*
- 4a76.** On the diffraction image of waves, redistribution of incident wave energy occurs (divided into parts). Is the electronic wave (de Broil wave) divided into parts during the diffraction of electrons?  
 A. *Also divided into parts*  
 B. *Turns into phonon*  
 C. *Turns into the energy of crystalline lattice*  
 D. *None of the answers is correct*  
 E. *No, is not divided into parts as the electron, as a whole, appears in the diffraction image maximum*
- 4a77.** If the effective mass were gravitation characteristic, in what direction would the hole move in the Earth gravitation field?  
 A. *Vertically upward*  
 B. *Wouldn't move*  
 C. *Would move chaotic*  
 D. *Vertically downward*  
 E. *None of the answers is correct*
- 4a78.** The energetic distance between the donor and acceptor levels is smaller than the band gap width ( $E_d - E_a < E$ ). Why is the probability of electron transition from the acceptor to donor (or the opposite) very small compared with the probability of band-band?  
 A. *Because the donor levels are not free*  
 B. *Because the acceptor levels are not free*  
 C. *Because the width between the dopand-dopand is large*  
 D. *Because the number of free places in bands is large*  
 E. *None of the answers is correct*
- 4a79.** Complementary MOSFET is characterized by:.  
 A. *The presence of only p-type channel*  
 B. *The presence of only n-type channel*  
 C. *The presence of p- and n-type channels*  
 D. *The absence of channel*
- 4a80.** Silicon thermal dioxide thickness grows:  
 A. *Only on the silicon substrate surface*  
 B. *On the silicon substrate surface and in the near-surface region*  
 C. *Only in the silicon near-surface region*  
 D. *In the volume of silicon substrate*
- 4a81.** The buffer  $n^+$  buried layer in the n-p-n-type bipolar transistor can be built in for:  
 A. *To decrease the volume resistance of collector region*  
 B. *To increase the volume resistance of collector region*  
 C. *To increase collector-emitter junction resistance*  
 D. *To reduce collector-emitter junction resistance*  
 E. *To reduce the lifetime of minority charge carriers in collector region*
- 4a82.** What are the silicon thermal oxidation processes for limiting the oxidation rate?  
 A. *Deposition of oxidant ( $O_2$ ,  $H_2O$ ) particles on Si surface*  
 B. *Oxidant diffusion through  $SiO_2$  layer to the Si- $SiO_2$  interface*

- C. Oxidation chemical reaction with Si and origination of the new SiO<sub>2</sub> layer
- D. By the A), and B) phases simultaneously
- E. Diffusion of the reaction steam results into the Si external surface

**4a83.** Ion-implantation doping advantages compared with diffusion process (mark the wrong answer).

- A. Exact (precise) regulation of impurities distribution profile
- B. A wider range of impurities
- C. Small sideways
- D. Low temperature of process
- E. Profiles of deep distribution of impurities

**4a84.** The silicon thermal oxidation process control is performed by the following technological parameters (mark the wrong answer).

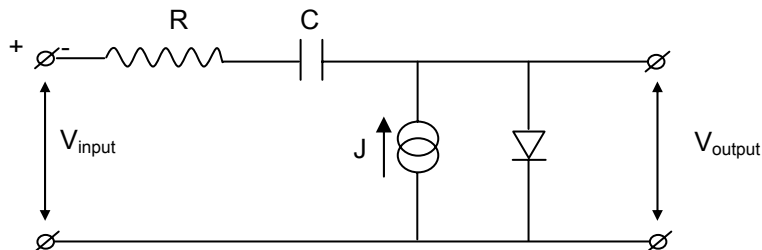
- A. Oxidation temperature
- B. Oxidation time
- C. Velocity of the carrier gases
- D. Steam concentration
- E. Impurity concentration in the carrier gases

**b) Problems**

**4b1.**

Semiconductor diode is often used in reducers as a variable resistor (see the circuit). In that case the diode's bias is given by means of J constant current source, and the connection between input and output signals is realized by the help of C capacitance, the reactive resistance of which is relatively smaller compared with R resistance.

Calculate and draw the dependence on J current expressed in decibels according to voltage signal depletion ( $20\lg(V_{\text{output}}/V_{\text{input}})$ ), when the current ranges from 0.01 mA to 10 mA. Use  $R=10^3$  Ohm in calculations, and diode saturation current  $J_s=10^{-6}$  A.



**4b2.**

Calculate the capacitance of p-n junction that is characterized by linear distribution of impurities:  $N_A - N_D = kx$ , where  $k = 10^{10} \text{ m}^{-1}$ ,  $\epsilon\epsilon_0 = 200 \text{ pF/m}$ , junction area  $A=10^{-7} \text{ m}^2$ , the difference of contact potential  $\Psi = 0.3\text{V}$ , and the opposite deflection  $V = 5\text{V}$ .

**4b3.**

Field transistor's n-channel of p<sup>+</sup>-n-junction is characterized by arbitrary distribution of channel width impurities  $N_D(x)$ . Show that the transconductance of a such transistor  $\rho_m = \frac{\partial I_D}{\partial V_G}$  equals

$$\rho_m = \frac{2z\mu}{L} [Q(h_2) - Q(h_1)],$$

where  $h_1$  and  $h_2$  are depletion layer widths at source and drain, accordingly,

and  $Q(y) = e \int_0^y N_D(y) dy$ ,  $z$  is channel width,  $L$  – its length, and  $\mu = \text{const}$  - electron mobility.

**4b4.**

The semiconductor, the Holy constant of which equals  $3.33 \cdot 10^{-4} \text{ m/Cl}$ , and specific resistance  $8.93 \cdot 10^{-3} \text{ Ohm.m}$ , is located in magnetic field, the induction of which equals 0.5 Tl. Define the Holy angle.

**4b5.**

On the photovoltaic cell, the integral sensibility of which is 100 uA/lm, 0.15 lm light flow falls. Resistor with 400 kOhm resistance is successively connected to the photovoltaic cell, the signal on which is given to the amplifier, which in its turn is controlled by 10 mA current and 220 V voltage controlling relay. Define the gain constant according to voltage and power.

**4b6.**

The ideal diode the opposite saturation current of which is 8 uA, is successively connected to 10 V emf and 1 kOhm resistor. At room temperature define diode's direct current and voltage drop on it.

**4b7.**

In a field transistor, the maximum value of channel current equals 2 mA, gate cutoff voltage - 5 V. Define channel current and slope of transistor characteristic in case of the following voltage values of the gate: a) - 5 V, b) 0 V, c) -2,5 V.

**4b8.**

Silicon p-n junction is given. Define p-n junction layer's

a)  $d_p$  and  $d_n$  widths of both common  $d$  and  $p$  &  $n$  regions,

b) contact  $\varphi_c$  difference of potentials,

if the following is known: intrinsic charge density of silicon  $n_i = 1,4 \cdot 10^{10} \text{ cm}^{-3}$ , dielectric transparency of vacuum  $\epsilon_0 = 8,85 \cdot 10^{-14} \text{ F/cm}$ , dielectric transparency of silicon  $\epsilon = 12$ , electron charge  $q = 1,6 \cdot 10^{-19} \text{ coulomb}$ , Boltzmann constant  $k = 1,38 \cdot 10^{-23} \text{ J/}^\circ\text{K}$ , temperature  $T = 300\text{K}$ , conductances in n and p regions  $\sigma_n = 10 \text{ Ohm.cm}$  and  $\sigma_p = 5 \text{ Ohm.cm}$ , mobility of electrons and holes  $\mu_n = 1300 \frac{\text{cm}^2}{\text{V}\cdot\text{v}}$ ,  $\mu_p = 500 \frac{\text{cm}^2}{\text{V}\cdot\text{v}}$ :

Impurity atoms in the given temperature are considered fully ionized.

**4b9.**

Define silicon ideal p-n junction photodiode's

a)  $j_s$  density of saturation current,

b) open circuit  $V_{o.c}$  voltage,

If the following is given:  $n_i = 1,4 \cdot 10^{10} \text{ cm}^{-3}$ ,  $\mu_n = 1300 \frac{\text{cm}^2}{\text{V}\cdot\text{v}}$ ,  $\mu_p = 500 \frac{\text{cm}^2}{\text{V}\cdot\text{v}}$ ,  $kT = 0,026 \text{ eV}$ ,  $q = 1,6 \cdot 10^{-19} \text{ q}$ ,

$N_d = 10^{15} \text{ cm}^{-3}$ ,  $N_a = 5 \cdot 10^{15} \text{ cm}^{-3}$ ,  $L_n = 100 \text{ um}$ ,  $L_p = 60 \text{ um}$ , absorption coefficient  $\alpha = 10^3 \text{ cm}^{-1}$ , base width  $W = 100 \text{ um}$ , external quantum output  $\beta = 0.7$ , light sensitive area  $S = 10^{-4} \text{ cm}^2$ , intensity of absorbed rays<sup>a</sup>

$\Phi = 10^{18} \frac{\text{q}}{\text{cm}^2 \cdot \text{v}}$ . Assume that the impurity atoms are ionized.

**4b10.**

Field effect silicon transistor with n-type channel and contrary p-n junctions is given.

Define

a)  $h_1/h_2$  ratio of  $h_1$  and  $h_2$  widths of the channel near the source and drain,

b) *cutoff voltage*,

if given:  $\epsilon = 12$ ,  $\epsilon_0 = 8,85 \cdot 10^{-14} \text{ F/cm}$ , the voltage applied to p-n junction from the supply source of the drain, near the source  $V_1 = 0,5 \text{ V}$ , near the drain  $V_2 = 1 \text{ V}$ , the voltage applied to the gate  $V_g = 0.5 \text{ V}$ ,  $\mu_n = 1300 \frac{\text{cm}^2}{\text{V}\cdot\text{v}}$ ,

$\rho_h = 5 \text{ Ohm.cm}$ , channel length  $\ell = 10^{-2} \text{ cm}$ , channel thickness  $\alpha = 2,5 \cdot 10^{-4} \text{ cm}$ , channel width  $b = 10^{-2} \text{ cm}$ .

**4b11.**

A field effect transistor with isolated gate is given which has built-in silicon n channel.

Define

a)  $C$  capacitance of the gate in depletion mode,

b) *cutoff voltage*  $V_{g0}$ ,

if given: thickness of isolator  $d = 0,5 \cdot 10^{-4} \text{ cm}$ ,  $\epsilon = 12$ ,  $\epsilon_0 = 8,85 \cdot 10^{-14} \text{ F/cm}$ ,  $\mu_n = 1300 \frac{\text{cm}^2}{\text{V}\cdot\text{v}}$ , channel length

$\ell = 10^{-2} \text{ cm}$ , width  $b = 10^{-2} \text{ cm}$ , thickness  $\alpha = 2 \cdot 10^{-4} \text{ cm}$ , electron charge  $q = 1,6 \cdot 10^{-19} \text{ cl}$ , voltage applied to the

gate  $|V_g|=3$ , voltage applied to the drain – 1V, density of energetic states in conducting band  $N_C \approx 10^{19} \text{ cm}^{-3}$ , position of Fermi level  $E_C - E_F = 0,2 \text{ V}$ , thermal energy  $kT=0,025 \text{ eV}$ .

**4b12.**

The barrier capacitance of an abrupt p-n junction is 200pF, when 2V reverse bias is applied towards it. What kind of reverse bias is required to apply in order to reduce its capacitance up to 50pF if the contact potential difference is  $\varphi_k = 0.82 \text{ V}$ .

**4b13.**

The conductivity of n-type channel of a field effect transistor, controlled by p-n junction, is  $320 \text{ Ohm}^{-1} \text{ m}^{-1}$ . The channel width is  $w = 8 \text{ } \mu \text{ m}$ , when “gate-source” voltage equals zero. Find the “pinch-off” voltage of transistor’s channel if the mobility of electrons is  $2000 \text{ cm}^2 / \text{V} \cdot \text{s}$ , and the dielectric constant of the semiconductor equals 13.

**4b14.**

It is known that the electrical field in the short channel field effect transistor can reach up to several kV/cm, in case of which the carriers are “heated”, and their mobility becomes dependent on the electric field strength  $\varepsilon$ . Considering that the dependence has the following form:

$$\mu = \frac{\mu_n}{1 + \frac{\varepsilon}{\varepsilon_c}}$$

( $\mu_n$  - low field mobility of electrons,  $\varepsilon_c$  -critical value of electrical field) find the drain current dependence on its and the gate’s voltages. Also consider that the channel length, width and depth are given.

**4b15.**

Boron diffusion is provided from a “limited source” with the total amount of impurities  $Q = 2,25 \cdot 10^{13} \text{ atom/cm}^2$  in  $t = 2$  hour. Diffusion coefficient equals  $D = 9,2 \cdot 10^{-13} \text{ cm}^2/\text{s}$ . Diffusion is provided into the bulk silicon substrate with the impurity concentration  $N_D = 1 \cdot 10^{16} \text{ atom/cm}^3$ . Calculate the depth of p-n junction  $x_j$  in micrometers.

**4b16.**

Consider a  $p^+ - n - p$  silicon transistor with doping levels of emitter, base and collector  $N_{AE} = 5 \cdot 10^{18} \text{ cm}^{-3}$ ,  $N_{DB} = 10^{16} \text{ cm}^{-3}$ ,  $N_{AK} = 10^{15} \text{ cm}^{-3}$ , base width  $W = 1 \text{ } \mu \text{ m}$ , cross sectional area is  $3 \text{ mm}^2$ , and applied voltages are  $U_{EB} = +0.5 \text{ V}$ ,  $U_{BK} = -5 \text{ V}$ . Calculate the width of quasi-neutral part of the base, minority carriers’ (holes) concentration near the emitter-base junction and total charge of minority carriers injected into the base ( $T=300\text{K}$ ).

**4b17.**

Calculate the bipolar diffusion coefficient of current carriers in intrinsic GaAs at 300K if electron and hole mobilities are equal  $8800 \text{ cm}^2/\text{V} \cdot \text{s}$  and  $400 \text{ cm}^2/\text{V} \cdot \text{s}$ , correspondingly.

**4b18.**

Find the potential barrier’s height existing for electrons in Schottky diode if the specific resistance of a semiconductor  $\rho = 1 \text{ Ohm} \cdot \text{cm}$ , electrons’ mobility  $3900 \text{ cm}^2 / \text{V} \cdot \text{s}$ , gold work function  $\varphi_{Au} = 5 \text{ eV}$ , semiconductor’s electron affinity  $\chi_{Gc} = 4 \text{ eV}$ , intrinsic concentration  $n_i = 2.5 \cdot 10^{13} \text{ cm}^{-3}$ , band-gap width  $E_g = 0.66 \text{ eV}$  ( $T=300\text{K}$ ).

**4b19.**

For an ideal p-n-p transistor, the current components are given by  $I_{ep}=3\text{mA}$ ,  $I_{en}=0.01\text{mA}$ ,  $I_{cp}=2.99\text{mA}$ ,  $I_{cn}=0.001\text{mA}$ . Determine:

- The emitter efficiency ( $\gamma$ ),
- The base transport coefficient ( $I_T$ ),
- The common-base current gain  $\bar{E}_0$  and  $I_{cB0}$ .



**4b20.**

In n-channel n<sup>+</sup>-multicrystal Si-SiO<sub>2</sub>-Si MOS transistor N<sub>a</sub>=10<sup>17</sup>cm<sup>-3</sup>, (Q<sub>ox</sub>/Q)=5\*10<sup>11</sup>cm<sup>-2</sup>, calculate the threshold voltage V<sub>T</sub>, if the oxide layer thickness is 5nm. What density of Bor ions is necessary to increase the threshold voltage up to 0.6V?

$$2\psi_B=0.84V, \epsilon_{SiO_2}=3.9, \psi_S=-0.98V, \epsilon_{Si}=11.9.$$

**4b21.**

Intrinsic Ge is in 3000 K temperature. How many percent will the specific conductance of the sample change if the temperature increases by 1%. Accept  $\Delta E = 0.72eV$ .

**4b22.**

Given mobilities of electrons and holes ( $\mu_n$  u  $\mu_p$ ), find the concentration of charge carriers corresponding to minimal specific conductance.

**4b23.**

How will charge carrier's lifetime in non-generated semiconductor change under the doped impurity concentration if it is known that T =3000K is constant, the mobility increases by 5%, and diffusion length decreases by 10 %?

**4b24.**

Using hydrogen atom model, calculate in semiconductor InSb crystal

- donors' ionization energy,
- the radius of electrons in basic state,
- the electron concentration density T = 4° K, when Nd = 1. 1014 cm<sup>-3</sup>.

The radius of basic orbits is  $r_H = 0.53 \overset{o}{A}$ , and bandgap of InSb  $E_g = 0.18eV$ ,  $\epsilon_s = 17$ ,  $m^* = 0.014m_o$ , ( $m_o$  is free electron mass),  $kT = 0.0258$  eV when T = 300 K, and ionization energy  $E_H = 13.6$  eV.

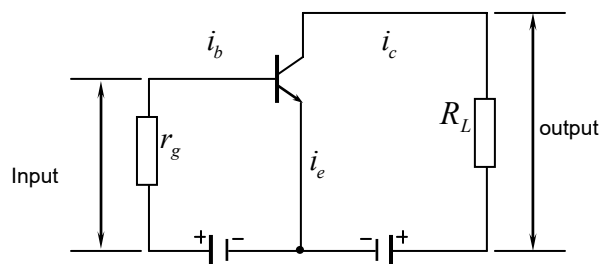
**4b25.**

What number of electrons must pass from one metal to another for  $V_k=1V$  contact potential difference to occur between them when the width of dielectric between metals is  $d=10^{-9}$  m,  $\epsilon_o = 8.85 \cdot 10^{-14}$  F/cm.

Compare the amount when there is metal-n type semiconductor contact for the same condition when in the surface of metal, concentration of electrons is  $n_{sm} \approx 10^{25}$  c<sup>-2</sup>, and in semiconductor  $n_{ss} \approx 10^{10}$  cm<sup>-2</sup>.

**4b26.**

A (n - p - n) transistor circuit connected by general emitter is shown. Calculate, according to power, amplification coefficient if  $\alpha = 0.98$ ,  $r_e = 20$  Ohm,  $r_b = 500$  Ohm,  $R_L = 30$  kOhm.

**4b27.**

In an intrinsic semiconductor electron concentration is  $1.3 \times 10^{16}$  cm<sup>-3</sup> at 400K and  $6.2 \times 10^{15}$  cm<sup>-3</sup> at 350K. Determine the forbidden band gap of material if it changes linearly via temperature.

**4b28.**

Determine the holes distribution in the n-type thin and long non-degenerated germanium wire in the case of point stationary injection of holes at point  $x = 0$ . Electric field intensity applied on the sample is  $E = 5$  V/cm, temperature is T = 300 K, hole diffusion length is  $L_p = 0.09$  cm.

**4b29.**

From the plane  $x = 0$  of homogeneous half-infinite ( $x \geq 0$ ) n-type semiconductor injected holes are stationary. Determine hole current density at  $x=0$  point if  $\Delta p(0) = 10^{13} \text{ cm}^{-3}$ , hole diffusion length is  $L_p = 0.07 \text{ cm}$ , hole diffusion coefficient is  $D_p = 49 \text{ cm}^2/\text{s}$ , injection coefficient is  $\xi = 0.4$ . Non-equilibrium carrier drift is neglected.

**4b30.**

The N type silicon sample has 4mm length, 1,5mm width, 1mm high and 80 Ohm resistivity. Determine the acceptor concentration of the sample if  $0.12 \text{ m}^2/\text{Vc}$  and  $0.025 \text{ m}^2/\text{Vc}$  are the electron and hole mobilities, respectively,  $2.5 \times 10^{16} \text{ m}^{-3}$  is the intrinsic concentration of current carrier.

**4b31.**

In the photodetector by surface p-n junction, the width of active layer, creating photocurrent, is  $d=1 \mu\text{m}$ ,  $F=10^{14} \text{ quantum}/\text{cm}^2 \cdot \text{s}$ , by  $\alpha = 10^4 \text{ cm}^{-1}$  and  $\alpha = 10^3 \text{ cm}^{-1}$  absorption coefficients, double wave ( $\lambda_1$  and  $\lambda_2$ ) radiation ( $h\nu \geq E_g$ ) are absorbed by photosensitive surface. Calculate the photocurrent ratio, created by those two wave absorption if photosensitive area is  $S = 10^{-4} \text{ cm}^2$ , quantum output  $\beta = 1$ , electron charge  $q = 1.6 \cdot 10^{-19} \text{ C}$ .

**4b32.**

Define the performance of photodiode if volume charge layer width is  $d=10^{-4} \text{ cm}$ , maximum speed of carriers' movement  $V_{\text{max}} = 5 \cdot 10^6 \text{ cm/s}$ , electron density in n-type base  $n = 5 \cdot 10^{15} \text{ cm}^{-3}$ , mobility  $\mu_n = 1,3 \cdot 10^3 \text{ cm}^2/\text{V} \cdot \text{s}$ , diffusion coefficient of minority carriers - holes  $D_p = 15,6 \text{ cm}^2/\text{v}$ , Holes mobility in base  $\mu_p = 600 \text{ cm}^2/\text{Vv}$ , electron charge  $q = 1.6 \cdot 10^{-19} \text{ C}$ , base width  $w = 3 \cdot 10^{-3} \text{ cm}$ , photosensitive area  $S = 10^{-2} \text{ cm}^2$ , dielectric permeability of semiconductor  $\epsilon = 12$ , vacuum  $\epsilon_0 = 8.86 \cdot 10^{-14} \text{ f/cm}$ .

**4b33.**

Define power density equivalent to photodiode noise if dark and light current sum  $I = I_d + I_L = 5,1 \cdot 10^{-7} \text{ A}$ , frequency band  $\Delta f = 1 \text{ Hz}$ , absorption radiation power  $P = 10^{-6} \text{ W}$ , photocurrent  $I_E = 5 \cdot 10^{-7} \text{ A}$ , photosensitive surface  $S = 10^{-2} \text{ cm}^2$ , electron charge  $q = 1.6 \cdot 10^{-19} \text{ C}$ .

**4b34.**

Define channel width shrinkage through p-n junction at the drain of field transistor, when source-drain domain affects are missing,  $V = +0.5 \text{ V}$  voltage has been applied to the drain, dielectric permeability of channel material  $\epsilon = 12$ , vacuum  $\epsilon_0 = 8.86 \cdot 10^{-14} \text{ f/cm}$ , contact difference of potentials  $\phi = 0.7 \text{ V}$ , donors' density in and n-channel  $N_d = 5 \cdot 10^{14} \text{ cm}^{-3}$ , electron charge  $q = 1.6 \cdot 10^{-19} \text{ C}$ .

**4b35**

Calculate the drain current of an NMOS Si transistor according to the following conditions: threshold voltage  $V_t = 1 \text{ V}$ , gate width  $W = 10 \mu\text{m}$ , gate length  $L = 1 \mu\text{m}$ , thickness of oxide layer  $t_{\text{ox}} = 10 \text{ nm}$ ,  $V_{\text{GS}} = 3 \text{ V}$  and  $V_{\text{DS}} = 5 \text{ V}$ . For calculation, use square model, surface mobility is  $300 \text{ cm}^2/\text{V}$  and  $V_{\text{BS}} = 0 \text{ V}$ . Also calculate  $g_m$  transconductance.

**4b36**

Define the space charge, the values of  $Q_{\text{sc}}$  charge and  $C_{\text{sc}}$  capacity for the following values of  $\phi_s$  surface potential:  $\phi_s = 0$ ;  $\phi_s = \phi_0$ ;  $\phi_s = 2\phi_0$ , for p- type of the silicon,  $\rho_{\text{Si}} = 10 \text{ Ohm} \cdot \text{cm}$ ,  $\mu_n = 1500 \text{ cm}^2/\text{V} \cdot \text{sec}$ ,  $\mu_p = 600 \text{ cm}^2/\text{V} \cdot \text{sec}$ ,  $\epsilon_s = 11.8$ ,  $\epsilon_0 = 8.85 \cdot 10^{-14} \text{ F/cm}$ ,  $n_i = 1.6 \cdot 10^{10} \text{ cm}^{-3}$ .

**4b37.**

Define the surface-state charge density  $Q_{\text{ss}}$  for p- type silicon  $N_a = 10^{18} \text{ cm}^{-3}$ ,  $T = 300 \text{ K}$ , according to the following values of surface potential:  $\phi_s = 0$ ;  $\phi_s = \phi_0$ ;  $\phi_s = 2\phi_0$ . Surface states are evenly distributed  $N_{\text{ss}} = 2 \cdot 10^{12} \text{ cm}^{-2} \cdot \text{eV}^{-1}$ . Compare the value of  $Q_{\text{ss}}$  with the appropriate charge of surface-state charge  $Q_{\text{sc}}$ .  $\epsilon_0 = 8.85 \cdot 10^{-14} \text{ F/cm}$ ,  $\epsilon_s = 11.8$ ,  $n_i = 1.6 \cdot 10^{10} \text{ cm}^{-3}$ .

**4b38.**

The density of donor in semiconductor which contains only donor impurity, is  $N_d$ , and their energy level is  $E_d$ . Find out the density of free electrons if the Fermi level coincides with  $E_d$ .

**4b39.**

Under the influence of light, homogenous distributed exceed charge carriers with the concentration of  $\Delta n$  occurred in semiconductor. The density of minority charge carriers is  $2.5 \cdot 10^{20} \text{ m}^{-3}$ , and initial speed decrease of the density is  $2.8 \cdot 10^{24} \text{ m}^{-3} \cdot \text{sec}^{-1}$ .

Define:

- The lifetime of minority charge carriers;
- The value of  $\Delta n$  after  $2 \mu\text{sec}$  when the light source had already been turned off.

**4b40.**

The mobility of electrons is  $0.38 \text{ m}^2/\text{V}\cdot\text{sec}$  in the germanium sample, and the mobility of holes is  $0.16 \text{ m}^2/\text{V}\cdot\text{sec}$ . The Hall effect is not seen in this sample. Which part of the current is conditioned by holes?

**4b41.**

As the concentration of impurity has changed, the mobility increased by 5% in non-degenerate semiconductor at constant temperature  $T=300 \text{ K}$ , and diffusion length increased by 10%. How did the lifetime of charge carriers change?

**4b42.**

The concentration of donors is  $N_d=2\cdot 10^{20} \text{ m}^{-3}$  in germanium sample. The effective mass of electron is  $m^*=1.57m_0$  ( $m_0$  is electron mass in vacuum). It can be considered that donor is dispersive center with  $5\cdot 10^{-2} \text{ }\mu\text{m}$  of radius. Find out the mean free path and time as well as electron mobility if  $T=300 \text{ K}$ .

**4b43.**

For the InSb crystal  $E_g = 0.23 \text{ eV}$  is the forbidden gap,  $\epsilon = 17$  is the dielectric permittivity,  $m = 0.015m_0$  is the electron effective mass. Determine the minimal concentration of donors when impurity band is originated.

**4b44.**

In the silicon with  $300\text{K}$  temperature electron mobility is  $\mu = 1500 \text{ cm}^2/\text{V}\cdot\text{c}$  and electron effective mass is  $m = 0.32m_0$ . Determine the electric field when band-to-band impact ionization takes place.

**4b45.**

Determine the coefficient of temperature expansion of semiconductor forbidden gap, if the product of electron and hole effective mass equals  $0.235m_0$ , intrinsic carriers' concentration equals  $2 \times 10^{16} \text{ cm}^{-3}$  at  $T_1 = 500\text{K}$  and  $8 \times 10^{12} \text{ cm}^{-3}$  at  $T_2 = 280\text{K}$ .

**4b46.**

Determine the energy of electron  $\epsilon$  that is reflected from the atomic (100) planes of the cubic crystalline lattice with constant  $a = 4 \text{ \AA}$  when the reflection angle is  $\theta = 45^\circ$ .

**4b47.**

Find silicon photodiode's idle state voltage if the radiation with  $F_0 = 10^{18} \frac{qv}{\text{cm}^2 \cdot \text{s}}$  intensity is observed in the active region of *p-n junction*. The external quantum output is 1, reflection coefficient 0. Electron and hole mobility equal  $\mu_n = 1300 \frac{\text{cm}^2}{\text{V}\cdot\text{v}}$ ,  $\mu_p = 500 \frac{\text{cm}^2}{\text{V}\cdot\text{v}}$  respectively, diffusion length of minority carriers  $L_n = 10^{-2} \text{ cm}$ ,  $L_p = 6 \cdot 10^{-3} \text{ cm}$ , electron charge  $q = 1.6 \cdot 10^{-19} \text{ C}$ , thermal energy  $kT = 0,026 \text{ eV}$ . Also given densities of majority carriers in *n* and *p* domains  $p_p = N_a = 10^{15} \text{ cm}^{-3}$ ,  $n_n = N_d = 5 \cdot 10^{15} \text{ cm}^{-3}$ , density of intrinsic charge  $n_i = p_i = 1.4 \cdot 10^{10} \text{ cm}^{-3}$ .

**4b48.**

Given silicon *p-n junction*. The position of Fermi level in *n* region is  $E_C - E_F = 0,2 \text{ eV}$ , and in *p* region it is  $E_F - E_V = 0,1 \text{ eV}$ . Density of energetic states in conduction and valence regions  $N_C = 2.8 \cdot 10^{19} \text{ cm}^{-3}$ ,  $N_V = 1.02 \cdot 10^{19} \text{ cm}^{-3}$ . Also given intrinsic carriers' density in a silicon  $n_i = p_i = 1.6 \cdot 10^{10} \text{ cm}^{-3}$  and room temperature energy  $kT = 0,026 \text{ eV}$ . Define the contact difference of *p-n junction* potentials.

**4b49.**

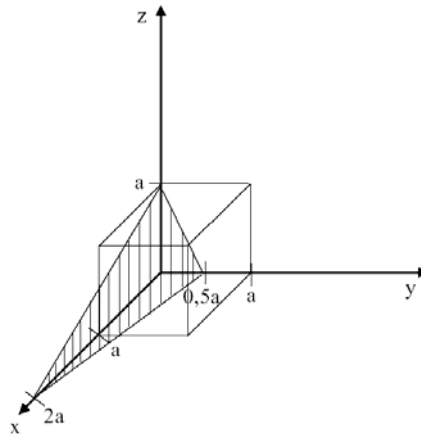
Define the charge capacitance of silicon structure of two backward potential barriers if the depletion layers have symmetric distribution and contact edge. The total width of depletion layers  $d = 10^{-4} \text{ cm}$ , and the area of *p-n junction*  $S \approx 10^{-4} \text{ cm}^2$ . Accept the dielectric transparency of vacuum  $\epsilon_0 = 8.86 \times 10^{-12} \text{ F/m}$ , and the one of Si  $\epsilon = 12$ .

**4b50.**

Barrier capacitance of *p-n junction* is  $C_1 = 100 \text{ pF}$  for  $2\text{V}$  backward bias voltage. What value will the capacitance take in case of decreasing  $V$  voltage twice? The height of potential barrier of *p-n junction* is  $\phi = 0.6\text{V}$ .

**4b51.**

Determine Miller indices of the shaded plane in a cubic crystal.

**4b52.**

n-type silicon has the following sizes: length is 10mm, width is 2mm and thickness is 1mm. Mobilities for electron and hole are 0.12 and 0.05m<sup>2</sup>/(V-s) respectively. The intrinsic carrier density is  $n_i = 1.5 \cdot 10^{16} \text{ m}^{-3}$  and the elementary charge is  $q = 1.6 \cdot 10^{-19} \text{ C}$ . Determine the donor impurities concentration  $N_d$  in the substrate when the resistance is  $R = 150 \Omega$ .

**4b53.**

Prove that semiconductor at the given temperature has minimum conductivity if the electron density is  $n = n_i \sqrt{\mu_p / \mu_n}$ . Here  $n$  and  $n_i$  are intrinsic densities and  $\mu_p$  and  $\mu_n$  are hole and electron motilities respectively.

**4b54.**

Conductivities of p and n regions for germanium p-n-junction are  $\sigma_p = 10^4 (\Omega \cdot m)^{-1}$  and  $\sigma_n = 10^2 (\Omega \cdot m)^{-1}$  respectively. Electron and hole mobilities are 0.39 and 0.19m<sup>2</sup>/(V-s), intrinsic carrier density is  $n_i = 2.5 \cdot 10^{19} \text{ m}^{-3}$  and elementary charge is  $q = 1.6 \cdot 10^{-19} \text{ C}$ . Calculate the diffusion potential at  $T = 300 \text{ K}$ .

**4b55.**

Silicon diode with saturation current  $I_0 = 25 \mu\text{A}$  operates at 0.1V forward voltage, when  $T = 300 \text{ K}$ . Determine the diode's DC resistance.

**4b56.**

In wet oxidation of silicon at 950°C the following data are obtained:

|                                        |       |       |       |       |       |
|----------------------------------------|-------|-------|-------|-------|-------|
| t (hour)                               | 0.11  | 0.30  | 0.40  | 0.50  | 0.60  |
| do (oxide thickness in $\mu\text{m}$ ) | 0.041 | 0.100 | 0.128 | 0.153 | 0.177 |

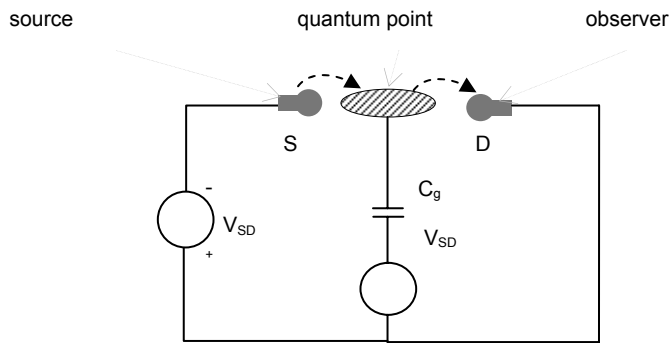
Show how to graphically determine the linear and parabolic rate constants from these experimental data.

**4b57.**

Boron diffusion is provided from a "limited source" with the total amount of impurities  $Q = 2,25 \cdot 10^{13} \text{ atom/cm}^2$  in  $t = 2$  hour. Diffusion coefficient equals  $D = 9,2 \cdot 10^{-13} \text{ cm}^2/\text{s}$ . Diffusion is provided into the bulk silicon substrate with the impurity concentration  $N_D = 1 \cdot 10^{16} \text{ atoms/cm}^3$ . Calculate the depth of p-n junction  $x_j$  in micrometers.

**4b58.**

The nanostructure of field transistor type consists of quantum point which is connected to two current conductors by tunnel current – electron's source and observer (Figure).



When  $V_{SD}$  voltage is applied between the source and the observer, current starts to flow through the net which is conditioned by the tunnel junction of electrons from the source into the quantum point, and then from the quantum point to the observer.

The second electrode of the transistor - gate,  $C_g$  is connected to the quantum point by capacitive link and it is possible to control the current that flows through the source-quantum point – observer by the applied  $V_g$  voltage.

Considering that the quantum point is  $r=10\text{nm}$  radius, and the cut set of current source and observer are of the same type, estimate, due to Coulomb blockade, how much it is necessary to change the gate voltage such that from the source to the quantum point after one electron tunnel junction, the second, the third and other electrical tunnel junctions are possible, due to which I-V dependence will look as Coulomb degree.

**4b59.**

Considering the silicon photodiode as an ideal photoreceiver (i.e. by an internal quantum output equal to one) find current and voltage values in its output when the receiver operates in the mode of photocurrent on one hand, and photoelshu on the other hand. The photoreceiver operates in a room temperature  $T=300\text{K}$ , light  $P = 10\text{mW}$  power  $\lambda=0,8 \mu\text{m}$  wavelength monochromatic light. It is known that backward bias current of the diode is  $I_0=10\text{nA}$ .

**4b60.**

In p-n-p bipolar transistor base, donor mixtures are distributed nonhomogeneously - exponentially c

$$N_D = N_0 \exp\left(-\frac{x}{L_0}\right), \text{ where } L_0 \text{ is the length characterizing that distribution, } W \text{ is the width of the base, and}$$

$L_0 < W$ . Estimate the tenseness of an internal electrical field in the base as well as the ratio of drift and diffusion times of moving through the hole base.

## 5. SEMICONDUCTOR TECHNOLOGY

### a) Test questions

- 5a1.** What is the number of photomasks defined by during the fabrication of the given integrated circuit?  
A. *Photomask wearability*  
B. *Number of simultaneously processing semiconductor wafers*  
C. *Number of topological layers formed on the wafer*  
D. *Minimum of feature size*  
E. *Production volume*
- 5a2.** What semiconductor material has the maximum band gap?  
A. *Si*  
B. *Ge*  
C. *GaAs*  
D. *SiC*  
E. *The correct answer is missing*
- 5a3.** The performance of a MOS transistor can be increased by:  
A. *Increasing the temperature*  
B. *Decreasing the temperature*  
C. *Changing the channel doping level*  
D. *Decreasing thickness of dielectric layer*  
E. *Increasing the gate voltage*
- 5a4.** Based on what semiconductor is it possible to make p-n junctions having higher operating temperatures?  
A. *Ge,  $E_g=0,66$  eV*  
B. *Si,  $E_g=1,12$  eV*  
C. *GaAs,  $E_g=1,45$  eV*  
D. *SiC,  $E_g= 3,1$  eV*  
E. *The correct answer is missing*
- 5a5.** The best resolution to form a topological pattern is  
A. *X-ray lithography*  
B. *Electron beam lithography*  
C. *Photolithography*  
D. *Chemical lithography*  
E. *All the answers are correct*
- 5a6.** What kind of p-n junction can be produced by epitaxial technology?  
A. *Linearly graded p-n junction*  
B. *p-n junction by exponent function*  
C. *Abrupt p-n junction*  
D. *All the answers are correct*  
E. *The correct answer is missing*
- 5a7.** Operating frequency of bipolar transistors can be increased by  
A. *Increasing emitter effectiveness*  
B. *Decreasing base width*  
C. *Applying collector large voltages*  
D. *Decreasing base impurity level*  
E. *Increasing signal frequency*
- 5a8.** Charge capacitance is larger than diffusion  
A. *Because it does not depend on the frequency*  
B. *In reverse-biased mode of p-n junction,*  
C. *In forward-biased mode of p-n junction*  
D. *Because it depends on the value of applied voltage*  
E. *Because it depends on the value of current*
- 5a9.** p-n junction current in reverse-biased mode can be reduced by:  
A. *The reduction of the density of impurities*  
B. *The reduction of temperature*  
C. *The reduction of contact potentials' difference*  
D. *The reduction of applied voltage*  
E. *The size reduction of p and n regions*

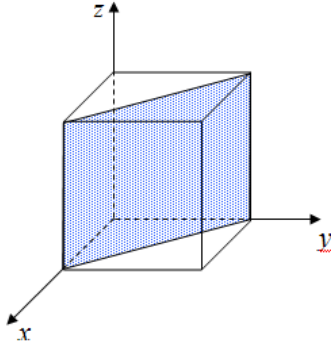
- 5a10.** Which semiconductor will have higher concentration of intrinsic carriers in the given temperature, for example  $T=300K$ ?
- Ge,  $E_g=0,66$  eV*
  - Si,  $E_g=1,12$  eV*
  - GaAs,  $E_g=1,45$  eV*
  - SiC  $E_g= 3,1$ eV*
  - All the answers are correct*
- 5a11.** Diffusion first step (drive in) realization provides:
- High surface density and large diffusion depth of impurities*
  - High surface density and small diffusion depth of impurities*
  - Low surface density and large diffusion depth of impurities*
  - Low surface density and small diffusion depth of impurities*
  - Low surface density of impurities*
- 5a12.** In semiconductor crystals dislocations are classified as:
- Schottky defects*
  - Linear structural defects*
  - Frenkel defects*
  - Surface structural defects*
  - Volume structural defects*
- 5a13.** Epitaxial growth technology of layers allows receiving:
- Linearly graded p-n junction*
  - p-n junction by exponent function*
  - Abrupt p-n junction*
  - p-n junction by arbitrary function*
  - p-n junction by quadratic function*
- 5a14.** Zone-melting method advantage of silicon monocrystal growth over Chokhralsi method is conditioned by:
- Absence of quartz melting tube*
  - Low level of thermal gradient*
  - High speed of monocrystal growth*
  - Low temperature of the process*
  - Presence of inexpensive equipment*
- 5a15.** For a diffusion from the infinite source the diffusion depth  $x_j$  depends on diffusion time  $t$  by the following expression:
- $x_j \sim t$*
  - $x_j \sim t^{1/2}$*
  - $x_j \sim t^2$*
  - $x_j \sim t^3$*
  - $x_j \sim \exp(t)$*
- 5a16.** The saturation of drain current on the output characteristic of p-n junction FET is determined by:
- Self-restriction effect if drain current increase*
  - Velocity saturation of channel majority carriers caused by drain voltage*
  - Density Impurities in the channel*
  - A and B*
  - All the answers are correct*
- 5a17.** What is diode's I-V characteristic linearity conditioned by:
- Ohmic contacts of diode*
  - Crystal structure of output material*
  - Contact potential difference of p-n junction*
  - Density of impurities in the base*
  - Width of output material band gap*
- 5a18.** The oxide layer of a MOS transistor is formed by the following method:
- Chemical*
  - Ion implantation*
  - Epitaxial deposition*
  - Thermal oxidation*
  - Diffusion*

- 5a19.** When a diode is forward biased, which carriers create current?
- Ions of impurity atoms*
  - Surface charges*
  - Majority carriers*
  - Minority carriers*
  - Free electrons of the base*
- 5a20.** Temperature of diffusion process in silicon IC technology is
- Less than 800°C*
  - higher than 1500°C*
  - In 1100...1300°C range*
  - Independent of temperature*
  - All the answers are correct*
- 5a21.** The current-voltage characteristic of p-n junction
- Is linear*
  - Is strictly non linear*
  - Forward current value smaller than reverse current*
  - Reverse current value higher than forward current*
  - Independent of temperature*
- 5a22.** In case of which connection does the bipolar transistor provide simultaneous amplification of current, voltage and power?
- Common collector*
  - Common base*
  - Common emitter*
  - When the transistor is used by separated base*
  - When emitter and base are short connected*
- 5a23.** What is the load resistance of a bipolar transistor look like connected by common base?
- Larger than the resistance of collector p-n junction*
  - Smaller than the resistance of collector p-n junction*
  - Larger than the resistance of emitter p-n junction*
  - Equal to base resistance*
  - Equal to the resistance of input contact*
- 5a24.** For an intrinsic semiconductor the electron concentration is  $10^{14} \text{ cm}^{-3}$ . What is the hole concentration?
- Higher than  $10^{14} \text{ cm}^{-3}$*
  - Less than  $10^{14} \text{ cm}^{-3}$*
  - Equal to electron concentration*
  - All the answers are correct*
  - All the answers are wrong*
- 5a25.** Semiconductor structures of  $A_3B_5$  (GaAs, InP, InAs, GaSb) type are made by:
- Heteroepitaxial growth method*
  - Diffusion method*
  - Homoepitaxial growth method*
  - Ion Implantation method*
  - Vacuum deposition method*
- 5a26.** Speed of silicon thermal oxidation is limited by:
- Speed of surface adsorption of oxidants ( $O_2, H_2O$ )*
  - Speed of diffusion oxidants through the  $SiO_2$  layer to Si- $SiO_2$  interface*
  - Speed of oxidation reaction with silicon*
  - Speed of diffusion of gas results of silicon surface reaction*
  - All the answers are correct*
- 5a27.** Thickness of silicon dioxide  $d$  is dependent on duration  $t$  of high-temperature oxidation by the following expression:
- $d \sim t^{1/2}$*
  - $d \sim t$*
  - $d \sim t^2$*
  - $d \sim t^3$*
  - $d \sim \exp(t)$*



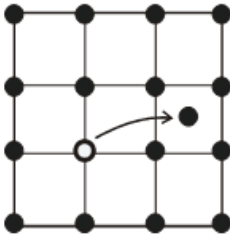
- 5a28. What is the technology roadmap of photolithography process?  
 B. Photoresist coating - alignment - development - exposure - etching  
 C. Photoresist coating - development - alignment - exposure - etching  
 D. Photoresist coating - alignment - exposure - etching - development  
 E. Photoresist coating - exposure - alignment - etching - development  
 E. Photoresist coating - alignment - exposure - development - etching

- 5a29. What crystal plane is shaded in cubic crystal?



- A. (001)  
 B. (100)  
 C. (101)  
 D. (110)  
 E. (111)

- 5a30. What structural defect is figured?



- A. Shottky defect  
 B. Linear structural defect  
 C. Frenkel defect  
 D. Surface structural defect  
 E. Volume structural defect.

- 5a31. Sub-collector n+ buried layer in n-p-n bipolar transistor structures is intended for:

- A. Reducing the bulk resistance of lateral collector  
 B. Increasing the bulk resistance of lateral collector  
 C. Increasing the transient resistance of collector-emitter  
 D. Reducing the transient resistance of collector-emitter  
 E. Reducing the minority-carrier lifetime in collector

- 5a32. To obtain n-type semiconductor silicon in microelectronic processing the following are used as an impurity:

- A. Elements of fifth group of periodic table  
 B. Elements of forth group of periodic table  
 C. Elements of third group of periodic table  
 D. Elements of first group of periodic table  
 E. Elements of sixth group of periodic table

- 5a33. To obtain p-type semiconductor silicon in microelectronic processing the following are used as an impurity:

- A. Elements of forth group of periodic table  
 B. Elements of sixth group of periodic table  
 C. Elements of third group of periodic table  
 D. Elements of fifth group of periodic table  
 E. Elements of second group of periodic table

- 5a34. In IC technology the photolithographic process is meant for:

- A. Forming the picture of topological layer on the substrate surface  
 B. Getting thin metallic films on the substrate surface  
 C. Getting thin dielectric films on the substrate surface  
 D. Getting thick oxide films on the substrate surface  
 E. Getting thin metallic and dielectric films on the substrate surface

- 5a35. In bipolar semiconductor ICs the electric coupling between layers of the multilevel metalization is realized by:

- A. Diffusion structures  
 B. Metallic through holes  
 C. Electric coupling between layers is absent

- D. External metallic conductors*  
*E. The correct answer is missing*
- 5a36.** In bipolar semiconductor ICs the resistors are formed:  
*A. in terms of base region*  
*B. In terms of emitter region*  
*C. In terms of base- emitter junction*  
*D. A and B answers are correct*  
*E. A, B and C answers are correct*
- 5a37.** The design rules for ICs:  
*A. Are used to design applications specific integrated circuits*  
*B. Define the minimal sizes of the elements and spacing between them during layout design*  
*C. Are non dependent on level of manufacturing process*  
*D. Are created by designer using design expertise*  
*E. The correct answer is missing*
- 5a38.** In real p-n junctions beyond the defined value of the reverse voltage the reverse current:  
*A. Sharply decreases*  
*B. Slowly decreases*  
*C. Sharply rises*  
*D. Slowly rises*  
*E. Remains practically constant*
- 5a39.** In semiconductor ICs to form the doped regions with required type and conductance in the substrate the following methods are used:  
*A. Diffusion and ion implantation*  
*B. Ion implantation and thermal oxidation*  
*C. Ion etching and diffusion*  
*D. Diffusion and thermal oxidation*  
*E. The correct answer is missing*
- 5a40.** For the real p-n junction the main breakdown mechanisms are:  
*A. Thermal, tunnel and avalanche*  
*B. Tunnel, mechanical and thermal*  
*C. Thermal, radiation and chemical*  
*D. Avalanche, and electron-beam*  
*E. Thermal, electrochemical and tunnel*
- 5a41.** For a MOS structure transistor the minimal thickness of the oxide layer is limited by:  
*A. Capabilities of the manufacturing technology*  
*B. Values of the unwanted tunnel currents*  
*C. Critical electric field strength*  
*D. Defects concentration*  
*E. All the answers are correct*
- 5a42.** To compensate the radiation defects after the ion implantation doping the following processes are necessary:  
*A. The additional doping with the donor impurities*  
*B. The additional doping with the acceptor impurities*  
*C. Thermal treatment in (600....800)<sup>o</sup>C range*  
*D. Thermal treatment in (1300....1500)<sup>o</sup>C range*  
*E. The mechanical planarization of the crystal surface*
- 5a43.** In case of IC scaling by  $\alpha > 1$  factor the power density changes in the following way:  
*A. Decreases by  $\alpha$  time*  
*B. Increases by  $\alpha$  time*  
*C. Remains constant*  
*D. Increases by  $\alpha^2$  time*  
*E. Decreases by  $\alpha^2$  time*
- 5a44.** As a MOS-resistor forming region in MOS IC, the following are used:  
*A. Transistor's channel*  
*B. Transistor's source*  
*C. Transistor's drain*  
*D. Gate's insulator*  
*E. Metal gate*

- 5a45.** To reduce the resistivity of the polysilicon in IC technology the following is performed:  
 A. Impurities additional doping and silicide process  
 B. Ion etching and impurities additional doping  
 C. Silicide process and ion etching  
 D. Surface oxidation and ion etching  
 E. Silicide process and thermal oxidation
- 5a46.** For an n-type silicon the donor impurity concentration is  $N_d=10^{16} \text{ cm}^{-3}$ . What kind and which concentration of impurity will be used to form p-n junction in this sample?  
 A. Boron with  $N > 10^{16} \text{ cm}^{-3}$   
 B. Boron with  $N < 10^{16} \text{ cm}^{-3}$   
 C. Boron with  $N = 10^{16} \text{ cm}^{-3}$   
 D. Phosphorus with any concentration  
 E. Phosphorus with  $N > 10^{16} \text{ cm}^{-3}$
- 5a47.** For a p-type silicon the acceptor impurity concentration is  $N_a=10^{15} \text{ cm}^{-3}$ . What kind and which concentration of impurity will be used to form p-n junction in this sample?  
 A. Phosphorus with  $N = 10^{15} \text{ cm}^{-3}$   
 B. Phosphorus with  $N < 10^{15} \text{ cm}^{-3}$   
 C. Phosphorus with  $N > 10^{15} \text{ cm}^{-3}$   
 D. Boron with any concentration  
 E. Boron with  $N > 10^{15} \text{ cm}^{-3}$

## **b) Problems**

### **5b1.**

For a silicon p-n junction the specific resistances of p and n-regions are  $10^{-4} \text{ Ohm}\cdot\text{m}$  and  $10^{-2} \text{ Ohm}\cdot\text{m}$  correspondingly. Calculate the contact potential of the junction at a room temperature when  $T=300\text{K}$  if the mobility of holes and electrons are  $0,05 \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$  and  $0,13 \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$ . The intrinsic concentration at a room temperature equals  $1,38 \times 10^{16} \text{ m}^{-3}$ .

### **5b2.**

Calculate the density of electrons and holes in a p-Ge at a room temperature if the sample specific conductance equals  $100 \text{ S/cm}$ , mobility of holes is  $1900 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$ , and  $n_i=2,5 \times 10^{13} \text{ atom/cm}^3$ .

### **5b3.**

The silicon sample is doped with a donor impurity  $N_d=10^{17} \text{ atom/cm}^3$ . The sample length equals  $100 \mu\text{m}$ , width  $10 \mu\text{m}$ , and thickness  $1 \mu\text{m}$ . Calculate the resistance and sheet resistance.

### **5b4.**

What thickness of  $\text{SiO}_2$  layer is required to fabricate a MOS capacitor with a specific capacity of  $100 \text{ nF/cm}^2$  ( $\epsilon_{\text{SiO}_2}=3,8 \times 8,85 \times 10^{-14} \text{ F/cm}$ ).

What oxidation process (wet or dry) would be used to grow the gate high quality oxide?

### **5b5.**

The gate capacitance of the MOS transistor equals  $C$ . The capacitor structure is scaled by the factor  $\alpha = 2$ . How will the gate capacitance change?

### **5b6.**

Semiconductor's band gap width  $E_g = 0.7 \text{ eV}$  and the temperature does not change when  $T_1 = 250 \text{ K}$  changes to  $T_2 = 300 \text{ K}$ .  $V=0.4 \text{ V}$  forward bias voltage is applied to the p-n junction created in it. Define  $j_2/j_1$  change of current density in the mentioned range of temperature change if in case of  $T_2 = 300 \text{ K}$ ,  $kT = 0.026 \text{ eV}$ .

### **5b7.**

How will the channel of a p-n junction field effect transistor change when the drain voltage is  $V=+0.1 \text{ V}$  if the dielectric transparency of semiconductor  $\epsilon = 12$ , and for vacuum  $\epsilon_0 = 8.86 \cdot 10^{-12} \text{ F/m}$ , contact difference of potentials  $\phi_k=0.6 \text{ V}$ , electron charge  $q=1.6 \cdot 10^{-19} \text{ C}$ , and donor density in n channel  $N_d=10^{15} \text{ cm}^{-3}$ . Ignore the effects of source-drain and gate-drain domains.

### **5b8.**

Calculate the drain current of a silicon n-MOSFET for the following conditions:  $V_t = 1 \text{ V}$ , gate width  $W = 10 \mu\text{m}$ , gate length  $L = 1 \mu\text{m}$  and oxide thickness  $t_{\text{ox}} = 10 \text{ nm}$ . The device is biased with  $V_{\text{GS}} = 3 \text{ V}$  and  $V_{\text{DS}} = 5 \text{ V}$ . Use the device quadratic model, a surface mobility of  $300 \text{ cm}^2/\text{V}\cdot\text{s}$  and  $V_{\text{BS}} = 0 \text{ V}$ .

**5b9.**

For a silicon n-channel MOS FET the source-drain distance is  $1\mu\text{m}$  and the doping levels are  $N_d = 10^{20}\text{ cm}^{-3}$ . The substrate doping is  $N_a = 10^{16}\text{ cm}^{-3}$ . Assume that the source and substrate are grounded. At what  $V_D$  voltage on the drain the depletion widths of source and drain p-n junctions will meet (punch-through effect)?

**5b10.**

The donor impurity concentration in the silicon substrate changes linearly  $N=kx\text{ [cm}^{-3}\text{]}$ , where  $k=8 \times 10^{18}\text{ cm}^{-4}$  and  $x$  is a distance. Calculate electrons diffusion current density when the electric field is absent. Electrons mobility at room temperature ( $T=300\text{ K}$ ) is  $1200\text{ cm}^2/\text{V}\cdot\text{s}$ .

**5b11.**

For a silicon  $n^+ - p - n$  bipolar transistor the base width is  $W_B=0,6\mu\text{m}$ , the doping levels of emitter, collector and base are  $N_{de} = 10^{19}\text{ cm}^{-3}$ ,  $N_{dc} = 5 \times 10^{16}\text{ cm}^{-3}$  and  $N_{ab} = 5 \times 10^{15}\text{ cm}^{-3}$  respectively. Define the collector critical voltage, when the device becomes uncontrolled (punch-through in the base region). Assume  $T=300\text{ K}$  and intrinsic concentration is  $n_i = 1,5 \times 10^{10}\text{ cm}^{-3}$ .

**5b12.**

The intrinsic silicon wafer is doped with donor  $N_d = 10^{17}\text{ cm}^{-3}$  and acceptor  $N_a = 10^{16}\text{ cm}^{-3}$  impurities simultaneously. What is the silicon polarity of conductivity? Calculate the specific conductivity of the sample at room temperature  $T=300\text{ K}$ . The electrons mobility at room temperature is  $1400\text{ cm}^2/\text{V}\cdot\text{s}$ , and electron charge is  $1,6 \times 10^{-19}\text{ C}$ .

**5b13.**

Estimate the time delay in a  $n^+$  polysilicon interconnect for the following conditions: the line length  $l=1\text{mm}$ , width  $b=1\mu\text{m}$ , the sheet resistance of polysilicon  $\rho_{\square}=200\ \Omega/\square$ , and the specific capacity of the line to the substrate  $C_0=60\text{ aF}/\mu\text{m}^2$ .

**5b14.**

Estimate the time delay in the metal interconnect (metal 1) for the following conditions: the line length  $l=1\text{mm}$ , width  $b=200\text{ nm}$ , the metal sheet resistance  $\rho_{\square}=0,1\ \Omega/\square$ , and the specific capacity of the line to the substrate  $C_0=23\text{ aF}/\mu\text{m}^2$ .

**5b15.**

Consider a one-sided silicon p-n diode with  $N_d = 10^{16}\text{ cm}^{-3}$  and  $N_a = 10^{18}\text{ cm}^{-3}$ . Calculate the p-n junction barrier layer capacitance for applied reverse voltages  $U_1=0\text{ V}$  and  $U_2=-5\text{ V}$ . Assume the contact potential of p-n junction at room temperature is  $0,7\text{ V}$ , relative permittivity of silicon  $\epsilon_{Si}=11,8$ , dielectric constant  $\epsilon_0=8,85 \times 10^{-14}\text{ F/cm}$  and the junction area  $S_{p-n} = 10^{-5}\text{ cm}^2$ .

## 6. NUMERICAL METHODS AND OPTIMIZATION

### a) Test questions

**6a1.** Fragmentation consecutive algorithm, compared with iteration algorithm, has:

- A. *More accuracy*
- B. *Higher performance*
- C. *Less accuracy*
- D. *More machine time*
- E. *B and C together*

**6a2.** Which is the inverse polynomial of  $X^4+X+1$ ?

- A.  $X^3+X+1$
- B.  $X^4+X+1$
- C.  $X^4+X^2+1$
- D.  $X^4+X^3+1$
- E. *The correct answer is missing*

**6a3.** Which of the given intrinsic description forms more accurately describes an electrical circuit?

- A. *The graph of commutation schema*
- B. *Complex list*
- C. *Adjacency matrix*
- D. *A and B equally*
- E. *B and C equally*

**6a4.** The following problem of linear programming:

$$2x_1 + 3x_2 \rightarrow \max$$

$$\begin{cases} 7x_1 + 6x_2 \leq 42 \\ -x_1 + 5x_2 \leq 15 \\ x_1 \geq 0, x_2 \geq 0 : \end{cases}$$

- A. *Does not have solution*
- B. *Has one solution*
- C. *Has two solutions*
- D. *Has solutions with infinite set*
- E. *Has unlimited solutions*

**6a5.** In case of which values of parameter  $\lambda$  the following problem has infinite set of solutions?

$$f(X) = \lambda x_1 + x_2 \rightarrow \min_{X \in D}$$

$$D: \begin{cases} x_1 + 2x_2 \geq 2, \\ x_1 - 4x_2 \leq 2, \\ 6x_1 + 5x_2 \leq 30, \\ x_1 \geq 0, x_2 \geq 0: \end{cases}$$

- A. 2
- B. 1/2
- C. 1
- D. 1/4
- E. 1/8

**6a6.** In case of which values of  $\lambda_1, \lambda_2$  parameters the following problem has one solution?

$$f(X) = \lambda_1 x_1 + \lambda_2 x_2 \rightarrow \min_{X \in D}$$

$$D: \begin{cases} x_1 + 2x_2 \geq 2, \\ x_1 - 4x_2 \leq 2, \\ 6x_1 + 5x_2 \leq 30, \\ x_1 \geq 0, x_2 \geq 0: \end{cases}$$

- A.  $\lambda_1 = 1, \lambda_2 = 4$
- B.  $\lambda_1 = 1/2, \lambda_2 = 1$
- C.  $\lambda_1 = 2, \lambda_2 = 4$
- D.  $\lambda_1 = 4, \lambda_2 = 8$
- E.  $\lambda_1 = 1, \lambda_2 = 2$

6a7. In case of what values of  $\lambda$  parameters the following problem does not have a solution?

$$f(X) = x_1 + x_2 + x_3 \rightarrow \max_{X \in D},$$

$$D : \begin{cases} x_1 + x_3 = 2, \\ x_1 + 2\lambda x_2 + x_3 = 0, \\ x_1 \geq 0, x_2 \geq 0, x_3 > 0: \end{cases}$$

- A. 0
- B. 5
- C. 1
- D. 1/2
- E. 2

6a8. In case of what values of  $b_{21}$  and  $b_{22}$  elements

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & 2 \\ 4 & 2 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 \\ b_{21} & b_{22} \\ 0 & 1 \end{bmatrix}$$

matrix system will not be fully controllable?

- A.  $b_{21} \neq 0, b_{22} = 0$
- B.  $b_{21} = 0, b_{22} = 0$
- C.  $b_{21} \neq 0, b_{22} \neq 0$
- D.  $b_{21} = 0, b_{22} \neq 0$
- E. The correct answer is missing

6a9. In case of which values of  $b_{11}$  and  $b_{12}$  elements

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 0 \\ 0 & 2 & 4 \end{bmatrix}, B = \begin{bmatrix} b_{11} & b_{12} \\ 1 & 2 \\ 2 & 0 \end{bmatrix}$$

matrix system will not be fully controllable?

- A.  $b_{11} = 0, b_{12} = 0$
- B.  $b_{11} = 1, b_{12} = 2$
- C.  $b_{11} = 2, b_{12} = 0$
- D.  $b_{11} = 1, b_{12} = 0$
- E.  $b_{11} = 0, b_{12} = 1$

6a10. In case of which values of  $c_{11}$  and  $c_{12}$  elements

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 2 & 0 \\ 1 & 1 & 1 \end{bmatrix}, C = \begin{bmatrix} c_{11} & c_{12} & 0 \\ 1 & 2 & 3 \end{bmatrix}$$

matrix system will not be fully observable?

- A.  $c_{11} = 0, c_{12} = 0$
- B.  $c_{11} \neq 0, c_{12} = 0$
- C.  $c_{11} = 0, c_{12} \neq 0$
- D.  $c_{11} \neq 0, c_{12} \neq 0$
- E. The correct answer is missing

6a11. In case of what values of  $b_{11}$ ,  $b_{21}$  and  $b_{31}$  elements

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{bmatrix}, B = \begin{bmatrix} b_{11} & 0 \\ b_{21} & 2 \\ b_{31} & 1 \end{bmatrix}$$

matrix system will not be normal system?

- A.  $b_{11} = 0, b_{21} = 0, b_{31} = 0$
- B.  $b_{11} = 0, b_{21} \neq 0, b_{31} = 1$
- C.  $b_{11} \neq 0, b_{21} = 0, b_{31} = 2$
- D.  $b_{11} = 1, b_{21} = 1, b_{31} = 1$
- E.  $b_{11} = 2, b_{21} \neq 1, b_{31} = 0$

**6a12.** For the given game model: for which values of  $p$  and  $q$  the expression  $(i_0, j_0) = (2, 2)$  is the best strategy?

$$\begin{bmatrix} 1 & q & 6 \\ p & 5 & 10 \\ 6 & 2 & 3 \end{bmatrix}$$

- A.  $p \leq 5$  and  $q \geq 5$
- B.  $p \leq 5$  and  $q \leq 5$
- C.  $p \geq 5$  and  $q \leq 5$
- D.  $p \geq 5$  and  $q \geq 5$
- E. The correct answer is missing

**6a13.** For the given game model, for which values of  $n$  and  $m$   $(i_0, j_0) = (2, 2)$  is the best strategy?

$$\begin{bmatrix} 0 & m & 5 \\ n & 4 & 9 \\ 5 & 1 & 2 \end{bmatrix}$$

- A.  $n \leq 4$  and  $m \geq 4$
- B.  $n \leq 4$  and  $m \leq 4$
- C.  $n \geq 4$  and  $m \leq 4$
- D.  $n \geq 4$  and  $m \geq 4$
- E. The correct answer is missing

**6a14.** For the given game model, in case of which values of  $p$  and  $q$   $(i_0, j_0) = (2, 2)$  will be the best strategy?

$$\begin{bmatrix} 4 & 6 & 3 \\ 15 & 11 & q \\ 2 & p & 9 \end{bmatrix}$$

- A.  $p \geq 11$  and  $q \leq 11$
- B.  $p \leq 11$  and  $q \leq 11$
- C.  $p \geq 11$  and  $q \geq 11$
- D.  $p \leq 11$  and  $q \geq 11$
- E. The correct answer is missing

**6a15.** For the given game model, for which values of  $n$  and  $m$ ,  $(i_0, j_0) = (2, 2)$  is the best strategy?

$$\begin{bmatrix} 5 & m & 10 \\ n & 8 & 14 \\ 10 & 6 & 7 \end{bmatrix}$$

- A.  $n \leq 8$  and  $m \geq 8$
- B.  $n \leq 8$  and  $m \leq 8$
- C.  $n \geq 8$  and  $m \leq 8$
- D.  $n \geq 8$  and  $m \geq 8$
- E. The correct answer is missing

**6a16.** What is the form of  $u_{1opt}(t)$ ,  $u_{2opt}(t)$  if

$$I = \int_0^T u_2^2(t) dt \rightarrow \min_{u_1(t), u_2(t)},$$

$$\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{pmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \cdot \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \cdot \begin{pmatrix} u_1(t) \\ u_2(t) \end{pmatrix},$$

$$c(u_1(t), u_2(t)) = u_1^2(t) + u_2^3(t) \leq 0,$$

whereas  $\psi_1(t)$  and  $\psi_2(t)$  are conjugated variables, and  $\mu$  - is a special multiplier.

- A.  $u_{1opt}(t) = -\frac{1}{2} \cdot \frac{\psi_2(t)}{\mu}$ ,  $u_{2opt}(t) = \frac{1 \pm \sqrt{1 + 12(\psi_1(t) + \psi_2(t))}}{6\mu}$
- B.  $u_{1opt}(t) = \psi_1(t) + \psi_2(t)$ ,  $u_{2opt}(t) = \frac{1}{2\mu}(\psi_1(t) - \psi_2(t))$
- C.  $u_{1opt}(t) = \frac{\psi_1(t)}{\mu}$ ,  $u_{2opt}(t) = \psi_1(t) \cdot \psi_2(t)$
- D.  $u_{1opt}(t) = \psi_1(t)$ ,  $u_{2opt}(t) = \frac{\psi_1(t)}{\psi_2(t)} \cdot \mu$
- E. *The correct answer is missing*

**6a17.** Which is the system of complement variables if

$$I = \int_0^T u^2(t) dt \rightarrow \min_{u(t)},$$

$$\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{pmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \cdot \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} + \begin{pmatrix} 1 \\ 2 \end{pmatrix} \cdot u(t).$$

- A.  $\dot{\psi}_1(t) = -\psi_1(t) + \psi_2(t)$ ,  $\dot{\psi}_2(t) = \psi_1(t) - \psi_2(t)$
- B.  $\dot{\psi}_1(t) = \psi_1(t) \cdot \psi_2(t)$ ,  $\dot{\psi}_2(t) = \frac{\psi_1(t)}{\psi_2(t)}$
- C.  $\dot{\psi}_1(t) = -\psi_1(t) \cdot \psi_2(t)$ ,  $\dot{\psi}_2(t) = -\frac{\psi_1(t)}{\psi_2(t)}$
- D.  $\dot{\psi}_1(t) = -2 \cdot \psi_1(t) - \psi_2(t)$ ,  $\dot{\psi}_2(t) = -\psi_1(t) - 2 \cdot \psi_2(t)$
- E. *The correct answer is missing*

**6a18.** What is the form of  $u_{opt}(t)$  if

$$I = \int_0^T x_1^2(t) dt \rightarrow \min_{u(t)},$$

$$\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{pmatrix} = \begin{bmatrix} 2 & 2 \\ 1 & 2 \end{bmatrix} \cdot \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \cdot u^2(t),$$

$$c(x_1(t)) = x_1^2(t) + x_1(t) \leq 0,$$

whereas  $\psi_1(t)$  and  $\psi_2(t)$  are conjugated variables, and  $\mu$  - is a special multiplier.

- A.  $u_{opt}(t) = \frac{\psi_1(t)}{\mu + \psi_2(t)}$
- B.  $u_{opt}(t) = \frac{\psi_2(t)}{\mu + \psi_1(t)}$
- C.  $u_{opt}(t) = \psi_1(t) + \frac{\psi_2(t)}{\mu}$
- D.  $u_{opt}(t) = -2 \cdot (1 + 2x_1(t)) \cdot \frac{\mu}{\psi_2(t)}$
- E. *The correct answer is missing*



6a19. What category is  $c(x_1(t)) = x_1^2(t) + x_1(t) \leq 0$  limitation if

$$I = \int_0^T x_1^2(t) dt \rightarrow \min_{u(t)},$$

$$\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{pmatrix} = \begin{bmatrix} 2 & 2 \\ 1 & 2 \end{bmatrix} \cdot \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \cdot u^2(t).$$

- A. 1<sup>st</sup> category
- B. 2<sup>nd</sup> category
- C. 3<sup>rd</sup> category
- D. 4<sup>th</sup> category
- E. 7<sup>th</sup> category

6a20. If  $j^{\text{th}}$  component of optimal solution of dual canonic problem of linear programming equals zero ( $y_j=0$ ), then direct problem's appropriate limitation is:

- A.  $> 0$
- B.  $< 0$
- C.  $= 0$
- D.  $\geq 0$
- E. The correct answer is missing

6a21. If  $j^{\text{th}}$  component of optimal solution of dual canonic problem of linear programming doesn't equal zero ( $y_j \neq 0$ ), then direct problem's appropriate limitation is:

- A.  $> 0$ ,
- B.  $= 0$ ,
- C.  $< 0$ ,
- D.  $= 8$ ,
- E. The correct answer is missing

6a22. For the solution of canonic problem of linear programming which condition must take place (n is the number of variables of direct problem, and m is the number of limitations)?

- A.  $x_i^{\text{direct}} = -(\text{sd})_{m+i}^{\text{dual}}$
- B.  $x_i^{\text{direct}} = (\text{sd})_{n+i}^{\text{dual}}$
- C.  $x_i^{\text{direct}} = -(\text{sd})_{n+i}^{\text{dual}}$
- D.  $x_i^{\text{direct}} = (\text{sd})_{m+i}^{\text{dual}}$
- E. The correct answer is missing

6a23. Which condition is valid for the optimal solution of canonical problems of linear programming (n-is the number of variables of direct problem and m-is the number of limitations)?

- A.  $y_i^{\text{dual}} = (\text{sd})_{n+i}^{\text{direct}}$
- B.  $y_i^{\text{dual}} = -(\text{sd})_{n+i}^{\text{direct}}$
- C.  $y_i^{\text{dual}} = -(\text{sd})_{m+i}^{\text{direct}}$
- D.  $y_i^{\text{dual}} = (\text{sd})_{m+i}^{\text{direct}}$
- E. The correct answer is missing

6a24. In case of which value of  $a_{33}$  cell

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & 4 \\ 1 & 0 & a_{33} \end{bmatrix}$$

matrix can be reduced to diagonal form?

- A.  $a_{33} = 1$ ,
- B.  $a_{33} = 2$ ,
- C.  $a_{33} \neq 1$ ,
- D.  $a_{33} \neq 1$  or  $a_{33} \neq 2$ ,
- E. The correct answer is missing

- 6a25.**  $P_2$  coefficient of characteristic polynomial  $P_0 \cdot \lambda^3 + P_1 \cdot \lambda^2 + P_2 \cdot \lambda + P_3 = 0$  of the following matrix

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 1 & 2 \\ 1 & 0 & 3 \end{bmatrix}$$

equals

- A. +11  
 B. 6  
 C. -11  
 D. 0  
 E. -6
- 6a26.** What does  $\Psi^T(t) \cdot \Psi(t)|_{t=2}$  equal if it is known that

$$\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{pmatrix} = \begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & 3 \\ -2 & -3 & 0 \end{bmatrix} \cdot \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix}, \begin{matrix} x_1(0) = 1, \\ x_2(0) = 2, \\ x_3(0) = 0, \end{matrix}$$

and  $\Psi(t) = (\psi_1(t), \psi_2(t), \psi_3(t))^T$  is complement variable vector?

- A. -5  
 B. -3  
 C. 0  
 D. 3  
 E. 5
- 6a27.** What does  $X^T(t) \cdot X(t)|_{t=3}$  equal if it is known that

$$\begin{pmatrix} \dot{\psi}_1(t) \\ \dot{\psi}_2(t) \\ \dot{\psi}_3(t) \end{pmatrix} = \begin{bmatrix} 0 & 2 & -1 \\ -2 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix} \cdot \begin{pmatrix} \psi_1(t) \\ \psi_2(t) \\ \psi_3(t) \end{pmatrix}, \begin{matrix} \psi_1(0) = 2, \\ \psi_2(0) = 1, \\ \psi_3(0) = 3, \end{matrix}$$

and  $X(t) = (x_1(t), x_2(t), x_3(t))^T$  is the variable vector of the state?

- A. 6  
 B. 8  
 C. 11  
 D. 14  
 E. 17
- 6a28.** The remainder of the division of the polynomial  $15x^4 - 14x^3 + 8x^2 - 7x - 2$  by the binomial  $x - 1$  is:
- A. 3  
 B. 4  
 C. 0  
 D. -8  
 E. -2

- 6a29.** What point belongs to the graph of the function  $f(x) = 3x^4 - 2x + 1$ ?

- A. (5,-1)  
 B. (0,3)  
 C. (-1,6)  
 D. (1,4)  
 E. (-2,-1)

- 6a30.** What value does the derivative of the function  $f(x) = x(x-1)(x-2)(x-3)(x-4)(x-5)$  have in the point  $x = 0$ ?

- A. -200  
 B. -120  
 C. -50  
 D. 100  
 E. 150

6a31. What number is the eigenvalue of the matrix  $A = \begin{pmatrix} 5 & 1 \\ 2 & 4 \end{pmatrix}$ ?

- A. 5
- B. 2
- C. -1
- D. 3
- E. 8

6a32. For what value of  $\alpha$  parameter  $M = \begin{pmatrix} 3 & 7 & 1 \\ 2 & -3 & 0 \\ 1 & \alpha & 1 \end{pmatrix}$  matrix does not have inverse?

- A. 3
- B. 0
- C. -2
- D. 10
- E. 15

6a33. For what value of  $A$  parameter  $f(x) = \begin{cases} \sin 3x, & x \neq 0 \\ x, & x = 0 \end{cases}$  function will be continuous?

- A. 0
- B. 1
- C. -1
- D. 4
- E. 3

6a34. Arrange the following integrals in ascending order.

$$I_1 = \int_1^2 \frac{dx}{\sqrt{x+1}} \quad I_2 = \int_1^2 \frac{2 \sin x}{\sqrt[3]{x}} dx \quad I_3 = \int_1^2 \frac{dx}{x + e^x}$$

- A.  $I_3, I_1, I_2$
- B.  $I_2, I_3, I_1$
- C.  $I_1, I_2, I_3$
- D.  $I_2, I_1, I_3$
- E.  $I_1, I_3, I_2$

6a35. How many real roots does  $f'(x) = 0$  equation have where  $f(x) = (x-1)(x-2)(x-3)(x-4)(x-5)$

- A. 4
- B. 3
- C. 2
- D. 1
- E. 0

6a36. For what value of  $a_{22}$  element  $A = \begin{bmatrix} 2 & 0 & 0 \\ 1 & a_{22} & 0 \\ 2 & 1 & 1 \end{bmatrix}$  matrix can be reduced to diagonal form?

- A.  $a_{22} = 2$
- B.  $a_{22} = 1$
- C.  $a_{22} \neq 1$
- D.  $a_{22} \neq 2$  and  $a_{22} \neq 1$
- E. The correct answer is missing

6a37. For what values of  $c_{11}$  and  $c_{21}$  elements  $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ ,  $C = \begin{bmatrix} c_{11} & 1 & 0 \\ c_{21} & 1 & 2 \end{bmatrix}$  matrix system will not be

fully observable?

- A.  $c_{11} \neq 0, c_{21} = 0$
- B.  $c_{11} = 0, c_{21} = 0$

- C.  $c_{11}=0, c_{21}\neq 0$
- D.  $c_{11}\neq 0, c_{21}\neq 0$
- E. *The correct answer is missing*

6a38. What category is  $c(x_1(t)) = x_1^2(t) + x_1(t) \leq 0$  limitation if

$$I = \int_0^T x_1^2(t) dt \rightarrow \min_{u(t)},$$

$$\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{pmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \cdot \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} + \begin{pmatrix} 0 \\ 2 \end{pmatrix} \cdot u^3(t).$$

- A. 1<sup>st</sup> category
- B. 3<sup>rd</sup> category
- C. 4<sup>th</sup> category
- D. 2<sup>nd</sup> category
- E. 5<sup>th</sup> category

6a39. For what values of  $b_{31}$  and  $b_{32}$  elements  $A = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 2 & 0 \\ 0 & 1 & 3 \end{bmatrix}$ ,  $B = \begin{bmatrix} 1 & 2 \\ 2 & 0 \\ b_{31} & b_{32} \end{bmatrix}$  matrix system will not be

fully controllable?

- A.  $b_{31}=1, b_{32}=2$ ,
- B.  $b_{31}=2, b_{32}=0$ ,
- C.  $b_{31}=0, b_{32}=0$ ,
- D.  $b_{31}=1, b_{32}=0$ ,
- E.  $b_{31}=0, b_{32}=1$ ,

6a40. What is the view of  $u_{1opt}(t), u_{2opt}(t)$  if

$$I = \int_0^T (u_1^2(t) + u_2^2(t)) dt \rightarrow \min_{u_1(t), u_2(t)},$$

$$\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{pmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \cdot \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} + \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \cdot \begin{pmatrix} u_1(t) \\ u_2(t) \end{pmatrix},$$

- A.  $u_{1opt}(t) = \psi_1(t) + \psi_2(t), u_{2opt}(t) = \psi_1(t) - \psi_2(t)$
- B.  $u_{1opt}(t) = \psi_1(t) \cdot \psi_2(t), u_{2opt}(t) = \frac{\psi_1(t)}{\psi_2(t)}$
- C.  $u_{1opt}(t) = \frac{\psi_2(t)}{\psi_1(t)}, u_{2opt}(t) = 0$
- D.  $u_{1opt}(t) = \frac{1}{2}(\psi_1(t) + \psi_2(t)), u_{2opt}(t) = \frac{1}{2}\psi_2(t)$
- E. *The correct answer is missing*

6a41. Which statement is correct?

- A. *Direct and dual problems have the same number of variables*
- B. *The objective functions of direct and dual problems have the same value*
- C. *Direct and dual problems have the same number of limitations*
- D. *None*
- E. *All the answers are correct*

6a42. For what value of  $a_{22}$  element  $A = \begin{bmatrix} 2 & 0 & 0 \\ 2 & a_{22} & 0 \\ 2 & 2 & 1 \end{bmatrix}$  matrix can be reduced to diagonal form?

- A.  $a_{22} = 2$
- B.  $a_{22} = 1$
- C.  $a_{22} \neq 1$
- D.  $a_{22} \neq 2$  or  $a_{22} \neq 1$
- E. *The correct answer is missing*

**6a43.** For what values of  $c_{11}$  and  $c_{21}$  elements  $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ ,  $C = \begin{bmatrix} c_{11} & 2 & 0 \\ c_{21} & 3 & 2 \end{bmatrix}$  matrix system will not be

fully observable?

- A.  $c_{11} \neq 0, c_{21} = 0$
- B.  $c_{11} = 0, c_{21} = 0$
- C.  $c_{11} = 0, c_{21} \neq 0$
- D.  $c_{11} \neq 0, c_{21} \neq 0$
- E. *The correct answer is missing*

**6a44.** What is  $X^T(t) \cdot X(t)|_{t=3}$  equal to, if known that

$$\begin{pmatrix} \dot{\psi}_1(t) \\ \dot{\psi}_2(t) \\ \dot{\psi}_3(t) \end{pmatrix} = \begin{bmatrix} 0 & 2 & -1 \\ -2 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix} \cdot \begin{pmatrix} \psi_1(t) \\ \psi_2(t) \\ \psi_3(t) \end{pmatrix}, \begin{matrix} \psi_1(0) = 0, \\ \psi_2(0) = 1, \\ \psi_3(0) = 3, \end{matrix}$$

and  $X(t) = (x_1(t), x_2(t), x_3(t))^T$  is conjugate variable vector?

- A. 0
- B. 2
- C. 6
- D. 10
- E. 14

**6a45.** Which is the conjugate variable system if

$$I = \int_0^T u^2(t) dt \rightarrow \min_{u(t)},$$

$$\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{pmatrix} = \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix} \cdot \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \end{pmatrix} \cdot u(t)?$$

- A.  $\dot{\psi}_1(t) = -\psi_1(t) - 3\psi_2(t), \dot{\psi}_2(t) = -3\psi_1(t) - \psi_2(t)$
- B.  $\dot{\psi}_1(t) = -\psi_1(t) + \psi_2(t), \dot{\psi}_2(t) = \psi_1(t) - \psi_2(t)$
- C.  $\dot{\psi}_1(t) = \psi_1(t) \cdot \psi_2(t), \dot{\psi}_2(t) = \frac{\psi_1(t)}{\psi_2(t)}$
- D.  $\dot{\psi}_1(t) = -\frac{\psi_2(t)}{\psi_1(t)}, \dot{\psi}_2(t) = \psi_1(t) \cdot \psi_2(t)$
- E. *The correct answer is missing*

**6a46.** What does  $\Psi^T(t) \cdot \Psi(t)|_{t=2}$  equal to if known that

$$\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{pmatrix} = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 2 \\ 1 & -2 & 0 \end{bmatrix} \cdot \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix}, \begin{matrix} x_1(0) = 1, \\ x_2(0) = 2, \\ x_3(0) = -3, \end{matrix}$$

and  $\Psi(t) = (\psi_1(t), \psi_2(t), \psi_3(t))^T$  is the conjugate variable vector?

- A. 14
- B. 10
- C. 6
- D. 2
- E. 0

**6a47.**  $P_3$  coefficient of characteristic polynomial  $P_0 \cdot \lambda^3 + P_1 \cdot \lambda^2 + P_2 \cdot \lambda + P_3 = 0$  of the following matrix

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 1 \\ 2 & 0 & 4 \end{bmatrix}$$

equals

- A. 12
- B. 5
- C. 4

- D. 3  
E. 1

**6a48.** For which values of  $b_{11}$ ,  $b_{21}$  and  $b_{31}$  elements  $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 1 \\ 1 & 1 & 2 \end{bmatrix}$ ,  $B = \begin{bmatrix} b_{11} & 1 \\ b_{21} & 2 \\ b_{31} & 3 \end{bmatrix}$  matrix system will not be

normal system?

- A.  $b_{11} \neq 0, b_{21} = 0, b_{31} \neq 0$   
 B.  $b_{11} = 0, b_{21} = 0, b_{31} \neq 0$   
 C.  $b_{11} = 0, b_{21} = 0, b_{31} = 0$   
 D.  $b_{11} = 1, b_{21} = 0, b_{31} = 2$   
 E.  $b_{11} = 2, b_{21} = 1, b_{31} = 0$

**6a49.** How many real roots has the equation  $\frac{df}{dx}(x) = 0$  for the function  $f(x) = x^6 - 6x^4 + x + 2$ ?

- A. 0  
 B. 1  
 C. 2  
 D. 3  
 E. 4

**6a50.** Calculate the integral  $\int_1^4 |x - 2| dx$ .

- A. 3  
 B. 2.5  
 C. 2  
 D. 1.5  
 E. 1

**6a51.** For which  $\alpha$  the rank of matrix  $A = \begin{pmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & 1 & \alpha \end{pmatrix}$  is equal to 2?

- A. -2  
 B. 0  
 C. 1  
 D. 3  
 E. All the answers are correct

**6a52.** How many significant digits of  $\lg 2$  should be taken for determination of the roots of equation  $x^2 - 2x + \lg 2 = 0$  with 4 digit accuracy?

- A. 2  
 B. 4  
 C. 6  
 D. 8  
 E. 10

**6a53.** Assume  $M = \begin{pmatrix} 2 & 4 \\ 1 & 2 \end{pmatrix}$ . The equality  $M^6 = kM$  holds if  $k$  is:

- A.  $2^6$   
 B.  $2^8$   
 C.  $2^{10}$   
 D.  $2^{12}$   
 E.  $2^{14}$

**6a54.** From the given points  $A, B, C, D$  which one is the closest to  $y = 3x + 2$  line?

- A.  $A(1, 2)$   
 B.  $B(3, 2)$   
 C.  $C(0, -1)$   
 D.  $D(-1, -2)$   
 E. C and D answers are correct

6a55. Find  $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{1}{n \left(1 + \frac{k}{n}\right)}$

- A. 1
- B.  $\ln 2$
- C. 0.5
- D. 0
- E. The correct answer is missing

6a56. Assume  $f(x) = e^{-x} \cos 2x$ . What second degree polynomial gives the best approximation of function  $f$  in the neighborhood of 0?

- A.  $P_2 = 1 + 0.2x + x^2$
- B.  $P_2 = 1 + x - 1.5x^2$
- C.  $P_2 = 1 - 2.5x + 3x^2$
- D.  $P_2 = 2 - x + 3x^2$
- E. The correct answer is missing

6a57. Assume  $f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2}$  for all  $x \neq 0$ . In that case  $f$  function is defined by the following expression:

- A.  $x^2 - 2$
- B.  $2x^2 + 1$
- C.  $x^2 + 4$
- D.  $4x - x^2$
- E.  $3x + x^2 + 1$

6a58. For  $f(x) = \begin{cases} e^x, & x > 0 \\ a + x, & x \leq 0 \end{cases}$  function to be constant,  $a$  constant value must equal

- A. 2
- B. 1
- C. 0
- D. -1
- E. -2

6a59. What does  $\lim_{n \rightarrow \infty} \sqrt[2n]{1 + x^{2n}}$  limit equal?

- A.  $|x| + 1$
- B.  $\min(2, |x|)$
- C.  $\max(1, |x|)$
- D. 1
- E.  $2|x|$

6a60. In case of what  $\beta$

$$\begin{cases} x_1 + x_2 + x_3 = 0 \\ x_1 + \beta x_2 + x_3 = 0 \\ x_1 + x_2 + 2x_3 = 0 \end{cases}$$

system has more than three solutions?

- A. 4
- B. 3
- C. 2
- D. 1
- E. 0

**6a61.** Find  $\lim_{n \rightarrow \infty} \int_0^{\infty} \frac{dx}{x^n + 1}$  limit

- A. 1
- B. 2
- C. 3
- D. 4
- E. 5

**6a62.** Assume random value distribution function is given by  $F(x) = A(B + \arctg 2x)$  formula. Define  $A$  and  $B$  constant values.

- A.  $A=1, B=0.5\pi$
- B.  $A=\pi^{-1}, B=0.5\pi$
- C.  $A=(2\pi)^{-1}, B=\pi$
- D.  $A=\pi^{-1}, B=1$
- E.  $A=2\pi^{-1}, B=\pi$

**6a63.** Assume  $A$  matrix looks as follows:

$$A = \begin{pmatrix} 0.2 & 5 \\ \alpha & 0.1 \end{pmatrix}$$

What value of  $\alpha$  of  $\bar{x} = A\bar{x}$  system can be solved by sequential approximation method?

- A. 2
- B. 1
- C. 0
- D. -1
- E. -2

**6a64.** Which of  $A(3,5), B(2,6), C(-1,1), D(2,0), E(5,0)$  points is the nearest to  $y = 3x + 1$ ?

- A. A
- B. B
- C. C
- D. D
- E. E

**6a65.** Arrange the integrals in ascending order:

$$I_1 = \int_0^1 e^{-x^2} dx, \quad I_2 = \int_0^1 x e^{-x^2} dx, \quad I_3 = \int_0^1 e^{-x^2} \sin x dx$$

- A.  $I_1, I_2, I_3$
- B.  $I_3, I_2, I_1$
- C.  $I_2, I_1, I_3$
- D.  $I_3, I_1, I_2$
- E.  $I_2, I_3, I_1$

**6a66.** For what value  $a$  the function

$$f(x) = \begin{cases} e^{2x}, & x > 0 \\ 1 + ax^2, & x \leq 0 \end{cases}$$

will be continuously differentiable?

- A. 2
- B. 1
- C. 0
- D. -1
- E. None



6a67. Find the limit  $\lim_{n \rightarrow \infty} \sqrt[2n]{1+x^{2n}}$

- A.  $|x|+1$
- B.  $\min(2,|x|)$
- C.  $\max(1,|x|)$
- D. 1
- E.  $x$

6a68. For what value  $\beta$   $\begin{cases} x_1 + x_2 + 3x_3 = 2 \\ x_1 + x_2 + x_3 = \beta \\ x_1 + x_2 + 2x_3 = 0 \end{cases}$  the system has a solution?

- A. -2
- B. -0.5
- C. 1
- D. -1
- E. 0

6a69. Find the limit  $\lim_{n \rightarrow \infty} \int_0^2 \frac{x^n - x^{-n}}{x^n + x^{-n}} dx$ .

- A. 1
- B. 2
- C. -2
- D. 0
- E. -1

6a70. For what values of the constants A, B the distribution function of the variate may be given by the formula  $F(x) = A + B \arctan 2x$ ?

- A.  $A = 0, B = -\pi^{-1}$
- B.  $A = \pi^{-1}, B = 0.5\pi$
- C.  $A = (2\pi)^{-1}, B = \pi$
- D.  $A = \pi^{-1}, B = 1$
- E. There are no such values

6a71. For what values  $\alpha$  all eigenvalues of the matrix  $A$  are real  $A = \begin{pmatrix} 2 & -1 \\ \alpha & 1 \end{pmatrix}$ ?

- A. -1
- B. 1
- C. 4
- D. 7
- E. 10

6a72. There are five points on the plane:  $A(3,5), B(2,6), C(-1,1), D(2,0), E(3,-2)$ . For what point the distance from the parabola  $y = x^2 - 1$  will be minimal?

- A. A
- B. B
- C. C
- D. D
- E. E

6a73. For what value of  $a_{22}$  element

$$A = \begin{bmatrix} 3 & 0 & 0 \\ 2 & a_{22} & 0 \\ 2 & 2 & 1 \end{bmatrix}$$

matrix can be reduced to diagonal form?

- A.  $a_{22} = 3$
- B.  $a_{22} \neq 3, a_{22} \neq 1$
- C.  $a_{22} = 1$
- D.  $a_{22} \neq 1$
- E. *The correct answer is missing*

**6a74.** For what values of  $b_{31}$  and  $b_{32}$  elements

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 2 & 0 \\ 0 & 1 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & 2 \\ 1 & 2 \\ b_{31} & b_{32} \end{bmatrix}$$

matrix system will not be fully controllable?

- A.  $b_{31}=0, b_{32}=0$
  - B.  $b_{31}=2, b_{32}=0$
  - C.  $b_{31}=1, b_{32}=0$
  - D.  $b_{31}=0, b_{32}=1$
  - E.  $b_{31}=1, b_{32}=2$
- 6a75.** Which is the conjugate variable system if

$$I = \int_0^T u^2(t) dt \rightarrow \min_{u(t)},$$

$$\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{pmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \cdot \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} + \begin{pmatrix} 1 \\ 2 \end{pmatrix} \cdot u(t).$$

- A.  $\dot{\psi}_1(t) = -\psi_1(t) + \psi_2(t), \dot{\psi}_2(t) = \psi_1(t) - \psi_2(t)$
- B.  $\dot{\psi}_1(t) = -\psi_1(t) - 2\psi_2(t), \dot{\psi}_2(t) = -2\psi_1(t) - \psi_2(t)$
- C.  $\dot{\psi}_1(t) = \psi_1(t) \cdot \psi_2(t), \dot{\psi}_2(t) = \frac{\psi_1(t)}{\psi_2(t)}$
- D.  $\dot{\psi}_1(t) = -\frac{\psi_2(t)}{\psi_1(t)}, \dot{\psi}_2(t) = \psi_1(t) \cdot \psi_2(t)$
- E. *The correct answer is missing*

**6a76.**  $P_3$  coefficient of characteristic polynomial  $P_0 \cdot \lambda^3 + P_1 \cdot \lambda^2 + P_2 \cdot \lambda + P_3 = 0$  of the following matrix

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 1 \\ 2 & 0 & 2 \end{bmatrix}$$

equals:

- A. 5
  - B. 1
  - C. 4
  - D. 3
  - E. 6
- 6a77.** For what values of  $c_{11}$  and  $c_{21}$  elements

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}, \quad C = \begin{bmatrix} c_{11} & 4 & 0 \\ c_{21} & 1 & 2 \end{bmatrix}$$

matrix system will not be fully observable?

- A.  $c_{11} \neq 0, c_{21} = 0$
- B.  $c_{11} = 0, c_{21} \neq 0$
- C.  $c_{11} \neq 0, c_{21} \neq 0$
- D.  $c_{11} = 0, c_{21} = 0$
- E. *The correct answer is missing*

**6a78.** What is  $X^T(t) \cdot X(t)|_{t=3}$  equal to, if known that

$$\begin{pmatrix} \dot{\Psi}_1(t) \\ \dot{\Psi}_2(t) \\ \dot{\Psi}_3(t) \end{pmatrix} = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 2 \\ 1 & -2 & 0 \end{bmatrix} \cdot \begin{pmatrix} \Psi_1(t) \\ \Psi_2(t) \\ \Psi_3(t) \end{pmatrix}, \begin{matrix} \Psi_1(0) = 0, \\ \Psi_2(0) = 1, \\ \Psi_3(0) = 2, \end{matrix}$$

and  $\Psi(t) = (\Psi_1(t), \Psi_2(t), \Psi_3(t))^T$  is conjugate variable vector?

- A. 2
- B. 5
- C. 6
- D. 14
- E. 0

**6a79.** What is  $\Psi^T(t) \cdot \Psi(t)|_{t=2}$  equal to, if known that

$$\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{pmatrix} = \begin{bmatrix} 0 & 2 & -1 \\ -2 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix} \cdot \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix}, \begin{matrix} x_1(0) = 3, \\ x_2(0) = 1, \\ x_3(0) = 0, \end{matrix}$$

and  $\Psi(t) = (\psi_1(t), \psi_2(t), \psi_3(t))^T$  is conjugate variable vector?

- A. 2
- B. 14
- C. 10
- D. 6
- E. 0

**6a80.** For which values of  $b_{11}$ ,  $b_{21}$  and  $b_{31}$  elements

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}, B = \begin{bmatrix} b_{11} & 1 \\ b_{21} & 3 \\ b_{31} & 2 \end{bmatrix}$$

matrix system will not be normal system?

- A.  $b_{11} \neq 0, b_{21} = 0, b_{31} \neq 0$
- B.  $b_{11} = 0, b_{21} = 0, b_{31} \neq 0$
- C.  $b_{11} = 0, b_{21} = 0, b_{31} = 0$
- D.  $b_{11} = 1, b_{21} = 0, b_{31} = 2$
- E.  $b_{11} = 2, b_{21} = 1, b_{31} = 0$

## **b) Problems**

**6b1.**

The following matrix game is given:

$$H = \begin{bmatrix} 3 & -2 & 4 \\ -1 & 4 & 2 \\ 2 & 2 & 6 \end{bmatrix}$$

Find the worth of game, optimal strategies and characterize the game. If necessary, construct corresponding problems of mathematical programming.

**6b2.**

Define the first two members of Taylor series  $\Phi(t, t_0)$  if

$$A(t) = \begin{bmatrix} t & t^2 & t^3 \\ 1 & t & (2-1) \\ t^2 & t & 1 \end{bmatrix}, t_0 = 2$$

**6b3.**

Define  $\Phi(t)$  if

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 3 & 0 \\ 2 & 1 & 1 \end{bmatrix}.$$

**6b4.**

Reduce the system to diagonal form if

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & 0 \\ 2 & 1 & 3 \end{bmatrix}:$$

**6b5.**

The following matrix is given:

$$A = \begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

What is the constant term of characteristic polynomial equal to?

**6b6.**

The following matrix is given:

$$A = \begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

What is  $\|A\|_2$  norm equal to?

**6b7.**

What is the coefficient of  $A_{3 \times 3}$  matrix's characteristic polynomial term containing  $\lambda^2$ , if  $\lambda_1 = 1, \lambda_2 = 2, \lambda_3 = 3$  are the eigenvalues of  $A_{3 \times 3}$ .

**6b8.**

What equals  $A_{4 \times 4}$  matrix's characteristic polynomial coefficient of the term which contains  $\lambda^2$ , if  $\lambda_1 = 0, \lambda_2 = 1, \lambda_3 = 2, \lambda_4 = 3$  are the eigenvalues of  $A_{4 \times 4}$ ?

**6b9.**

The table of  $\log x$  function in  $[0, 1000]$  interval is built by the help of linear interpolation. What  $h$  step should be selected for the error not to exceed 0.001? Consider cases of constant and variable steps (by dividing the interval into several subintervals). Estimate the minimum number of nodes which is necessary to provide the given accuracy.

**6b10.**

Using generalized trapezoid rule, calculate

$$\int_0^1 \frac{e^x}{x^2 + b^2} dx$$

integral with  $O(h^2)$  accuracy ( $h$  is division step). How should this formula be applied to provide the same accuracy if  $b \ll 1$ ?

**6b11.**

Apply quadrature formula for the calculation of the integral

$$\int_0^1 \frac{f(x)}{\sqrt{x}} dx$$

with  $10^{-4}$  accuracy, providing  $|f''(x)| \leq 1$ .

**6b12.**

Define the initial approximation domains of  $x_0$  for which  $x_{n+1} = \frac{x_n^3 + 1}{20}$  iterations converge.

**6b13.**

$f(x) = \frac{5}{1+100x^2}$  function is replaced by  $y = \alpha x + \beta$  linear function in segment  $[-10, 10]$ . Define coefficients  $\alpha$  and  $\beta$  to provide the smallest error. Can such  $\alpha$  and  $\beta$  uniquely be found? If yes, find it; if no – consider possible cases.

**6b14.**

Construct an approximate formula (multiple-application rectangle rule) for the calculation of the integral  $\int_1^{\infty} \frac{f(x) dx}{1+x^2}$  with  $\varepsilon = 0,01$  accuracy. The function  $f$  on interval  $[1, \infty)$  is continuously differentiable and bounded.

**6b15.**

Continuously differentiable function  $f$  is given on  $[a, b]$  segment and  $a < x_1 < x_2 < b$ . Find the polynomial  $P_3$  of the third order so that in  $x_1$  and  $x_2$  points its and its derivative's values coincide with  $f(x_1), f(x_2)$  and  $f'(x_1), f'(x_2)$  values respectively. Is it possible to generalize the obtained formula for arbitrary number of points  $a < x_1 < \dots < x_2 < b$ ?

**6b16.**

Find the roots of the equation  $x^4 - 10x + 1 = 0$  with  $\varepsilon = 0,01$  accuracy. Choose the fastest algorithm.

**6b17.**

Assume  $f(x) = \frac{1}{1+x} + \frac{1}{3-x}$ . Find  $g(x) = ax^2 + bx + c$  quadratic function, which gives the best approximation for the function  $f$  in  $[0, 2]$  interval. Consider different cases.

**6b18.**

Assume

$$\Delta_n = \det \begin{pmatrix} 2 & 1 & 0 & 0 & \dots & 0 \\ 1 & 2 & 1 & 0 & \dots & 0 \\ 0 & 1 & 2 & 1 & \dots & 0 \\ 0 & 0 & 1 & 2 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \dots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 2 \end{pmatrix}. \text{ Calculate the limit } \lim_{n \rightarrow \infty} \frac{\Delta_n}{n}.$$

**6b19.**

The triangle  $ABC$  is given by the three vertices  $A(x_1, y_1), B(x_2, y_2), C(x_3, y_3)$ . How to check if the given point  $D$  with coordinates  $D(x_0, y_0)$  lies in interior of that triangle? Describe the algorithm. Is it possible to generalize this algorithm for the case of

- a) convex polygon
- b) arbitrary polygon?

**6b20.**

$x_j, j=1, 2$ , and  $y_k, k=0, \dots, 4$  real numbers are given. Find the forth degree polynomial  $P_4$  in a way that  $P_4^{(k)}(x_1) = y_k, k=0, 1, 2, 3; P_4(x_2) = y_4$ . Generalize the result: find the  $n$ -th degree polynomial  $P_n$  so that  $P_n^{(k)}(x_1) = y_k, k=0, \dots, m; P_n^{(j)}(x_2) = y_{m+1+j}, j=0, \dots, n-m-1$ .

**6b21.**

The following matrix is given:  $A = \begin{bmatrix} 1 & -3 & 0 \\ 0 & 1 & 2 \\ -1 & 0 & 1 \end{bmatrix}$  What does the free term of characteristic polynomial equal?

**6b22.**

Given  $A = \begin{bmatrix} 1 & -3 & 0 \\ 0 & 1 & 2 \\ -1 & 0 & 1 \end{bmatrix}$  matrix. What does  $\|A\|_2$  norm equal?

**6b23.**

What does  $A_{3 \times 3}$  matrix's characteristic polynomial coefficient of the term which contains  $\lambda$  equal if

$$\lambda_1 = 1, \lambda_2 = 4, \lambda_3 = 2.$$

**6b24.**

What does  $A_{4 \times 4}$  matrix's characteristic polynomial coefficient of the term which contains  $\lambda^2$  equal if

$$\lambda_1 = 4, \lambda_2 = 1, \lambda_3 = 2, \lambda_4 = 3.$$

**6b25.**

Find the  $n$  degree polynomial  $P_n$  which meets the following conditions:

$$P_n^{(j)}(1) = y_j, \quad j = 0, 1, 2, \quad P_n^{(k)}(0) = z_k, \quad k = 0, 1.$$

Find the smallest  $n$  for which the problem has a solution for arbitrary values  $y_j$  and  $z_k$ . For this  $n$  find the polynomial  $P_n$  in explicit form.

**6b26.**

On the segment  $[-1, 1]$  find the best uniform approximation polynomial of order one  $P_1$  for  $f(x) = x^3$  function, i.e. find  $P_1$  polynomial for which the uniform norm  $\|f - P_1\| = \max_{x \in [-1, 1]} |f(x) - P_1(x)|$  is minimal.

**6b27.**

Find the eigenvalues of the matrix  $A$  which satisfies the condition  $A^2 = 0$  ( $0$  is zero matrix). What may be said about the eigenvalues of the matrix  $A$  which satisfies the condition  $A^n = 0$  where  $n$  is a natural number. Justify the answer.

**6b28.**

Calculate  $\ln 2$  with 0.001 accuracy, using  $\ln(1+x) = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{x^k}{k}$  series for  $|x| < 1$ . Find more optimal solution, i.e. a method of solution which requires the smallest number of summands of series.

**6b29.**

The following matrix is given  $A = \begin{bmatrix} 2 & -6 & 0 \\ 0 & 2 & 4 \\ -2 & 0 & 2 \end{bmatrix}$ . What does the free term of characteristic polynomial equal?

**6b30.**

What does  $A_{4 \times 4}$  matrix's characteristic polynomial coefficient of the term which contains  $\lambda^2$  equal if

$$\lambda_1 = 2, \lambda_2 = 0.5, \lambda_3 = 1, \lambda_4 = 1.5:$$

**6b31.**

Given the matrix of state variables of controlling system.

$$A = \begin{bmatrix} -2 & 0 \\ 1 & -1 \end{bmatrix}$$

Define  $\Phi(t)$  fundamental matrix of replacement.

**6b32.**

Find out full controllability of the system described by the following equations.

$$\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{pmatrix} = \begin{bmatrix} 0.5 & 0 & 1 \\ 0.5 & 0.5 & 0 \\ 0 & 0.5 & 0.5 \end{bmatrix} \cdot \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix} + \begin{bmatrix} 0 & 0.5 \\ 0.5 & 0 \\ 0.5 & 1 \end{bmatrix} \cdot \begin{pmatrix} u_1(t) \\ u_2(t) \end{pmatrix},$$

**6b33.**

Estimate an accuracy of the rectangle's symmetric rule

$$I = \int_a^b f(x)dx \approx f\left(\frac{a+b}{2}\right)(b-a) = \tilde{I},$$

supposing that the function  $f$  two times continuously differentiable on the segment  $[a, b]$ .

**6b34.**

Applying the rectangle's symmetric rule

$$\int_a^b f(x)dx \approx f\left(\frac{a+b}{2}\right)(b-a) = \tilde{I}_1,$$

and the trapezoidal rule

$$\int_a^b f(x)dx \approx \frac{f(a)+f(b)}{2}(b-a) = \tilde{I}_2,$$

to the integral  $I = \int_{\ln a}^{\ln b} e^x dx$ , where  $0 < a < b$ , prove an inequalities

$$\sqrt{ab} < \frac{b-a}{\ln b - \ln a} < \frac{a+b}{2}$$

(geometrical mean is less than logarithmical mean, which is less than arithmetical mean).

**6b35.**

A generalized Rolle's theorem. Suppose the function  $f$  is continuous on  $[a, b]$  and  $n+1$  times continuously differentiable on  $(a, b)$ . Let  $x_0 \in (a, b)$  be a point at which  $f(x_0) = f'(x_0) = \dots = f^{(n)}(x_0) = 0$ . Then for arbitrary point  $x \in (a, b)$ ,  $x \neq x_0$  at which  $f(x) = 0$  there is a point  $\eta \in (x, x_0)$ , such that  $f^{(n+1)}(\eta) = 0$ .

**6b36.**

Define the sequence of positive numbers:  $x_0 > 0$ ,  $x_{n+1} = \frac{1}{1+x_n}$  for  $n \geq 0$ .

Prove that this sequence is converges and find the limit.

**6b37.**

Calculate the integral  $I = \int_0^{0.5} \frac{\sin x}{x} dx$  with accuracy 0,001. That is, find number  $I^*$  such that  $|I - I^*| < 0,001$ .

**6b38.**

The following matrix is given:

$$A = \begin{bmatrix} 2 & -6 & 0 \\ 0 & 2 & 4 \\ -2 & 0 & 2 \end{bmatrix}$$

What does the free term of characteristic polynomial equal?

**6b39.**

What does  $A_{4 \times 4}$  matrix's characteristic polynomial coefficient of the term which contains  $\lambda^2$  equal if  $\lambda_1 = 2, \lambda_2 = 0.5, \lambda_3 = 1, \lambda_4 = 3$ .

**6b40.**

Given the matrix of state variables of controlling system.

$$A = \begin{bmatrix} -1 & 0 \\ 2 & -2 \end{bmatrix}$$

Define  $\Phi(t)$  fundamental matrix of replacement.

**6b41.**

Find out full controllability of the system described by the following equations.

$$\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{pmatrix} = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \cdot \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 1 & 2 \end{bmatrix} \cdot \begin{pmatrix} u_1(t) \\ u_2(t) \end{pmatrix}$$

## 7. DISCRETE MATHEMATICS AND THEORY OF COMBINATIONS

### a) Test questions

- 7a1.** The set is countable if it is  
A. *Finite*  
B. *Equivalent to some subset of finite set*  
C. *Equivalent to any subset of natural number set*  
D. *Equivalent to any infinite set*  
E. *Equivalent to any infinite subset of natural number set*
- 7a2.** The graph includes Euler cycle if and only if  
A. *It is connected and the degrees of all nodes are even*  
B. *It is connected and the degrees of some nodes are odd*  
C. *The degrees of all nodes are even*  
D. *It is not connected and the degrees of some nodes are even*  
E. *It is connected and the degrees of all nodes are odd*
- 7a3.** If for  $f(x_1, \dots, x_1, \dots, x_n)$  Boolean function  $\omega_i^f = 1$  ( $i=1, 2, \dots, n$ ), it is  
A. *Constant 1 function*  
B. *Linear function*  
C. *Self-dual function*  
D. *Monotone function*  
E. *Constant 0 function*
- 7a4.** If Boolean function is monotonous, then  
A. *Its short disjunctive normal form does not contain negation of variables*  
B. *It is not self-dual*  
C. *Its short disjunctive normal form does not coincide with its minimal disjunctive normal form*  
D. *it is also a threshold*  
E. *it is not a threshold*
- 7a6.** In the given design styles in what sequence does the performance increase in case of other same parameters a) library; b) gate matrix; c) full custom; d) programmable matrix;  
A. *d - b - a - c*  
B. *a - b - c - d*  
C. *b - d - a - d*  
D. *c - a - b - d*  
E. *d - c - b - a*
- 7a7.** Which of the presented description forms of an electrical circuit is more convenient for the realization of sequential placing algorithm?  
A. *Graph of commutation scheme*  
B. *Complex list*  
C. *Adjacency matrix*  
D. *A and B equally*  
E. *B and C equally*
- 7a8.** In case of the presented design styles, in what order does the density of cells' placing increase? a) standard IC design; b) library design; c) gate matrix design; d) full custom design; e) programmable matrix design.  
A. *b - a - c - e - d*  
B. *a - b - c - d - e*  
C. *d - c - b - e - a*  
D. *a - d - b - c - e*  
E. *d - b - c - e - a*
- 7a9.** The activity of  $X_3$  argument of  $(x_1 \vee x_2) \oplus (x_3 \vee x_4 x_5)$  function equals  
A. *1/4*  
B. *1/8*  
C. *3/8*  
D. *5/8*  
E. *3/4*



**7a10.** If for  $f(x_1, \dots, x_n)$  Boolean function  $\omega_{1,2,\dots,n}^f = 1$ , then  $f$  is

- A. Constant 0 function
- B. Constant 1 function
- C. Monotone function
- D. Self-dual function
- E. Linear function

**7a11.** The activity of  $X_1(\bar{X}_2 \vee X_3(X_4 \vee X_5 \bar{X}_6))$  function's  $X_1$  argument equals

- A. 25/32
- B. 27/32
- C. 21/32
- D. 13/16
- E. 19/32

**7a12.**  $X_1(\bar{X}_2 \vee X_3(X_4 \vee X_5 \bar{X}_6))$  function's norm equals

- A. 11/64
- B. 13/32
- D. 17/64
- D. 21/64
- E. 23/64

**7a13.**  $(x_1 \vee x_2) \oplus (x_3 \vee x_4 x_5)$  function's norm equals

- A. 7/16
- B. 9/16
- C. 13/16
- D. 11/16
- E. 5/16

**7a14.**  $f(x_1, \dots, x_n)$  is dual if

- A.  $\exists(\alpha_1, \dots, \alpha_n) \bar{f}(\bar{\alpha}_1, \dots, \bar{\alpha}_n) = f(\alpha_1, \dots, \alpha_n)$
- B.  $\forall(\alpha_1, \dots, \alpha_n) \bar{f}(\bar{\alpha}_1, \dots, \bar{\alpha}_n) = f(\alpha_1, \dots, \alpha_n)$
- C.  $\exists(\alpha_1, \dots, \alpha_n) \bar{f}(\bar{\alpha}_1, \dots, \bar{\alpha}_n) = \bar{f}(\alpha_1, \dots, \alpha_n)$
- D.  $\exists(\alpha_1, \dots, \alpha_n) f(\bar{\alpha}_1, \dots, \bar{\alpha}_n) = f(\alpha_1, \dots, \alpha_n)$
- E.  $\forall(\alpha_1, \dots, \alpha_n) f(\bar{\alpha}_1, \dots, \bar{\alpha}_n) = f(\alpha_1, \dots, \alpha_n)$

**7a15.** The activity of the combination of arguments  $X_3$  and  $X_4$  for the function  $(x_1 x_2) \oplus (x_3 \vee x_4) x_5$  equals:

- A. 1/4
- B. 7/16
- C. 3/4
- D. 11/16
- E. 1/8

**7a16.** If  $f(x_1, \dots, x_n)$  is threshold function

- A.  $\bar{f}(x_1, \dots, x_n)$  is not threshold function
- B.  $f(\bar{x}_1, \dots, \bar{x}_n)$  is not threshold function
- C.  $f(\bar{x}_1, \dots, \bar{x}_n)$  is threshold function
- D.  $x \oplus f(x_1, \dots, x_n)$  is threshold function
- E.  $x \oplus \bar{f}(x_1, \dots, x_n)$  is threshold function

**7a17.** Binary relation is the equivalence relation if it is

- A. Transitive, symmetric but not reflexive
- B. Not symmetric, reflexive and transitive

- C. Symmetric, reflexive but not transitive
- D. Reflexive, transitive and symmetric
- E. Not symmetric, not reflexive and transitive

**7a18.**  $(x_3 \vee x_2) \oplus (\bar{x}_1 \vee x_4 x_5)$  function's norm equals

- A. 9/16
- B. 11/16
- C. 13/16
- D. 7/16
- E. 5/16

**7a19.** The activity of  $(x_3 \vee x_2) \oplus (x_1 \vee x_4 x_5)$  function's  $x_3$  argument equals

- A. 1/4
- B. 1/8
- C. 3/8
- D. 5/8
- E. 1/2

**7a20.** How many connectivity components does the complementation of the graph which has 4-connected component have?

- A. 1
- B. 2
- C. 3
- D. 4.
- E. The correct answer is missing

**7a21.** Given an n-input, m-output combinational circuit C, depending on input variables  $x_1, x_2, \dots, x_n$  and output variables  $y_1, y_2, \dots, y_m$  implementing functions  $f_i(x_1, x_2, \dots, x_n), 1 \leq i \leq m$ , and a combinational circuit C\* implementing functions  $f_j^*(x_1, x_2, \dots, x_n), 1 \leq j \leq m$ , and obtained from C when a fault F (for example, single stuck-at-0 or stuck-at-1) occurs on its arbitrary line A. Set T of input vectors is a test with respect to the set  $\Phi$  of faults if

- A. For any  $f \in \Phi$  and for any an input vector  $(\alpha_1, \alpha_2, \dots, \alpha_n)$  such that  $\alpha_i \in \{0, 1\}, 1 \leq i \leq n$ , and for any  $j, 1 \leq j \leq m, f^*(\alpha_1, \alpha_2, \dots, \alpha_n) \equiv f_j(\alpha_1, \alpha_2, \dots, \alpha_n)$ ;
- B. For any  $f \in \Phi$  and for any input vector  $(\alpha_1, \alpha_2, \dots, \alpha_n)$  such that  $\alpha_i \in \{0, 1\}, 1 \leq i \leq n$ , and for any  $j, 1 \leq j \leq m, f_j^*(\alpha_1, \alpha_2, \dots, \alpha_n) \neq f_j(\alpha_1, \alpha_2, \dots, \alpha_n)$ ;
- C. For any  $f \in \Phi$  there exists an input vector  $(\alpha_1, \alpha_2, \dots, \alpha_n)$  such that  $\alpha_i \in \{0, 1\}, 1 \leq i \leq n$ , and there exists a  $j, 1 \leq j \leq m$ , such that  $f_j^*(\alpha_1, \alpha_2, \dots, \alpha_n) \leq f_j(\alpha_1, \alpha_2, \dots, \alpha_n)$ ;
- D. For any  $f \in \Phi$  there exists such an input vector  $(\alpha_1, \alpha_2, \dots, \alpha_n)$  such that  $\alpha_i \in \{0, 1\}, 1 \leq i \leq n$ , and there exists a  $j, 1 \leq j \leq m$ , such that  $f_j^*(\alpha_1, \alpha_2, \dots, \alpha_n) \geq f_j(\alpha_1, \alpha_2, \dots, \alpha_n)$ .
- E. For any  $f \in \Phi$  there exists an input vector  $(\alpha_1, \alpha_2, \dots, \alpha_n)$  such that  $\alpha_i \in \{0, 1\}, 1 \leq i \leq n$ , and there exists a  $j, 1 \leq j \leq m$ , such that  $f_j^*(\alpha_1, \alpha_2, \dots, \alpha_n) \neq f_j(\alpha_1, \alpha_2, \dots, \alpha_n)$ ;

**7a22.** Given an n-input, m-output combinational circuit C, depending on input variables  $x_1, x_2, \dots, x_n$  and output variables  $y_1, y_2, \dots, y_m$  implementing functions  $f_i(x_1, x_2, \dots, x_n), 1 \leq i \leq m$ , and a combinational circuit C\* implementing functions  $f_j^*(x_1, x_2, \dots, x_n), 1 \leq j \leq m$ , and obtained from C when a fault F (for example, single stuck-at-0 or stuck-at-1) occurs on its arbitrary line A. Set  $(\beta_1, \beta_2, \dots, \beta_n)$  detects fault F if

- A.  $\exists j, 1 \leq j \leq m$ , and  $f^*(\beta_1, \beta_2, \dots, \beta_n) \neq f_j(\beta_1, \beta_2, \dots, \beta_n)$ ;
- B. for  $\forall (\alpha_1, \alpha_2, \dots, \alpha_n)$  input patterns,  $\alpha_i \in \{0, 1\}, 1 \leq i \leq n$ , and for  $\forall j, 1 \leq j \leq m, f_j^*(\alpha_1, \alpha_2, \dots, \alpha_n) \equiv f_j(\beta_1, \beta_2, \dots, \beta_n)$ ;
- C. for  $\forall (\alpha_1, \alpha_2, \dots, \alpha_n)$  input patterns, for  $\alpha_i \in \{0, 1\}, 1 \leq i \leq n$ , and  $\forall j, 1 \leq j \leq m, f_j^*(\alpha_1, \alpha_2, \dots, \alpha_n) \neq f_j(\beta_1, \beta_2, \dots, \beta_n)$ ;
- D.  $\exists j, 1 \leq j \leq m$ , so that  $f_j^*(\beta_1, \beta_2, \dots, \beta_n) \leq f_j(\beta_1, \beta_2, \dots, \beta_n)$ ;
- E.  $\exists j, 1 \leq j \leq m$ , so that  $f_j^*(\beta_1, \beta_2, \dots, \beta_n) \geq f_j(\beta_1, \beta_2, \dots, \beta_n)$ .

**7a23.** Given an n-input, m-output combinational circuit C, depending on input variables  $x_1, x_2, \dots, x_n$  and output variables  $y_1, y_2, \dots, y_m$  implementing functions  $f_i(x_1, x_2, \dots, x_n), 1 \leq i \leq m$ , and a combinational circuit C\* implementing functions  $f_j^*(x_1, x_2, \dots, x_n), 1 \leq j \leq m$ , and obtained from C when a fault F (for example, single stuck-at-0 or stuck-at-1) occurs on its arbitrary line A. F fault cannot be detected with respect to some class faults if

- A.  $\exists (\alpha_1, \alpha_2, \dots, \alpha_n)$  such input pattern,  $\alpha_i \in \{0, 1\}, 1 \leq i \leq n$ , and  $\exists j, 1 \leq j \leq m$ , and  $f^*(\alpha_1, \alpha_2, \dots, \alpha_n) \neq f_j(\alpha_1, \alpha_2, \dots, \alpha_n)$ ;
- B. 1. F fault cannot be detected or

2.  $\exists G \in \Phi$  fault for which  $\exists$  line B, for which in case of  $\forall (\alpha_1, \alpha_2, \dots, \alpha_n)$  input pattern,  $\alpha_i \in \{0, 1\}, 1 \leq i \leq n$ , and  $\forall j, 1 \leq j \leq m$ , and  $f^*(\alpha_1, \alpha_2, \dots, \alpha_n) = f_j^{**}(\alpha_1, \alpha_2, \dots, \alpha_n)$ , where  $f_j^{**}$  is the function obtained from C when a fault G occurs on its line B

C. 1. fault F is detectable;

2. for  $\forall (\alpha_1, \alpha_2, \dots, \alpha_n)$  input patterns, for  $\alpha_i \in \{0, 1\}, 1 \leq i \leq n$ , and  $\forall j, 1 \leq j \leq m, f_j^*(\alpha_1, \alpha_2, \dots, \alpha_n) \neq f_j^{**}(\alpha_1, \alpha_2, \dots, \alpha_n)$ ;

D. 1. fault F is detectable;

2.  $\exists (\alpha_1, \alpha_2, \dots, \alpha_n)$  such input pattern,  $\alpha_i \in \{0, 1\}, 1 \leq i \leq n$ , and  $\exists j, 1 \leq j \leq m$ , so that  $f_j^*(\alpha_1, \alpha_2, \dots, \alpha_n) \leq f_j^{**}(\alpha_1, \alpha_2, \dots, \alpha_n)$ ;

E. 1. fault F is detectable;

2.  $\exists (\alpha_1, \alpha_2, \dots, \alpha_n)$  such input pattern,  $\alpha_i \in \{0, 1\}, 1 \leq i \leq n$ , and  $\exists j, 1 \leq j \leq m$ , so that  $f_j^*(\alpha_1, \alpha_2, \dots, \alpha_n) \geq f_j^{**}(\alpha_1, \alpha_2, \dots, \alpha_n)$ .

**7a24.** Given an n-input, m-output combinational circuit C, depending on input variables  $x_1, x_2, \dots, x_n$  and output variables  $y_1, y_2, \dots, y_m$  implementing functions  $f_i(x_1, x_2, \dots, x_n), 1 \leq i \leq m$ , and a combinational circuit C\* implementing functions  $f_j^*(x_1, x_2, \dots, x_n), 1 \leq j \leq m$ , and obtained from C when a fault F (for example, single stuck-at-0 or stuck-at-1) occurs on its arbitrary line A. F fault is not redundant if

A. for  $\forall (\alpha_1, \alpha_2, \dots, \alpha_n)$  input patterns, for  $\alpha_i \in \{0, 1\}, 1 \leq i \leq n$ , and  $\forall j, 1 \leq j \leq m, f_j^*(\alpha_1, \alpha_2, \dots, \alpha_n) \neq f_j(\alpha_1, \alpha_2, \dots, \alpha_n)$ ;

B.  $\exists$  input pattern  $(\alpha_1, \alpha_2, \dots, \alpha_n)$  so that  $\alpha_i \in \{0, 1\}, 1 \leq i \leq n$ , and  $\exists j, 1 \leq j \leq m$ , for which  $f^*(\alpha_1, \alpha_2, \dots, \alpha_n) \neq f_j(\alpha_1, \alpha_2, \dots, \alpha_n)$ ;

C.  $\exists$  such input pattern  $(\alpha_1, \alpha_2, \dots, \alpha_n), \alpha_i \in \{0, 1\}, 1 \leq i \leq n$ , and  $\exists j, 1 \leq j \leq m$ , that  $f_j^*(\alpha_1, \alpha_2, \dots, \alpha_n) = f_j(\alpha_1, \alpha_2, \dots, \alpha_n)$ ;

D.  $\exists$  such input pattern  $(\alpha_1, \alpha_2, \dots, \alpha_n), \alpha_i \in \{0, 1\}, 1 \leq i \leq n$ , and  $\exists j, 1 \leq j \leq m$ , so that  $f_j^*(\alpha_1, \alpha_2, \dots, \alpha_n) \leq f_j(\alpha_1, \alpha_2, \dots, \alpha_n)$ ;

E. for  $\forall$  input pattern  $(\alpha_1, \alpha_2, \dots, \alpha_n), \alpha_i \in \{0, 1\}, 1 \leq i \leq n, \exists j, 1 \leq j \leq m$ , so that  $f_j^*(\alpha_1, \alpha_2, \dots, \alpha_n) \geq f_j(\alpha_1, \alpha_2, \dots, \alpha_n)$ .

**7a25.** Given an n-input, m-output combinational circuit C, depending on input variables  $x_1, x_2, \dots, x_n$  and output variables  $y_1, y_2, \dots, y_m$  implementing functions  $f_i(x_1, x_2, \dots, x_n), 1 \leq i \leq m$ , and a combinational circuit C\* implementing functions  $f_j^*(x_1, x_2, \dots, x_n), 1 \leq j \leq m$ , and obtained from C when a fault F (for example, single stuck-at-0 or stuck-at-1) occurs on its arbitrary line A. Set T is not a test with respect to set  $\Phi$  of faults if

A.  $\exists f \in \Phi$  so that for  $\forall (\alpha_1, \alpha_2, \dots, \alpha_n)$  input patterns, for  $\alpha_i \in \{0, 1\}, 1 \leq i \leq n$ , and  $\forall j, 1 \leq j \leq m, f^*(\alpha_1, \alpha_2, \dots, \alpha_n) \neq f_j(\alpha_1, \alpha_2, \dots, \alpha_n)$ ;

B. for  $\forall f \in \Phi$ , for  $\forall$  input pattern  $(\alpha_1, \alpha_2, \dots, \alpha_n)$ , for  $\alpha_i \in \{0, 1\}, 1 \leq i \leq n$ , and  $\forall j, 1 \leq j \leq m, f_j^*(\alpha_1, \alpha_2, \dots, \alpha_n) \neq f_j(\alpha_1, \alpha_2, \dots, \alpha_n)$ ;

C. for  $\forall f \in \Phi, \exists$  such input pattern  $(\alpha_1, \alpha_2, \dots, \alpha_n), \alpha_i \in \{0, 1\}, 1 \leq i \leq n$ , and  $\exists j, 1 \leq j \leq m$ , so that  $f_j^*(\alpha_1, \alpha_2, \dots, \alpha_n) \leq f_j(\alpha_1, \alpha_2, \dots, \alpha_n)$ ;

D. for  $\forall f \in \Phi, \exists$  such input pattern  $(\alpha_1, \alpha_2, \dots, \alpha_n), \alpha_i \in \{0, 1\}, 1 \leq i \leq n$ , and  $\exists j, 1 \leq j \leq m$ , so that  $f_j^*(\alpha_1, \alpha_2, \dots, \alpha_n) \geq f_j(\alpha_1, \alpha_2, \dots, \alpha_n)$ .

E.  $\exists f \in \Phi$ , for which, and for  $\forall (\alpha_1, \alpha_2, \dots, \alpha_n)$  input pattern,  $\alpha_i \in \{0, 1\}, 1 \leq i \leq n$ , and  $\forall j, 1 \leq j \leq m, f_j^*(\alpha_1, \alpha_2, \dots, \alpha_n) = f_j(\alpha_1, \alpha_2, \dots, \alpha_n)$ .

**7a26.** In a combinational circuit which has N lines, any number of stuck-at-0 or stuck-at-1 faults can occur. Find all possible multiple (not single) number of faults.

A.  $2^N$

B.  $3^N$

C.  $3^N - 1$

D.  $3^N - 2^N - 1$

E.  $2N$

**7a27.** What is a technical object model? B technical object is called A technical object model if through its experiments it is possible to have an idea about A technical object's

A. design

B. properties

C. parameters

D. circuit

E. structure

- 7a28.** In what sequence are the following stages of digital IC design executed?  
a) layout design, b) behavioral-level design, c) logic design, d) RTL design  
A. *c-a-b-d*  
B. *d-b-c-a*  
C. *b-d-c-a*  
D. *c-a-d-b*  
E. *a-b-c-d*
- 7a29.** In component level of IC design what kind of mathematical method is used?  
A. *Probability theory*  
B. *Theory of queue system*  
C. *Boolean algebra*  
D. *Differential equations system*  
E. *Partial differential equations system*
- 7a30.** In case of the given elemental bases, in what sequence does the consumption power increase if the other parameters are similar? a) bipolar; b) CMOS; c) N-MOS  
A. *b - c-a*  
B. *a-c-b*  
C. *a-b-c*  
D. *b-a-c*  
E. *c-b-a*
- 7a31.** Which of the given answers more contributes to the increase of performance in digital circuits?  
A. *Decrease of load capacitance*  
B. *Increase of load capacitance*  
C. *Decrease of technological sizes*  
D. *Decrease of supply voltage*  
E. *A. and C. together*
- 7a32.** Which of the given answers more characterizes the advantage of a MOS elemental base in comparison to bipolar?  
A. *Little consumption power*  
B. *High performance*  
C. *Simplicity of technology*  
D. *A and C together*  
E. *A and B together*
- 7a33.** A password must contain 2 numerals from the list {0, 1, 2, 3, 4, 5, 6, 7, 8, 9} and 4 letters from the list {a, A, b, B, c, C, d, D, e, E}. The first symbol must be a letter. Symbols may be repetitive. How many passwords can be generated meeting those conditions?  
A. *1 000 000*  
B. *2 000 000*  
C. *5 000 000*  
D. *8 000 000*  
E. *10 000 000*
- 7a34.** A password must contain 2 numerals from the list {0, 1, 2, 3, 4, 5, 6, 7, 8, 9} and 2 letters from the list {a, A, b, B, c, C, d, D, e, E}. The first symbol must be a letter. Symbols may be repetitive. How many passwords can be generated meeting those conditions?  
A. *100 000*  
B. *400 000*  
C. *600 000*  
D. *800 000*  
E. *1 000 000*
- 7a35.** A password must contain 2 numerals from the list {0, 1, 2, 3, 4, 5, 6, 7, 8, 9} and 3 letters from the list {a, A, b, B, c, C, d, D, e, E}. The first symbol must be a letter. Symbols may be repetitive. How many passwords can be generated meeting those conditions?  
A. *600 000*  
B. *800 000*  
C. *1 000 000*  
D. *1 200 000*  
E. *1 800 000*

- 7a36.** A password must contain 3 numerals from the list {0, 1, 2, 3, 4, 5, 6, 7, 8, 9} and 3 letters from the list {a, A, b, B, c, C, d, D, e, E}. The first symbol must be a letter. Symbols may be repetitive. How many passwords can be generated meeting those conditions?  
 A. 100 000  
 B. 1 000 000  
 C. 10 000 000  
 D. 100 000 000  
 E. 1 000 000 000
- 7a37.**  $\bar{x}_1(x_2x_3 \vee x_4(x_5 \oplus \bar{x}_6))$  norm of function equals:  
 A. 3/32  
 B. 9/32  
 C. 5/36  
 D. 7/32  
 E. 11/32
- 7a38.**  $x_1x_2(\bar{x}_3 \oplus x_4(\bar{x}_5 \vee x_6))$  norm of function equals:  
 A. 3/16  
 B. 1/8  
 C. 5/8  
 D. 1/16  
 E. 3/4
- 7a39.**  $(x_1 \vee x_2) \oplus x_3(\bar{x}_4 \vee \bar{x}_5)\bar{x}_6$  norm of function equals:  
 A. 21/32  
 B. 25/32  
 C. 27/32  
 D. 11/32  
 E. 19/32
- 7a40.**  $x_1$  argument activity of  $x_1\bar{x}_2(x_3 \vee x_4\bar{x}_5x_6)$  function equals:  
 A. 5/32  
 B. 9/32  
 C. 9/16  
 D. 7/16  
 E. 19/32
- 7a41.**  $x_3$  argument activity of  $x_1\bar{x}_2(x_3 \vee x_4\bar{x}_5x_6)$  function equals:  
 A. 5/16  
 B. 11/32  
 C. 7/32  
 D. 5/32  
 E. 13/16
- 7a42.**  $x_4$  argument activity of  $x_1\bar{x}_2(x_3 \vee x_4\bar{x}_5x_6)$  function equals:  
 A. 13/32  
 B. 1/32  
 C. 9/32  
 D. 7/32  
 E. 11/32
- 7a43.** The activity of the combination of  $x_4$  and  $x_5$  arguments for  $(\bar{x}_1 \vee x_2) \oplus (x_3 \vee x_4\bar{x}_5)$  function equals:  
 A. 3/4  
 B. 1/4  
 C. 5/8  
 D. 5/16  
 E. 3/8
- 7a44.** The activity of the combination of  $x_3$  and  $x_4$  arguments for  $(\bar{x}_1x_2) \oplus (x_3 \vee \bar{x}_4)x_5$  function equals:  
 A. 1/8  
 B. 7/8  
 C. 1/4  
 D. 3/4  
 E. 5/8

- 7a45.** The activity of the combination of  $x_4$  and  $x_5$  arguments for  $(\bar{x}_1 \vee \bar{x}_2) \oplus (x_3 \vee \bar{x}_4 \bar{x}_5)$  function equals:
- A. 3/4
  - B. 1/4
  - C. 5/8
  - D. 1/8
  - E. 5/4

- 7a46.** This table defines AND function for 5-valued logic of D-algorithm. Fill in the missing values.

| AND       | 0 | 1         | x | D | $\bar{D}$ |
|-----------|---|-----------|---|---|-----------|
| 0         | 0 | 0         | 0 | 0 | 0         |
| 1         | 0 | 1         |   | D | $\bar{D}$ |
| x         | 0 |           |   |   |           |
| D         | 0 | D         |   | D | 0         |
| $\bar{D}$ | 0 | $\bar{D}$ |   | 0 | z         |

- A. All the missing values must be 0
  - B. All the missing values must be 1
  - C. All the missing values must be D
  - D. All the missing values must be  $\sim D$  (inverse of D)
  - E. All the missing values must be x
- 7a47.** This table defines OR function for 5-valued logic of D-algorithm. Fill in the missing values.

| OR        | 0         | 1 | x | D | $\bar{D}$ |
|-----------|-----------|---|---|---|-----------|
| 0         | 0         | 1 |   | D | $\bar{D}$ |
| 1         | 1         | 1 | 1 | 1 | 1         |
| x         |           | 1 |   | 1 |           |
| D         | D         | 1 |   | D | 1         |
| $\bar{D}$ | $\bar{D}$ | 1 |   | 1 | $\bar{D}$ |

- A. All the missing values must be 0
  - B. All the missing values must be 1
  - C. All the missing values must be D
  - D. All the missing values must be  $\sim D$  (inverse of D)
  - E. All the missing values must be x
- 7a48.** The combinational circuit is divided into components each of which must be verified pseudo exhaustively: Let  $C=\{C_1, C_2, \dots, C_k\}$  be a division of n-input combinational circuit, with respective  $n_1, n_2, \dots, n_k$  inputs of k components. In that case which of the 5 variants does the number of input sets of pseudo exhaustive test correspond to?
- A.  $n_1+n_2+\dots+n_k$
  - B.  $n_1 \cdot n_2 \cdot \dots \cdot n_k$
  - C.  $2^{n_1+n_2+\dots+n_k}$
  - D.  $2^{n_1} + 2^{n_2} + \dots + 2^{n_k}$
  - E.  $n_1 2^{n_1} + n_2 2^{n_2} + \dots + n_k 2^{n_k}$
- 7a49.** The combinational circuit is divided into components each of which must be verified exhaustively. Let  $C=\{C_1, C_2, \dots, C_k\}$  be a division of n-input combinational circuit, with respective  $n_1, n_2, \dots, n_k$  inputs of k components. In that case which of the 5 variants does the number of input sets of exhaustive test correspond to?
- A.  $n_1+n_2+\dots+n_k$
  - B.  $n_1 \cdot n_2 \cdot \dots \cdot n_k$
  - C.  $2^{n_1+n_2+\dots+n_k}$
  - D.  $2^{n_1} + 2^{n_2} + \dots + 2^{n_k}$
  - E.  $n_1 2^{n_1} + n_2 2^{n_2} + \dots + n_k 2^{n_k}$

- 7a50.**  $(x_1 \oplus x_2)(x_3 x_4 \vee (x_5 \oplus x_6))$  function's norm equals:  
 A. 9/16  
 B. 3/16  
 C. 5/16  
 D. 11/16  
 E. 1/16
- 7a51.**  $\bar{x}_1 x_2 \oplus (x_3 \vee x_4 \bar{x}_5 \bar{x}_6)$  function's norm equals:  
 A. 19/32  
 B. 17/32  
 C. 11/16  
 D. 19/32  
 E. 22/16
- 7a52.**  $(\bar{x}_1 x_2 \vee x_3) x_4 (x_5 \oplus \bar{x}_6)$  function's norm equals:  
 A. 7/64  
 B. 21/32  
 C. 5/32  
 D. 13/16  
 E. 15/32
- 7a53.** The activity of argument  $x_1$  for the function  $(x_1 x_2 \oplus \bar{x}_3) \vee x_4$  is equal to:  
 A. 5/8  
 B. 3/4  
 C. 1/4  
 D. 7/4  
 E. 5/4
- 7a54.** The activity of argument  $x_3$  for the function  $(x_1 \bar{x}_2 \oplus \bar{x}_3) \vee x_4$  is equal to:  
 A. 1/4  
 B. 1/2  
 C. 3/8  
 D. 5/8  
 E. 3/16
- 7a55.** The activity of argument  $x_5$  for the function  $(x_1 \bar{x}_2 \oplus \bar{x}_3) \vee x_4 x_5$  is equal to:  
 A. 3/4  
 B. 5/8  
 C. 1/4  
 D. 3/8  
 E. 5/16
- 7a56.** The activity of the combination of arguments  $x_4$  and  $x_5$  for the function  $x_1 x_2 \oplus (x_3 \vee x_4 \vee x_5)$  is equal to:  
 A. 5/8  
 B. 3/2  
 C. 1/2  
 D. 5/2  
 E. 1/8
- 7a57.** The activity of the combination of arguments  $x_1$  and  $x_2$  for the function  $(x_1 \vee x_2) \oplus (\bar{x}_3 \vee \bar{x}_4 \vee \bar{x}_5)$  is equal to:  
 A. 5/8  
 B. 1/2  
 C. 3/4  
 D. 3/8  
 E. 5/16

- 7a58.** The activity of the combination of arguments  $x_1$ ,  $x_2$  and  $x_3$  for the function  $x_1 x_2 x_3 (\overline{x_4} \vee \overline{x_5})$  is equal to:
- A.  $5/8$
  - B.  $3/16$
  - C.  $7/16$
  - D.  $5/16$
  - E.  $3/8$
- 7a59.** What design stage does the technological mapping follow?
- A. *High level synthesis*
  - B. *Logic synthesis*
  - C. *Physical synthesis*
  - D. *Circuit design*
  - E. *Technological design*
- 7a60.** In what sequence do the effects of design solutions increase on the final quality of the design a) conceptual, b) structural and c) parametrical synthesis steps?
- A. *a - b - c*
  - B. *c - b - a*
  - C. *b - c - a*
  - D. *b - a - c*
  - E. *c - a - b*
- 7a61.** What is the difference of electrical “short” or “long” interconnects at most characterized by?
- A. *Signal amplitude*
  - B. *Signal power*
  - C. *Signal edge increase*
  - D. *Current power*
  - E. *A. and B. together*
- 7a62.** Which of the numerated devices are used in high level synthesis algorithms? a) register; b) multiplexer; c) flip-flop; d) transistor; e) inverter; f) driver
- A. *a-b-e*
  - B. *a-c-f*
  - C. *a-b-d-f*
  - D. *a-b-f*
  - E. *c-d-e-f*
- 7a63.** Let Q be the number of integers from the segment  $[1, 2, \dots, 2010]$  that are divisible by 2 and 3 but not by 5. Find the number closest to Q.
- A. *150*
  - B. *200*
  - C. *250*
  - D. *300*
  - E. *350*
- 7a64.** Let Q be the number of integers from the segment  $[1, 2, \dots, 2010]$  that are divisible by 3 and 5 but not by 7. Find the number closest to Q.
- A. *100*
  - B. *120*
  - C. *150*
  - D. *170*
  - E. *200*
- 7a65.** Let Q be the number of integers from the segment  $[1, 2, \dots, 2010]$  that are divisible by 3 and 7 but not by 11. Find the number closest to Q.
- A. *70*
  - B. *80*
  - C. *90*
  - D. *100*
  - E. *110*



- 7a66.** Let Q be the number of integers from the segment [1,2, ... , 2010] that are divisible by 5 and 7 but not by 11. Find the number closest to Q.
- A. 15  
B. 20  
C. 50  
D. 70  
E. 80

**b) Problems**

**7b1.**

Construct Zhegalkin polynomial for the threshold function with threshold  $w=3$  and weights of variables  $\xi_1=2, \xi_2=2, \xi_3=2$ .

**7b2.**

Verify the completeness of functions'  $\{x_1 \vee x_2, x_1 \rightarrow x_2, \overline{x_1 x_2 x_3}\}$  system.

**7b3.**

Using Post theorem verify the completeness of functions'  $\{1, \overline{0}, \overline{x}, (x_1 \rightarrow x_2) \rightarrow x_3\}$  system.

**7b4.**

Construct Zhegalkin polynomial of  $(x_1 \rightarrow x_2)^{\overline{x_3}}$  function.

**7b5.**

Prove that for N bit number it is possible to calculate the number of 1-s for  $O(\log N)$  time.

**7b6.**

Prove that in N bit number it is possible to calculate the number of 1s in  $O(M)$  time where M is the number of 1s.

**7b7.**

Prove that in unordered array  $K^{\text{th}}$  search can be executed in linear time.

**7b8.**

Prove that ordering based on binary heap is executed in  $O(N \log N)$  time.

**7b9.**

Construct complete disjunctive normal form of the Boolean function given by 11011011 table.

**7b10.**

Construct Zhegalkin polynomial of the Boolean function given by 11011101 table.

**7b11.**

Cayley code of G tree by numbered nodes is given  $h(G)=(3,5,4,4,5,6,7,8)$ . Reconstruct the tree.

**7b12.**

Check the completeness of  $\{x_1^{x_2} \vee x_2^{x_3}, x_1 \overline{x_2}, x_1 \mapsto x_2\}$  system of functions.

**7b13.**

Cayley code of G tree by numbered nodes is given  $h(G)=(3,2,4,4,5,6,8,8)$ . Reconstruct the tree.

**7b14.**

Check the completeness of  $\{x_1^{x_3} \vee x_2^{x_1}, x_1 \rightarrow \overline{x_2}, x_1 \oplus x_2\}$  system of functions.

**7b15.**

Calculate arguments'  $\xi_1 = 2, \xi_2 = 5, \xi_3 = 7, \xi_4 = 10$  balances and argument activities having  $w=10$  threshold function.

**7b16.**

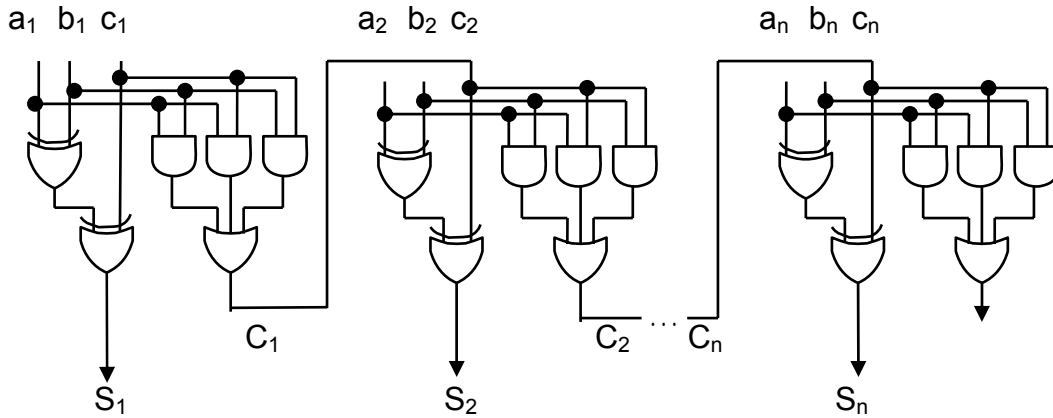
Check the completeness of functions'  $\{x_1 x_2, x_1 \rightarrow x_2, x_1 \oplus x_2, x_1 \vee x_2\}$  system.

**7b17.**

Given an n-input Boolean function  $f(x_1, x_2, \dots, x_n) = x_1 \oplus x_2 \oplus \dots \oplus x_n$ . Construct the Binary Decision Diagram for  $f(x_1, \dots, x_i, \dots, x_n)$ .

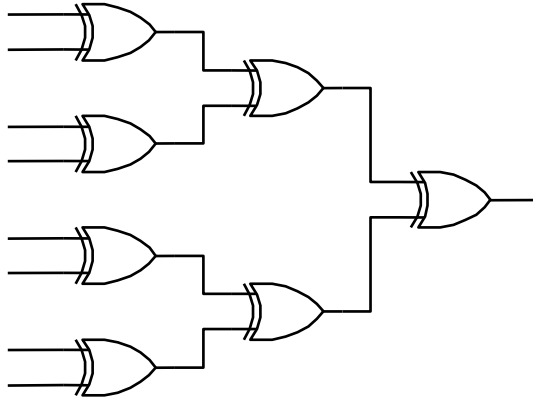
**7b18.**

Figure depicts the circuit of an n-bit adder. Prove that all possible single stuck-at-0 and stuck-at-1 faults on all lines of that circuit can be detected by means of only 8 test vectors.



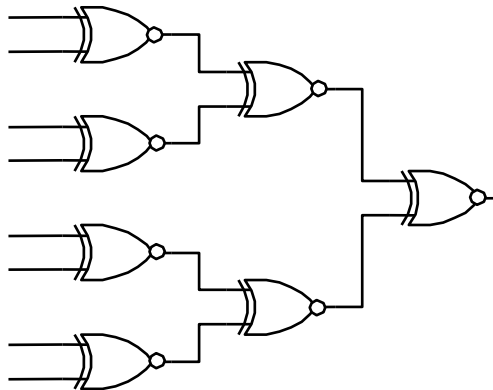
**7b19.**

Figure depicts the circuit of an N-input parity tree. Prove that all possible single stuck-at-0 and stuck-at-1 faults on all lines in that circuit can be detected by means of only 4 test vectors.



**7b20.**

Figure depicts the circuit of N-input tree with negated Modulo 2 elements (gates XOR). Prove that all possible single stuck-at-0 and stuck-at-1 faults on all lines of that circuit can be detected by means of only 4 test vectors.

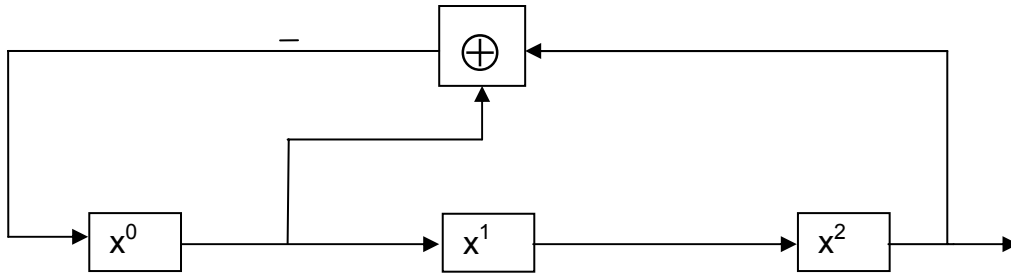


**7b21.**

The given combinational circuit has 1000000 lines. Constant 0 or constant 1 faults of random numbers can occur there. Find the number of all possible multiple faults.

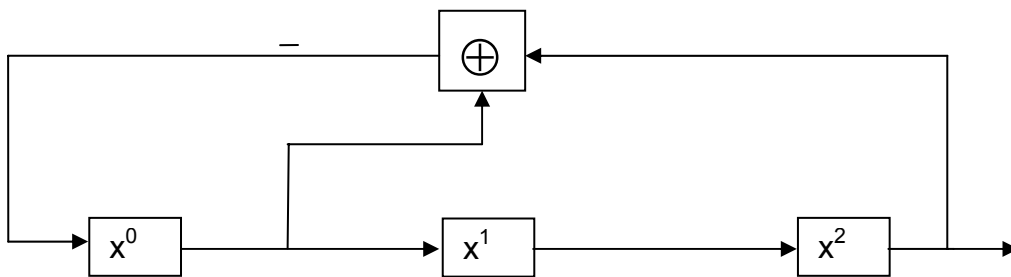
**7b22.**

Given a Linear Feedback Shift Register corresponding to characteristic polynomial  $1+x+x^3$ . Find the subsets of all patterns which are generated by the given LFSR.



**7b23.**

Given a Linear Feedback Shift Register corresponding to characteristic polynomial  $1+x+x^3$ . Transform the circuit, adding elementary cell(s) in a way that the obtained new circuit generates one set, consisting of all patterns.



**7b24.**

Construct Zhegalkin polynomial of the Boolean function given by  $f = x_1 \oplus x_3^{x_1 \vee x_2}$ .

**7b25.**

By root tree's 000101001111 code, reconstruct that tree.

**7b26.**

Calculate arguments activities of  $x_1x_2 \oplus (x_1 \vee x_2x_3)$  function.

**7b27.**

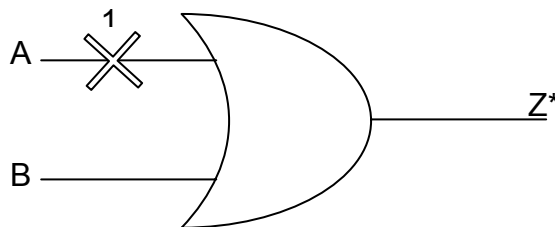
Cayley code of G tree by numbered nodes is given  $h(G)=(2, 3, 3, 4, 5, 5, 8)$

Reconstruct the tree.

**7b28.**

Let  $T(F)$  be a test set for a fault  $F$ , a set of all possible input patterns detecting  $F$ .

Definition: Faults  $F_1$  and  $F_2$  from  $T(F)$  are called equivalent faults if  $T(F_1) \subseteq T(F_2)$  and  $T(F_2) \subseteq T(F_1)$ , i.e.,  $T(F_1) = T(F_2)$ .



Find the faults that are equivalent to the mentioned in the figure fault "line A stuck-at-1".

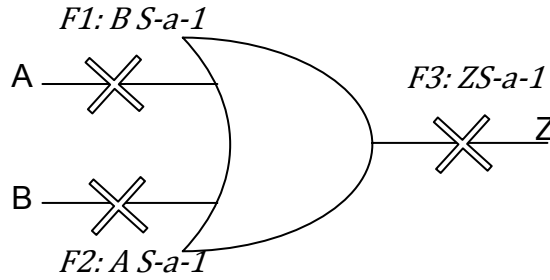
**7b29.**

The combinational circuit is partitioned into components each to be tested exhaustively. Let  $C=\{C_1, C_2, \dots, C_5\}$  be a partition of a combinational circuit with  $n$  inputs into 5 components with 12, 14, 16, 18, 20 primary inputs. Calculate the number of input patterns for the pseudoexhaustive test with respect to the partition.

**7b30.**

Let  $T(F)$  be a test set for a fault  $F$ , a set of all possible input patterns detecting  $F$ .

Definition: Faults  $F_1$  and  $F_2$  from  $T(F)$  are called equivalent faults if  $T(F_1) \subseteq T(F_2)$  and  $T(F_2) \subseteq T(F_1)$ , i.e.,  $T(F_1) = T(F_2)$ .

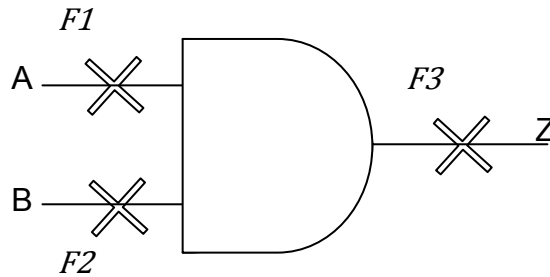


Prove that faults  $F_1$ ,  $F_2$  and  $F_3$  are equivalent.

**7b31.**

Suppose, given a 2-input, 1-output combinational circuit depending on input variables  $A$ ,  $B$  and  $Z$  output variable implementing the function of conjunction  $Z = A \cdot B$ . Let  $T(F)$  be a test set for a single fault  $F$ , a set of all possible input patterns detecting  $F$ .

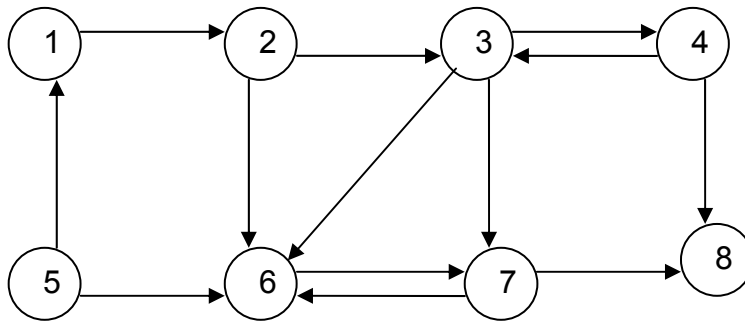
Definition: Faults  $F_1$  and  $F_2$  from  $T(F)$  are called equivalent faults if  $T(F_1) \subseteq T(F_2)$  and  $T(F_2) \subseteq T(F_1)$ , i.e.,  $T(F_1) = T(F_2)$ .



Prove that faults  $F_1$ ,  $F_2$  and  $F_3$  are equivalent.

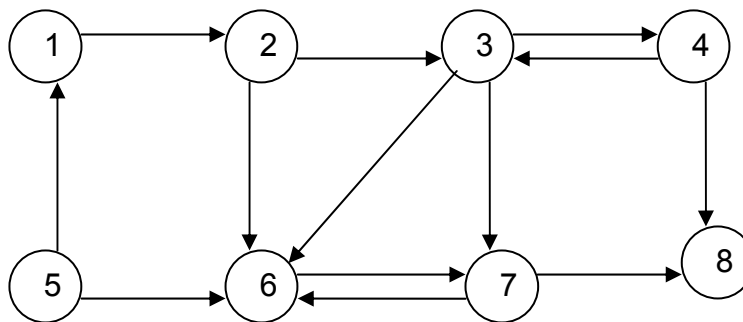
**7b32.**

In the given directed graph, realize search according to depth starting from the 2<sup>nd</sup> vertex.



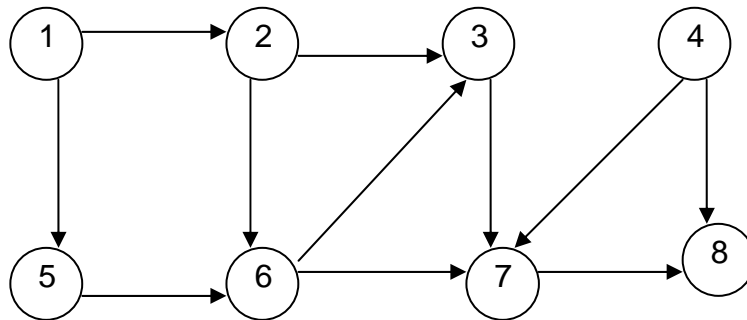
**7b33.**

In the given directed graph, separate strongly connected components.



**7b34.**

For the given directed graph, perform topological sorting.



**7b35.**

In 7 x 7 discrete field realize Lee algorithm by the following initial conditions:

The connecting contact coordinates A(1;2) , B(7;5).

Objective functions are:  $f_1$  – number of crossings;  $f_2$  – connection length.

Both objective functions are minimizing and respectively have the following importance coefficients  $a_1=2$ ,  $a_2=1$ .

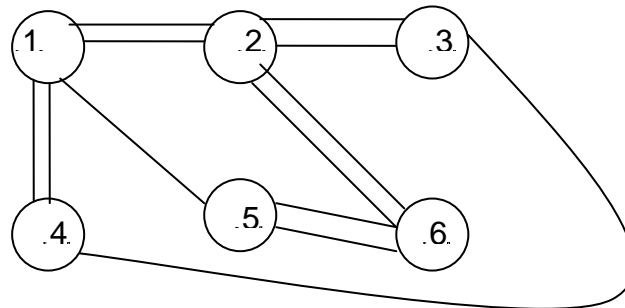
The discrete values of already occupied interconnections are: (2,5); (3,5); (4,1); (4,2); (4,3); (4,4); (4,5); (6,1); (6,3); (6,4); (6,5); (6,6)

**7b36.**

Given net's contact coordinates, having 5 contacts in a discrete field in conditional units: a(2,9); b(8,8); c(2,5); d(8,3); e(2,2): It is required to construct the orthogonal distance matrix of contacts and by its help compute the minimal tree by Prim's algorithm, realizing the given net.

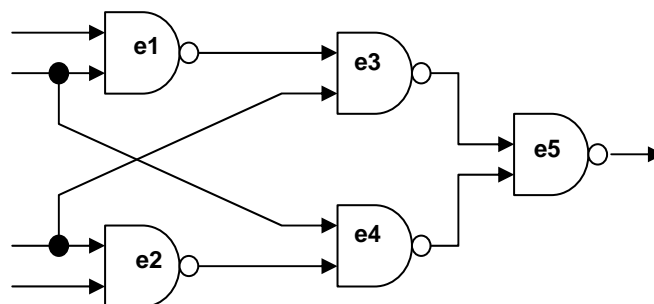
**7b37.**

For the given graph construct adjacency matrix and using partitioning sequential algorithm, partition into 2 equal parts. As an optimality criterion, take the minimum of connection numbers between the parts.



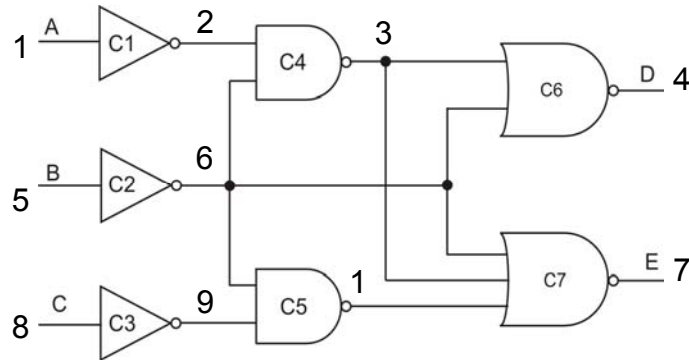
**7b38.**

Perform initial placement of the given circuit on the line using adjacency matrix, and using minimum interconnects length condition. As an initial element, take e1 cell.



**7b39.**

Construct the timing graph of the given circuit and compute its critical path delays if the cell delays are given in conditional units:  $C_1=C_3=5$ ,  $C_2=10$ ,  $C_5=20$ ,  $C_4=C_6=C_7= 30$ .



**7b40.**

Construct Zhegalkin polynomial of  $f = x_1 \oplus x_2^{x_1 \vee x_3}$  Boolean function.

**7b41.**

Reconstruct the tree of root tree by 001010001111 code.

**7b42.**

Calculate argument activities of  $\overline{x_1 x_3} \oplus (x_1 \vee x_2)$  function.

**7b43.**

$f = (x_1, x_2, x_3)$  threshold function's variable weights are:  $\xi_1=3$ ,  $\xi_2=4$ ,  $\xi_3=6$ , and the threshold  $w=5$ .

Construct the perfect disjunctive normal form of  $f = (x_1, x_2, x_3)$  function.

**7b44.**

Check the completeness of  $\{\overline{x_1} \rightarrow x_2, x_1 x_3, x_1 \vee x_2\}$  system of functions.

## 8. OBJECT-ORIENTED PROGRAMMING

### a) Test questions

- 8a1.** A function is formed that displays data (on the screen) as well as writes them in the file. Referring to the scenario, which one of the following function declarations satisfies that need?
- A. `void print(cout);`
  - B. `void print(ostream &os);`
  - C. `void print(istream &is);`
  - D. `void print(ofstream ofs);`
  - E. `void print(istream is);`
- 8a2.** After performing the below mentioned code, which is the value of n?  
int n = !(15 & !7)
- A. *false*
  - B. 1
  - C. 3
  - D. 5
  - E. *It contains an error. After using ! operator, constant values should be definitely taken in brackets, e.g. !(5)*
- 8a3.** Small function is present, which is often called from several special places. How can the implementation of the codes be accelerated which use the given function?
- A. *Make function virtual*
  - B. *Replace floating point computation of integer numbers, to use FPU device*
  - C. *Reduce the use of automat variables*
  - D. *Make all the variables of that function volatile*
  - E. *Make function inline*
- 8a4.** Which is the difference between non-specialized function's member and constructor?
- A. *Constructor can return values, and member-functions - no*
  - B. *Member-functions can define values, constructors - no.*
  - C. *Constructor can define values, and member-functions - no*
  - D. *Member-functions can return values, and constructors - no*
  - E. *Constructor can announce values, and member-functions - no*
- 8a5.** `const int x= 0xFFFE;`  
`int y = 2;`  
`int z = x&& y;`  
Which is the value of z, in terms of the above code?
- A. 0
  - B. 1
  - C. 2
  - D. 3
  - E. 4
- 8a6.** Which of the below mentioned statements concerning overloading of ++ operator is true?
- A. *It is impossible to overload prefix ++ operator*
  - B. *It is impossible to overload postfix ++ operator*
  - C. *It is necessary to use additional int type parameter to overload ++ postfix operator*
  - D. *It is always necessary to create prefix ++ operator*
  - E. *It is impossible to overload both prefix and postfix ++ operators for one class.*
- 8a7.** Binary search complexity is
- A.  $O(\log N)$ , where N is the number of elements
  - B.  $O(N \log N)$
  - C.  $O(N)$
  - D.  $O(N^2)$
  - E. *The correct answer is missing*
- 8a8.** The complexity of Prim's algorithm is
- A.  $O(|E|)$
  - B.  $(|V|^2)$
  - C.  $O(|E| * |V|)$
  - D.  $O(|E| + |V|)$
  - E. *The correct answer is missing*

- 8a9.** When the class is inherited from base as public, which of the below mentioned statements is right?
- All the members of base class become public members of inherited class
  - All the members of inherited class become public members of base class
  - All the protected members of base class are protected members of inherited class
  - All the members of base class become protected members of inherited class
  - All the members of inherited class become private members of base class
- 8a10.** The ability to invoke a method of an object without knowing exactly what type of object is being acted upon is known as which one of the following?
- Encapsulation
  - Class relationship
  - Inheritance
  - Polymorphism
  - Friend relationship
- 8a11.**
- ```
class X
{
    int I;
public:
    int f() const;
};
int X::f() const {return I++;}
```
- Where is an error in the above written code?
- X::f* member-function must be static
 - X::f* member-function is constant but changes non-mutable data-member of the object
 - X::I* data-member lacks access specifier
 - X::f* member-function cannot change as it lacks access specifier
 - It is not possible to change the integer I as it lacks access specifier
- 8a12.** The complexity of quick sorting algorithm for the worst case will be
- $O(\log N)$, where N is the number of elements
 - $O(N \log N)$
 - $O(N)$
 - $O(N^2)$
 - The correct answer is missing
- 8a13.**
- ```
class Base {
public:
 Base();
 ~Base();
 int getBaseNum();
private:
 int baseNum;
};

class A : public Base{
public:
 A();
 ~A();
 float getBaseNum();
private:
 float baseNum;
};
```
- Which concept is presented through the code in the example?
- Recursion
  - Polymorphism
  - Inheritance
  - Reloading of functions
  - Virtual functions
- 8a14.**
- ```
int i = 4, x = 0;
do {
    x++;
} while(i--);
```
- What is the value of X after performing the above written code?

- A. 5
- B. 4
- C. 0
- D. Infinite
- E. There is syntax error while cycle cannot be formed as mentioned above

- 8a15.** The complexity of Floyd algorithm is
- A. $O(N^3)$, where N is the number of nodes
 - B. $O(N \log N)$
 - C. $O(N)$
 - D. $O(N^2)$
 - E. The correct answer is missing

- 8a16.** The complexity of subline search in Knut-Morris-Pratt algorithm on average will be (n – length of the line, m – length of the subline)
- A. $O(n)$
 - B. $O(m)$
 - C. $O(n+m)$
 - D. $O(n*m)$
 - E. The correct answer is missing

- 8a17.**
- ```
class MyClass {
public:
 MyClass();
 virtual void MyFunction()=0;
};
```

Which is the below written statements is correct for the given code?

- A. *MyClass is a pure virtual class*
  - B. *Class definition is wrong*
  - C. *MyClass is a virtual base class*
  - D. *Function returns value*
  - E. *MyClass is an abstract class*
- 8a18.** Which of the below written statements is correct for function overloading?
- A. Although the return type can be modified, the types of the parameters can as well. The actual number of parameters cannot change.
  - B. *Function overloading is possible in both C++ and C*
  - C. Templates and namespaces should be used to replace occurrences of function overloading.
  - D. Overloaded functions may not be declared as "inline."
  - E. The compiler uses only the parameter list to distinguish functions of the same name declared in the same scope.
- 8a19.** The complexity of Dexter algorithm is
- A.  $O(|E|)$
  - B.  $O(|E|*|V|)$
  - C.  $O(|V|^2)$
  - D.  $O(|E|+|V|)$
  - E. The correct answer is missing
- 8a20.** Assume an algorithm determines the number of triangles formed by  $n$  points in the plane. What is the maximal possible output of A for  $n=7$ ?
- A. 35
  - B. 27
  - C. 17
  - D. 15
  - E. The correct answer is missing
- 8a21.** What is the difference between class and struct?
- A. *Class must contain constructor, and struct may not have it*
  - B. *Struct does not have inheritance opportunity*
  - C. *Struct does not have destructor*
  - D. *By default input specifiers are distinguished*
  - E. *No difference*

- 8a22.** Which is the value of z in case of the code below?  
`const int x= 012;`  
`int z = 1 << x;`  
 A. 0  
 B. 1024  
 C. 4096  
 D. 2048  
 E. *It contains an error and will get compilation error*
- 8a23.** Which of the mentioned cycles is infinite?  
 A. `for (int i=1;i>23;i++);`  
 B. `for (int i=0;i>=1;i++);`  
 C. `for (int i=10;i>6;i++);`  
 D. `for (int i=5;i>15;i++);`  
 E. *All the cycles are finite*
- 8a24.** After performing the code below, what will be displayed on the screen? (assume `sizeof(int) = 4`)  
`#include <iostream>`  
`using namespace std;`  
`void main(){`  
`int (*p)[10]= {NULL};`  
`int k= int((size_t)(p+1) - (size_t)p);`  
`cout<<k<<endl;`  
`}`  
 A. 1  
 B. 2  
 C. 4  
 D. 10  
 E. 40
- 8a25.** Which of the mentioned ones is not heredity access attribute  
 A. *public*  
 B. *private*  
 C. *virtual*  
 D. *protected*  
 E. *All are heredity access attribute*
- 8a26.** For a given pair of integers  $n, k$  it is calculated  $n^k$ . Assuming that calculation uses only multiplications, show the minimal number of multiplications among represented in the table that is enough to calculate  $5^{1024}$ .  
 A. 12  
 B. 11  
 C. 10  
 D. 9  
 E. 8
- 8a27.** After performing the code below, which is the value of n?  
`int i = 5;`  
`int n = i++-1;`  
 A. 6  
 B. 5  
 C. 4  
 D. 3  
 E. *It contains an error and will get compilation error*

**8a28.** After performing the code below, which is the value of b?

```
#include <iostream>
using namespace std;
void main () {
 int a = 0;
 int b = 2;
 switch (b) {
 case 1:
 a=1;
 break;
 case 2:
 int b=0;
 }
}
```

- A. 0
- B. 1
- C. 2
- D. *There is syntax error, switch is not possible to form as described above*
- E. *The program will work infinitely*

**8a29.** Which of the mentioned is not C++ data type?

- A. *unsigned long*
- B. *unsigned short*
- C. *unsigned char*
- D. *unsigned int*
- E. *All are C++ data types*

**8a30.** Which of the mentioned is not one of the basic clauses of object-oriented programming?

- A. *Encapsulation*
- B. *Typification*
- C. *Inheritance*
- D. *Polymorphism*
- E. *All the mentioned belong to the basic clauses of object-oriented programming.*

**8a31.** An algorithm is developed that for any given list of natural numbers  $L = \{a_1, \dots, a_n\}$  finds the maximum, minimum elements and the arithmetical average value of L. What is the minimal number of passes in reading L that is enough get the output in the algorithm?

- A. 5
- B. 4
- C. 3
- D. 2
- E. 1

**8a32.** After performing the code below, which is the value of n?

```
int n = 1 << 3 * 2 + 1;
```

- A. 1
- B. 17
- C. 65
- D. 128
- E. *It contains an error and will get compilation error*

**8a33.** To classify the following 1,2,3,5,4 sequence ascending, it is more appropriate to use

- A. *Quick Sort*
- B. *Bubble Sort*
- C. *Merge Sort*
- D. *Heap Sort*
- E. *No appropriate version*

**8a34.** After performing the code below, what will be displayed on the screen?

```
#include <iostream>
using namespace std;
double A;
void main ()
```

```

{
 int A;
 A=5;
 ::A = 2.5;
 cout<<A<<" "<<::A;
}

```

- A. 5 2.5
- B. 2.5 5
- C. 5 5
- D. 2.5 2.5
- E. The program contains a syntax error

**8a35.** How much is the complexity of algorithm to add element in binary tree?

- A.  $O(N^2)$ , where  $N$  is the number of nodes
- B.  $O(\log N)$
- C.  $O(N)$
- D.  $O(1)$
- E. The correct answer is missing

**8a36.** An algorithm is developed that for any given list of natural numbers  $L = \{a_1, \dots, a_n\}$  finds the sum

$\sum_{i=1}^n (a_i - \bar{a})^2$ , where  $\bar{a}$  denotes the arithmetical average of the elements list  $L$ : What is the floor

amount of passes enough to calculate the output?

- A. 1
- B. 2
- C. 3
- D. 4
- E. 5

**8a37.** Which of the statements is correct?

- A. Class cannot have 2 constructors
- B. Class cannot have 2 destructors
- C. Virtual destructor does not exist
- D. All the above mentioned statements are correct
- E. All the above mentioned statements are wrong

**8a38.** The complexity of Merge Sort algorithm in the worst case will be:

- A.  $O(N \log N)$
- B.  $O(N^2)$
- C.  $O(N)$
- D.  $O(N^3)$
- E. The correct answer is missing

**8a39.** After performing the code below, what will be displayed on the screen?

```

#include <stdio.h>
class a {
public: virtual void print() {printf("a");}
};
class b: public a{
public: virtual void print() {printf("b");}
};

```

```

int main(){
 b b0;
 a a0= b0, &a1= b0;
 a0.print();
 a1.print();
 return 0;
}

```

- A. aa
- B. ab
- C. bb
- D. ba
- E. The program contains syntax error

**8a40.** After performing the code below, what will be displayed on the screen?

```
#include <iostream>
using namespace std;
```

```
void main(){
 int c = 1;
 c= ++c + ++c;
 cout<<c;
}
```

- A. 2
- B. 3
- C. 4
- D. 5
- E. 6

**8a41.** What is the output of the following part of the program?

```
cout << (2 | 4 ^ ~3);
```

- A. 0
- B. 1
- C. 2
- D. 3

E. *the correct answer is missing*

**8a42.** How many errors are below?

```
class B {}
class A
 : public B
{
 int m_value
 int get_value(int = 0)
 {
 return m_value;
 }
}
int main () {
 A a();
 a = a;
 a.*get_value();
 return 0;
}
```

- A. 3
- B. 4
- C. 5
- D. 6
- E. 7

**8a43.** What is  $(7 \gg 1 \ll 1)$  expression value?

- A. 5
- B. 6
- C. 7
- D. 8

E. *the correct answer is missing*

**8a44.** What is the output of the following part of the program?

```
int n = 5;
switch(n) {
 case '5':
 cout << "A \n";
 break;
 case 5:
 cout << "B \n";
 default:
 cout << "C \n";
 break;
}
```

- A. A
- B. B
- C. C
- D. BC
- E. AC

8a45. Complexity of “merge” sorting algorithm

- A. linear
- B. constant
- C. logn
- D.  $n \cdot \log n$
- E. square

8a46. `int n = !(5 & 7)`

What is the value of n after performing the code above?

- A. false
- B. true
- C. 0
- D. 1

E. It has an error. Using ! operator, constant values must be written in brackets. For example, (!5)

8a47. Complexity of adding element in search balanced binary tree

- A. linear
- B. constant
- C. logn
- D.  $n \cdot \log n$
- E. square

8a48. `int i = 4, x = 0;`

```
do{
 x++;
}while(i--);
```

After executing the above written code, which is the value of X?

- A. 0
- B. 1
- C. 3
- D. 4
- E. 5

8a49. `class MyClass`

```
{
public:
 MyClass();
 virtual void MyFunction()=0;
};
```

Which statement is true for the above mentioned code?

- A. Class definition is wrong
- B. MyClass is a virtual class
- C. MyClass is a virtual base class
- D. MyClass is an abstract class
- E. the correct answer is missing

8a50. What is the value of  $(13 \gg 1 \ll 1)$  expression?

- A. 11
- B. 12
- C. 13
- D. 14
- E. 15

**8a51.** How many errors does the following part contain?

```
class A
 : int
{
 int m_value
 int get_value(int = 0)
{
 return m_value;
}
}
int main () {
 A a();
 a = a;
 a->get_value();
return 0;
}
```

- A. 2
- B. 3
- C. 4
- D. 5
- E. 6

**8a52.** After performing the code below, what will be displayed on the screen?

```
#include <iostream>
struct A {
 virtual void f(int x = 0) {
 std::cout << "A: " << x << std::endl;
 }
};
struct B : public A {
 virtual void f(int x = 1) {
 std::cout << "B: " << x << std::endl;
 }
};
int main()
{
 A* p = new B();
 p->f();
 return 0;
}
```

- A. A 0
- B. A 1
- C. B 0
- D. B 1
- E. The correct answer is missing

**8a53.** After performing the code below, what will be displayed on the screen?

```
#include <iostream>
struct A {
 virtual ~A() {
 std::cout << "A" << std::endl;
 }
};
struct B : public A {
 virtual ~B() {
 std::cout << "B" << std::endl;
 }
};
int main() {
 A* p = new B();
 delete p;
 return 0;
}
```

- A. A
- B. A B
- C. B
- D. B A
- E. *The correct answer is missing*

**8a54.** After performing the code below, what will be displayed on the screen?

```
#include <iostream>
#include <memory>
void f(std::auto_ptr<int> a) {
 *a = 3;
}
int main() {
 std::auto_ptr<int> p(new int(0));
 f(p);
 std::cout << *p << std::endl;
 return 0;
}
```

- A. 0
- B. 3
- C. *The program contains compile-time error*
- D. *The program contains run-time error*
- E. *The correct answer is missing*

**8a55.** After performing the code below, what will be displayed on the screen?

```
#include <iostream>
struct Base {
 virtual void f() {
 foo();
 }
 virtual void foo() = 0;
};
struct Derived : public Base {
 virtual void foo() { std::cout << "Derived" << std::endl; }
};
int main() {
 Base* pb = new Derived();
 pb->f();
 return 0;}
```

- A. *The program contains compile-time error as foo function has been called without description*
- B. *Derived*
- C. *Nothing*
- D. *The program contains run-time error*
- E. *The correct answer is missing*

**8a56.** After performing the code below, what will be displayed on the screen?

```
#include <iostream>
int main() {
 int x = 1, y = 1;
 if (x++ || y++) {
 x += 5;
 }
 std::cout << y << std::endl;
 return 0;
}
```

- A. 1
- B. 2
- C. 5
- D. 6
- E. *The correct answer is missing*



**8a57.** After performing the code below, what will be displayed on the screen?

```
#include <iostream>
struct A {
 int m_x;
 A(int x) : m_x(x) { std::cout << m_x << " "; };
};
struct B : virtual public A {
 B() : A(5) { }
};
struct C : virtual public A {
 C() : A(0) { }
};
struct D : public B, public C {
 D() : A(1) { }
};
int main() {
 A* p = new D();
 return 0;
}
```

- A. 5 0 1
- B. 0 1
- C. 1
- D. *The program contains compile-time error*
- E. *The correct answer is missing*

**8a58.** After performing the code below, what will be displayed on the screen?

```
#include <iostream>
void f(int& x) {
 x += 3;
 std::cout << x << std::endl;
}
int main() {
 f(0);
 return 0;
}
```

- A. 0
- B. 3
- C. *The program contains compile-time error*
- D. *The program contains run-time error*
- E. *The correct answer is missing*

**8a59.** After performing the code below, what will be displayed on the screen?

```
#include <iostream>
struct A {
 int mx;
 A(int x) : mx(0) { }
};
int main() {
 A arr[5];
 for (int i = 0; i < 3; ++i) {
 std::cout << arr[i].mx << std::endl;
 }
 return 0;
}
```

- A. 0 0 0
- B. 5 5 5
- C. 3 3 3
- D. 3 3 3 0 0
- E. *The correct answer is missing*

**8a60.** After performing the code below, what will be displayed on the screen?

```
#include <iostream>
struct Exc1 {
 void what() { std::cout << "Exc1" << std::endl; }
};
struct Exc2 : public Exc1 {
 void what() { std::cout << "Exc2" << std::endl; }
};
int main() {
 try {
 throw Exc2();
 } catch (Exc1& e) {
 std::cout << "First catch: ";
 e.what();
 } catch (Exc2& e) {
 std::cout << "Second catch: ";
 e.what();
 } catch(...) {
 std::cout << "Unknown exception" << std::endl;
 }
 return 0;
}
```

- A. *First catch Exc1*
- B. *First catch Exc2*
- C. *Second catch Exc2*
- D. *Unknown exception*
- E. *The correct answer is missing*

**8a61.** After performing the code below, what will be displayed on the screen?

```
#include <iostream>
template <int i>
struct A {
 static const int n = i*A<i-1>::n;
};
template <>
struct A<1> {
 static const int n = 1;
};
int main(){
 std::cout << A<5>::n << std::endl;
 return 0;
}
```

- A. *The program contains compile-time error*
- B. *5*
- C. *24*
- D. *120*
- E. *The correct answer is missing*

**8a62.** Can the overloaded operator of the class be declared virtual?

- A. *Yes*
- B. *No*
- C. *Only ++ and – operators*
- D. *Only if the class contains virtual destructor*
- E. *The correct answer is missing*

**8a63.** After performing the code below, what will be displayed on the screen?

```
#include <iostream>
#include <typeinfo>
struct A {};
struct B : public A {};
int main() {
 A* p = new B();
 std::cout << typeid(p).name() << std::endl;
 return 0;
}
```

- A. Struct A\*
- B. Struct B\*
- C. The program contains compile-time error
- D. The program contains run-time error
- E. The correct answer is missing

**8a64.** What design pattern is presented below?

```
#include <iostream>
struct A {
 virtual void f() {
 func();
 }
 virtual void func() = 0;
};
struct B : public A {
 virtual void func() {}
};
```

- A. Virtual constructor
- B. Template method
- C. Strategy
- D. The program contains compile-time error
- E. The correct answer is missing

**8a65.** What design pattern is presented below?

```
struct A {
 virtual A* clone() { return new A(*this); }
};
struct B : public A {
 A* clone() { return new B(*this); }
};
```

- A. Virtual constructor
- B. Strategy
- C. Template method
- D. None
- E. The correct answer is missing

**8a66.** Which of the following class methods is not generated by compiler by default – in case of being not defined by the programmer?

- A. Default constructor
- B. Copy constructor
- C. Assignment operator
- D. Destructor
- E. The correct answer is missing

**8a67.** After performing the code below, what will be displayed on the screen?

```
#include <iostream>
void f(int) { std::cout << "Integer" << std::endl; }
void f(unsigned short) { std::cout << "Unsigned Short" << std::endl; }
void f(unsigned int) { std::cout << "Unsigned Integer" << std::endl; }
int main() {
 short c = 5;
 f(c);
 return 0;
}
```

- A. Integer
- B. Unsigned Short
- C. Unsigned Integer
- D. The program contains compile-time error
- E. The correct answer is missing

**8a68.** After performing the code below, what will be displayed on the screen?

```
#include <iostream>
#include <algorithm>
#include <functional>
struct A {
 virtual void f() { std::cout << "A "; }
```

```

};
struct B : public A {
 virtual void f() { std::cout << "B "; }
};
int main() {
 A* a[] = { new A(), new B() };
 std::for_each(a, a+2, std::mem_fun(&A::f));
 return 0;
}

```

- A. A A
- B. A B
- C. B A
- D. B B
- E. *The correct answer is missing*

**8a69.** What design pattern is presented below?

```

struct AImpl {
 void f() { }
};
class A {
 AImpl* m_impl;
public:
 A() : m_impl(new AImpl()) { }
 void f() { m_impl->f(); }
};

```

- A. *Virtual constructor*
- B. *Strategy*
- C. *Template method*
- D. *Bridge*
- E. *The correct answer is missing*

**8a70.** What data-members can be initialized at declaration?

- A. *Static*
- B. *Static const*
- C. *Static const integral type*
- D. *None*
- E. *The correct answer is missing*

## **b) Problems**

**8b1.**

Develop a program that, given two symbol sequences, finds the length of the longest common subsequence of the given sequences

For example,

SYNOPSIS, SINOPSIS → 7

**8b2.**

Assume A is the known W. Ackermann function:

$$A(0, y) = y + 1;$$

$$A(x, 0) = A(x - 1, 1);$$

$$A(x + 1, y + 1) = A(x, A(x + 1, y)).$$

Redefine A in a way that it is computed modulo 7717.

$$AM(x, y) = A(x, y) \text{ Mod}(7717),$$

Restrict the enormous growth of A.

Develop a program that can compute AM(10,10) in several seconds and find the value of AM(10,10).

**8b3.**

Assume Fib is the Fibonacci function: 
$$Fib(n) = \begin{cases} 1, & \text{if } n = 1 \text{ or } n = 2 \\ Fib(n - 1) + Fib(n - 2), & \text{otherwise.} \end{cases}$$

Define the number Q as follows:  $Q = \sum_{k=1}^{11} (1/\text{Fib}(\text{Fib}(k)))$ .

Calculate the number Q with an accuracy guaranteeing that the sum of the first 36 decimal digits can be computed exactly.

**8b4.**

Develop an algorithm that for any given pair of natural numbers  $n \geq k$  constructs the set of all the subsets of  $\{1, 2, \dots, n\}$  having cardinality equal to k.

**8b5.**

Prove (using induction argument) that the following algorithm for exponentiation is correct.

```
function power (y,s)
comment Return y^z , where $y \in R, z \in N$
x:=1;
while z > 0 do
if z is odd then x := x * y;
z:= $\lfloor z/2 \rfloor$;
y:=y2;
return(x)
```

**8b6.**

Prove (using induction argument) that the following algorithm for the multiplication of natural numbers is correct.

```
function multiply (y, z)
comment Return y^z , where $y, z \in N$
x:=0;
while z > 0 do
x := x + y * (z mod 2);
y:=2y; z:= $\lfloor z/2 \rfloor$;
return(x)
```

**8b7.**

Prove (using induction argument) that the following algorithm for the multiplication of natural numbers is correct.

```
function multiply(y, z)
comment Return the product y^z .
If z = 0 then return(0) else
return(multiply(2y, $\lfloor z/2 \rfloor$) + y*(z mod 2))
```

**8b8.**

Write a program which, given a sequence of integers, finds the length of the longest increasing subsequence.

*Input*

Input contains length of sequence  $0 < N < 1000$ , then follows N integers, does not exceed 109 by absolute.

*Output*

Write one number in the output, length of the longest increasing subsequence.

*Example*

| Input     | Output |
|-----------|--------|
| 5         | 1      |
| 7 1 5 2 3 | 3      |

**8b9.**

Assume F is a Fibonacci sequence.

$F_0 = 1;$

$F_1 = 1;$

$F_N = F_{N-1} + F_{N-2} \quad N > 1$

Write a program to find N-th Fibonacci number modulo 1000007.  $0 < N < 109$

*Example*

$N = 5 \quad F_5 \% 1000007 = 8$

**8b10.**

Given a positive integer N.

Write a program to count the number of positive integers, not exceeding N and not divisible none of the given integers: 2,3,5.

*Input*

Input contains one integer N ( $1 \leq N \leq 2000000000$ ).

*Output*

Output one number, answer for the given N.

*Example*

|       |        |
|-------|--------|
| Input | Output |
| 10    | 2      |

**8b11.**

Develop a detailed flowchart for an algorithm computing the greatest common divisor for a pair of positive integers, avoiding deletion operation.

**8b12.**

Assume a digital image is given by means of  $M = \|m_{ij}\|$   $n \times n$  matrix where  $m_{ij}$  are non-negative integers.

a. Develop a detailed flowchart for an algorithm that finds the (rounded) coordinates of the centre of gravity of the image. Define needed auxiliary functions.

b. Evaluate the complexity of the developed algorithm.

**8b13.**

Assume for any integer  $m \geq 2$   $P(m)$  is a result of attaching consecutive positive integers represented in  $m$ -ary form. For example, the beginning part of  $P(3)$  looks as follows:

|     |   |     |    |     |    |     |    |     |     |     |
|-----|---|-----|----|-----|----|-----|----|-----|-----|-----|
| 1   | 2 | 10  | 11 | 12  | 20 | 21  | 22 | 100 | 101 | 110 |
| └─┘ |   | └─┘ |    | └─┘ |    | └─┘ |    | └─┘ |     | └─┘ |
| 1   | 2 | 3   | 4  | 5   | 6  | 7   | 8  | 9   | 10  |     |

Develop a detailed flowchart for an algorithm that given  $m \geq 2$  and  $n \geq 1$  finds the numeral allocated at  $n$ -th place in  $P(m)$ . Define needed auxiliary functions.

**8b14.**

A sequence is said to be a *polyndrome* if it has the central symmetry. Develop a detailed flowchart for an algorithm that given binary sequence S finds a longest segment of S that is a palindrome. If there are several such segments then it is enough to find one of them and the length of it.

**8b15.**

Print out all simple numbers not exceeding 1000. Natural number is called simple if it has exactly two dividers.

**8b16.**

Print out all perfect numbers not exceeding 1000. Natural number is called perfect if it equals the sum of all its dividers. For example,  $6 = 1 + 2 + 3$ .

**8b17.**

Check if the given  $n$  number is symmetric or not ( $n < 107$ ). The number is symmetric if it is read the same way from the beginning and the end. For example, 7586857.

**8b18.**

Let  $n \geq 2$  be an integer. A *partition* of  $n$  is a representation of  $n$  as a sum of positive integers without taking their order into account. For example, the partitions of 3 are:

1+1+1; 2+1; 3.

Develop a pseudocode for an algorithm generating the list of all the partitions of the given  $n$ .

**8b19.**

Binomial coefficients  $\binom{n}{m}$  are defined as follows:  $\binom{n}{m} = \frac{n!}{m!(n-m)!}$

Assume that admissible arithmetical operations are addition, subtraction, multiplication and division.

Assuming that multiplication and division operations have the same complexity, and neglecting addition and subtraction operations, do the following.

- Write a pseudocode for trivial, direct calculation of binomial coefficients.
- Propose a faster algorithm for calculation of binomial coefficients and develop a pseudocode.
- Evaluate the complexities  $C1(n,m)$  and  $C2(n,m)$  of the developed codes.
- Find, how faster your own algorithm's code works with respect to the trivial one for  $n=100$  and  $m=90$ .

**8b20.**

The array coefficients  $A_n = [a_n, a_{n-1}, \dots, a_1, a_0]$  are associated with a random polynomial  $A_n(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$ .

- A. Develop an algorithm that given polynomial coefficients arrays  $A_n = [a_n, a_{n-1}, \dots, a_1, a_0]$  and  $B_m = [b_m, b_{m-1}, \dots, b_1, b_0]$  calculates the array corresponding the product  $A_n(x) * B_m(x)$ .
- B. Develop a detailed flowchart for that algorithm.

**8b21.**

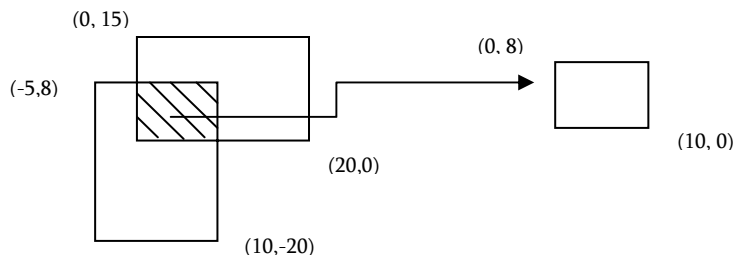
The sequence  $P(n)$  represents monotonically ordered integer powers and sums of those powers of the number 5. For simplification, the initial segment of  $P(n)$  is as follows:

1, 5, 6, 25, 26, 30, 37, 125, ...

Develop a detailed flowchart for an algorithm that calculates  $P(n)$ .

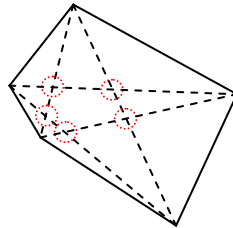
**8b22.**

Assume given two rectangles – with top left vertex and bottom right vertex coordinates. Implement a program that will detect the crossing of those rectangles.



**8b23.**

Assume the number of vertices of convex polygon is given. No three diagonals of a convex polygon cross in one point. Develop a program that finds the number of crossing points of polygon's diagonals.



The number of crossing points of diagonals is equal to 5.

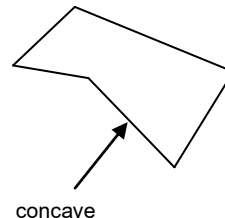
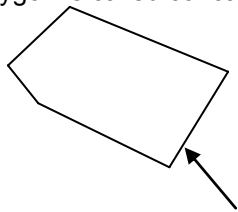
**8b24.**

Develop a program that will check whether the given polygon is convex or not. A convex polygon is a simple polygon interior of which is a convex set. The following properties of a simple polygon are all equivalent to convexity:

Every internal angle is less than 180 degrees.

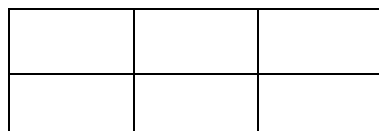
Every line segment between two vertices remains inside or on the boundary of the polygon.

Otherwise, the polygon is called concave.



**8b25.**

Assume given a  $M \times N$  rectangle in a unit grid. Develop a program that will find all rectangles within the given rectangle, the vertices of which are the nodes of the grid, and the sides of which are parallel to the coordinate axis.



Example: For the 2x3 rectangle given above, the result is 18 rectangles.

## 9. NANOELECTRONICS

### a) Test questions

- 9a1.** What are the conditions that minimum energy change of contact potential must satisfy to notice one electron-tunneling?  
A. *Must be smaller than temperature fluctuation*  
B. *Must be larger than quantum fluctuation*  
C. *Must be larger than temperature fluctuation*  
D. *Must be larger than potential barriers*  
E. *B) and C) together*
- 9a2.** Why is size quantization phenomena more accessible and easier observed in semiconductors than the metals?  
A. *Charge carriers' density in semiconductors is small*  
B. *Semiconductors can be doped by mixtures*  
C. *The effective electron mass and energy in semiconductors are small*  
D. *Charge carriers' density in metals is large*  
E. *Charge carriers' mobility in semiconductors is small*
- 9a3.** What model is required to be used considering size quantization phenomena in a MOS transistor in deep inverse mode?  
A. *Rectangular infinite well model*  
B. *Rectangular finite well model*  
C. *Short channel phenomena model*  
D. *Full depletion mode model*  
E. *Triangular well model*
- 9a4.** In order to overcome Coulomb blockade, voltage must be applied which should be  
A. *Larger than potential barriers value*  
B. *Smaller than potential barriers value*  
C. *Equal to*  
D. *Larger than*  
E. *A) and D) together*
- 9a5.** What are the zone marginal bendings in hyper networks (eg. GaAs) conditioned by?  
A. *Difference of bandgap prohibited zones*  
B. *Difference of output work*  
C. *Interrelation difference towards electron*  
D. *Concentration of mixtures*  
E. *Charged different depletion layers, following each other*
- 9a6.** Mark the wrong answer for carbon nanotubes:  
A. *Carbon nanotube is the third subtype of carbon*  
B. *Carbon nanotube is a molecule, consisting of one of the various atoms of carbon*  
C. *Carbon nanotube is a pipe with up to 1 nm in diameter and several  $\mu\text{m}$  roller tube*  
D. *Carbon atoms are arranged in the plane of separate layers in carbon nanotubes*  
E. *Carbon atoms are arranged on the vertices of regular six-angles on the walls of carbon nanotubes.*
- 9a7.** What is atomic force microscope operation based on?  
A. *The intermolecular interactions in case of angstrom sized distances between probe and the surface*  
B. *The intermolecular interactions in case of micrometer sized distances between probe and the surface*  
C. *The intermolecular interactions in case of several dozen nanometers sized distances between probe and the surface*  
D. *The registration of the tunnel current between probe and the surface*  
E. *The optical interactions between the probe and surface*



## b) Problems

### 9b1(13 points)

It is known that colloidal solutions (sols), for example, quantum dots, can agglomerate, forming composite complexes, consisting of 2 or more particles. One reason for this phenomenon is the excess surface energy and the power of molecular attraction, forcing other small particles to unite. To prevent agglomeration, it is possible to report the charge of the same name sign to nanoparticles, which will lead to their repulsion. How can nanoparticles be charged in a colloidal solution (**2 points**)? What could be the minimum and maximum charge of nanoparticles (**2 points**)? Suppose, for example, each of the forming sols of silicon nanocrystals (Si), having a spherical shape with a radius  $R = 1$  nm, and a positive charge  $q$ , equal in magnitude to twice the electron charge was reported. Will these particles form agglomerates in a collision in a colloidal solution in benzene at room temperature (**5 points**)? Will the result change if benzene is replaced with water (**3 points**)? Does the probability of agglomeration depend on size of the nanoparticle (**1 point**), their concentration (**1 point**), the temperature of the solution (**2 points**)?



Charged colloidal nanoparticles

### 9b2(10 points)

At present, many types of light sources are known. However, few know that the work of any of them is impossible without the use of nanotechnology. Moreover, the more informed a person is using nanotechnology to create a light source, the more sophisticated, versatile and reliable they become.

Historically, the first light source for humans was the fire. Sitting in a cave near the fire, the old man indifferently followed how under the influence of ascending air currents, nanoparticles ash and soot, smoke generators, moved into aerosol state and evaporated back.

- 1) Calculate the upward air flow  $V$  speed, which is necessary for translation of spherical nanoparticles of carbon with a diameter of 100 nm into aerosol state, if  $\mu$  (coefficient of internal friction of particles) is equal to 0.72, outhesia force  $F$  (which determines the particle sedimentation) 0.4 N, the resistance coefficient of particles equal to  $10^7$ , particle density is taken as  $1.17 \text{ g/cm}^3$ . (**2 points**)

*Note: for calculation use  $v = (2\mu F/\rho c S)^{1/2}$ , where  $S = \text{area of cross-section of the particle}$ .*

The next generation light sources were incandescent lamps. The work of such light sources is based on a tungsten (wolfram) filament that glows because of warm-up from passing of electric current through it. The lifetime of the lamp is small. However, it was found that the addition of halogens into a lamp (especially iodine) significantly prolongs the lifetime of the light source (the so-called halogen lamps). Special research has shown that the lifetime of the lamp in this case is due to the chemical transport reactions involving formed intermediate compounds nanoclusters of tungsten and halogen.

- 2) One of such nanoclusters has composition  $\text{W}_6\text{I}_{12}$  and is characterized by an ionic structure. Experimentally it is stated that under the action of silver nitrate, only 1/3 of the total amount of iodine can be precipitated of the nanoclusters. Suggest a nanocluster structure. Note that the cation in the nanocluster has high-symmetric structure. (**2 points**)

The main drawback of incandescent lamps - the enormous loss of energy in the form of useless heat dissipation. As an alternative to incandescent lamps mercury (Hg) lamps can be seen, in which the sources of light radiation are the mercury atoms, excited by a glow electric discharge. The main disadvantage of such lamps - the complexity of their utilization.

- 3) Suggest a reasonable means of utilizing mercury lamps by means of nanotechnology. Note that the proposed method should be simple, economically viable and exclude any risk for the environment. (**3 points**)

The most advanced light sources (LED lamps) are based on the luminescence of quantum dots. Their main peculiarity is that, by varying the size of nanoparticles of luminescent material, radiation with different energies and, consequently, with different wavelengths can be obtained.

- 4) What color will light LED lamp based on quantum dots of cadmium selenide radius of 3 nm? For calculation use  $(E_g)^2 = (E_0)^2 + [2 \times (\hbar/2\pi)^2 \times E_0 \times (\pi/r)^2] / m$ , where  $E_g$  – gap width for a quantum quantum dot,  $E_0$  - the band gap for the bulk sample,  $r$  - radius of the nanocrystal (m),  $m$  - electron effective mass. For CdSe  $E_0 = 2.88 \times 10^{-19}$  J,  $m = 1.09 \times 10^{-31}$  kg. **(3 points)**

**9b3 (8 points)**

In a nanoworld there is everything - even machines that can transport molecules, clusters and other nanogoods or simply ride without business. Imagine a nano-vehicle, in which the roles of front and rear pairs of wheels do the same nanotube, closed on both sides (Fig. 1).

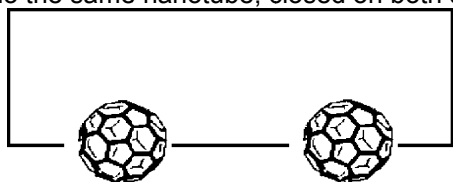


Fig.1. Nano-vehicle on nano-wheels



Fig.2. Section of a wheel (N = 8)

The wheels of such a vehicle are not perfect cylinders. They are composed of hexagons with a side of 0.14 nm, and the cross section is not a circle, but correct N-gon (Fig. 2).

When moving, the truck will keep on hopping, spending energy  $mgh$  on each jump, where  $m$  - mass of the truck,  $h$  - the height of the jump, which depends on the number of hexagons in a section of the wheel –  $N$ .

The mass of nano-truck for large  $N$  can be described by:  $m(N) = m_1 + m_2N + m_3N^2$ , where  $m_1 = 10\,000$  a.m.u. (atomic mass unit),  $m_2 = 700$  a.m.u.,  $m_3 = 25$  a.m.u.

- 1) Explain the type of dependence  $m(N)$  **(1 point)**.
- 2) Determine the dependence of the energy  $E$  required for one step, from  $N$  **(3 points)**.
- 3) Determine the value of  $N$ , for which the energy wastes in one step are minimal, and calculate these costs. **(4 points)**

Consider that for small angles, the approximate expressions for trigonometric functions can be used:  $\sin(x) \approx x$ ,  $\cos(x) \approx 1 - x^2/2$ .

**9b4 (13 points)**

A quantum dot is a semiconductor nanocrystal, in which the movement of charges is limited to three dimensions in space. In a bulk semiconductor material there exists valence band and conductivity band, separated from one another by band gap. If the electron energy increases, it passes into the conductivity band and a hole appears in valence band. In a quantum dot instead of bands, there are discrete levels, and the band gap ( $E_g$ ) in this case is the difference of higher energy filled and lowest unoccupied electronic levels.

- 1) Qualitatively sketch the energy band diagram for the bulk semiconductor and the quantum dot. In both figures, mark the band gap **(2 points)**.
- 2) What is a hole? **(1 point)**.  
It was found that wavelength of luminescence and band gap are related for quantum dots:  
 $(E_g)^2 = (E_0)^2 + [2 \times (\hbar/2\pi)^2 \times E_0 \times (\pi/r)^2] / m$ , where  $E_g$  – gap width for a quantum dot,  $E_0$  - the band gap for the bulk sample,  $r$  - radius of the nanocrystal (m),  $m$  - effective mass of electron. For cadmium selenide  $E_0 = 2.88 \times 10^{-19}$  J,  $m = 1.09 \times 10^{-31}$  kg.
- 3) What is the luminescence? **(1 point)**
- 4) Calculate what is the wavelength of the luminescence (assuming that it corresponds to the band gap) for a crystal with 1 cm radius, 1 nm equal to **(2 points)**.
- 5) What is the minimum size of the quantum dot that corresponds to the luminescence in the visible range? **(1 point)**. Find the required data.

One way of obtaining nanoparticles of cadmium selenide is the interaction of cadmium oleate  $\text{Cd}(\text{C}_{17}\text{H}_{33}\text{COO})_2$  and trioctylphosphine selenide  $\text{SeP}(\text{C}_8\text{H}_{17})_3$  in the environment of diphenyl ether  $(\text{C}_6\text{H}_5)_2\text{O}$ . The reaction is carried out by heating up to 200°C for 5 minutes in an argon atmosphere, then cooled to room temperature. The obtained quantum dots of cadmium selenide are precipitated with acetone.

- 6) Write the reaction equation of obtaining quantum dots as per the above written method **(1 point)**.
- 7) Why such specific conditions (argon atmosphere, reagents, solvents) are needed for the reaction? Maybe it is easier to pour hot water solutions of salts of cadmium and appropriate selenide? **(3 points)**.
- 8) Where quantum dots, based on cadmium selenide, can be (or are already) applied? **(2 points)**.

**9b5 (15 points)**

Once, two young friend-nanotechnologists asked, at first glance, a simple question: how does the de Broglie wave frequency  $\omega$  of free particle depend on wave vector  $k$ ? They decided to get the right formula, but each of them acted in his own way.

The first one thought this way. Write the well-known formula of connection (cyclic) frequency with the period:

$$\omega = \frac{2\pi}{T}. \text{ Express the period through the wavelength and speed: } T = \frac{\lambda}{v}. \text{ In the result, this is obtained:}$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi v}{\lambda} \quad (1)$$

Then apply the de Broglie relation for the pulse and wavelength:  $\lambda = \frac{h}{p}$ . Substitute it in (1), after which

consider  $h = 2\pi\hbar$ , and multiply the numerator and denominator by mass of the particle  $m$ . Then apply the definition of pulse  $\vec{p} = m\vec{v}$  and connection pulse by wave vector  $\vec{p} = \hbar\vec{k}$ . The chain of equalities is obtained:  $\omega = \frac{2\pi}{T} = \frac{2\pi v}{\lambda} = \frac{2\pi v p}{h} = \frac{pv}{\hbar} = \frac{pmv}{\hbar m} = \frac{\hbar^2 k^2}{\hbar m} = \frac{\hbar k^2}{m}$  (2) that is the desired dependence.

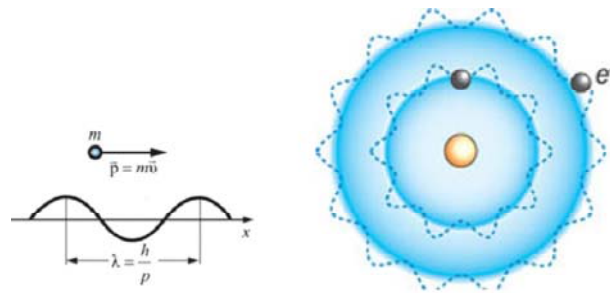
The second one argued differently. Energy and frequency are related by ratio  $E = \hbar\omega$ .

In case of a free particle the energy is  $E = \frac{mv^2}{2}$ , and the pulse equals to  $\vec{p} = m\vec{v}$ . From the last two

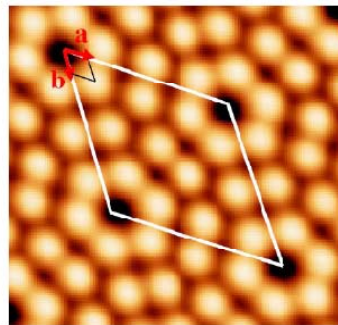
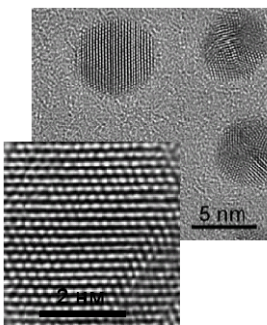
equalities  $E = \frac{p^2}{2m}$ . Considering that  $\vec{p} = \hbar\vec{k}$ , there is:  $\hbar\omega = E = \frac{\hbar^2 k^2}{2m}$ . Hence the answer follow

$$\omega = \frac{\hbar k^2}{2m} \quad (3)$$

To the surprise of the friends, their results (2) and (3) differ twice. Why? Find a mistake in the reasoning (or errors if there are several), and bring the correct formula to connect the frequency and wave vector. **(15 points)**



**9b6 (10 points)**



To date, it is well known that in a microcosm (or rather, on the atomic scale length) the laws of classical physics do not work, and quantum mechanics comes to replace them. Because of Heisenberg's uncertainty relation, the exact location or boundary of any object in space is impossible to determine. It makes no sense to talk about the exact location of an electron in an atom, as well as the boundary of the atom. Of course, it is possible to talk about orbital and electron density, but the density is, actually

the probability amplitude to detect the electron in the vicinity of a point in the elementary act of measurement, rather than a continuous distribution of charge density. Why on many images of micro- and nanostructures obtained by using different types of microscopes (AFM, TEM, AFM, STM), the atoms look like

balls or "clusters"? What a circumstance common for different types of microscopy, allows creating images illustrated in the figure? (10 points)

**9b7 (6 points)**

In the years of 1825-1827 George Simon Ohm discovered his famous law which connects the power of the current flowing through the conductor and the voltage applied to the ends of the conductor, through a factor - the conductivity (or resistance). This dependence is observed with sufficiently high accuracy for bulk conductors.

Recently, however, physicist-researchers with the help of a tunneling microscope were able to measure I-V characteristics of the contact of the tungsten(wolfram) wire with a diameter of 1 nm with a gold substrate. Experimental data is shown in the table.

*Table with experimental data*

| U, mV | I, mkA |
|-------|--------|
| 7.5   | 0.37   |
| 22.7  | 0.83   |
| 45.4  | 1.74   |
| 68    | 2.66   |
| 92.7  | 3.58   |
| 111.5 | 4.59   |
| 149.3 | 6.33   |
| 160.7 | 7.25   |
| 173.9 | 7.89   |
| 192.8 | 8.81   |
| 200.4 | 9.45   |
| 207.9 | 9.91   |
| 225   | 11.2   |
| 243.9 | 12.48  |
| 260.9 | 13.76  |
| 266.5 | 14.31  |

| U, mV | I, mkA |
|-------|--------|
| 276   | 14.95  |
| 293   | 16.15  |
| 300.6 | 16.79  |
| 310   | 17.34  |
| 323.3 | 17.98  |
| 330.8 | 18.62  |
| 342.2 | 20     |
| 351.6 | 21.56  |
| 361.1 | 22.29  |
| 374.3 | 23.67  |
| 385.6 | 25.23  |
| 397   | 26.61  |
| 412.1 | 27.8   |
| 421.6 | 29.45  |
| 431   | 30.92  |
| 442.3 | 32.39  |

| U, mV | I, mkA |
|-------|--------|
| 455.6 | 33.76  |
| 470.7 | 35.5   |
| 480.2 | 36.97  |
| 493.4 | 38.35  |
| 502.8 | 39.08  |
| 514.2 | 40.28  |
| 523.6 | 41.93  |
| 533.1 | 43.58  |
| 542.5 | 45.32  |
| 550.1 | 46.97  |
| 557.7 | 48.53  |
| 567.1 | 50.28  |
| 576.6 | 51.74  |
| 584.1 | 53.3   |
| 593.6 | 55.14  |
| 601.1 | 56.88  |

1. Why gold or tungsten wires are most commonly used? (2 points)
2. What effect was observed by physicists? (1 point) What is the difference between current-voltage characteristics of a tungsten wire of circular section of 1 mm diameter, 1 micron, 10 nm and 1 nm and a unit length (plot all these curves on a graph and explain). (2 points)
3. Plot the conductivity of the applied voltage according to point 3. Constant  $G_0$  is usually applied to the effect. What is this constant called? What is its dimension and value in the SI system? And for what is this value currently used? (3 points)

**9b8 (7 points)**

Porous silicon is an aggregate of nanocrystals and pores with sizes ranging from units to hundreds of nanometers. This material is currently subject to many research laboratories around the world because of its unique structural, optical, electronic and biological properties. Porous silicon is classified in accordance with the International Union of Pure and Applied Chemistry which determines the type of porous material depending on the size of pores.

*Table 1. Classification of porous silicon according to the size of its porous.*

| Porous silicon type      | Porous size         |
|--------------------------|---------------------|
| Microporous (Nanoporous) | $\leq 2 \text{ nm}$ |
| Mesoporous               | 2-50 nm             |
| Macroporous              | >50 nm              |

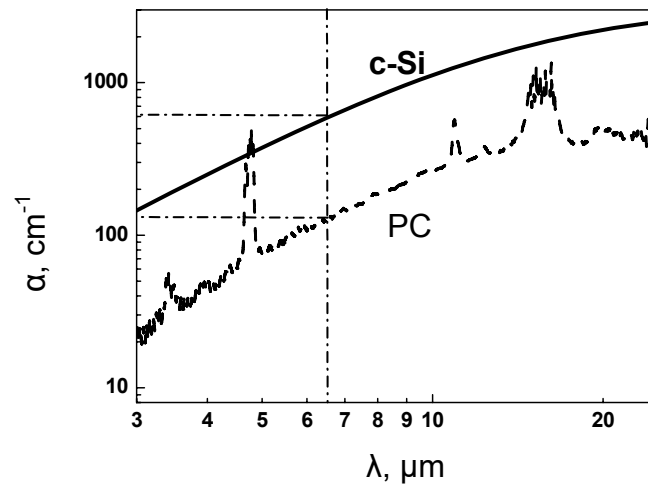
The existence of equilibrium free charge carriers in mesoporous silicon nanocrystals (meso-PC) was recently proved. It was found that in addition to the bands of local surface oscillations in the spectra of IR

transmission of films of meso-PC, absorption of infrared radiation due to the presence of free charge carriers (FCC) is observed.

- 1) How can the decrease in the concentration of free charge carriers (FCC) in mesoporous silicon nanocrystals compared with bulk silicon be explained? **(1 point)**
- 2) the figure shows the absorption coefficient  $\alpha$  of crystalline silicon (c-Si) and meso-PC on the wavelength  $\lambda$  of infrared radiation. Dependencies  $\alpha(\lambda)$  were determined on the basis of measured transmission spectra according to the relation

$$\alpha(\lambda) \approx -h^{-1} \ln[T(\lambda)], \quad (1)$$

where  $h$  – thickness of meso-PC,  $T(\lambda)$  - transmission (depending on  $T(\lambda)$  are obtained experimentally).



Figure, A dependence of the absorption coefficient of c-Si and meso-PC on the wavelength of infrared radiation.

Note that the transmission spectrum of crystalline silicon was removed under the same conditions as for the porous silicon. Specific resistance of crystalline silicon, the dependence of  $\alpha(\lambda)$  of which is shown in the figure, coincides with the specific resistance of the substrate with a-Si, on which the film of meso-PC was grown.

Assuming scattering times of holes in silicon nanocrystals with characteristic sizes, far from the conditions of the quantum size effect, are close to the values for the substrate c-Si. And also, the nature of the absorption for samples of meso-PS corresponds to the classical Drude model, determine the concentration of FCC in the meso-PC **(4 points)**, using the normalization of the spectrum of crystalline silicon with a known concentration of charge carriers equal to  $10^{20} \text{ cm}^{-3}$ .

The following expression is used to calculate FCC concentration:

$$\alpha(\nu) = N_{CH3} \frac{e^2 n \lambda^2}{4\pi^2 c^3 \epsilon_0 m^* \tau}, \quad (2)$$

where  $N_{FCC}$  – FCC concentration,  $n$  – refractive index of the sample ( $n_{c-Si} = 3.4$ ,  $n_{meso-PC} = 1.7$ ),  $\tau$  – scattering time of quasi-pulse of holes (consider  $\tau_{c-Si} = \tau_{meso-PC}$ ), take value  $A = 4\pi^2 c^3 \epsilon_0 m^* / e^2$  equal to *const* (here  $m^*$  - effective mass (for free holes in c-Si  $m_p^* = 0.37m_0$ ,  $m_0 = 9.1 \cdot 10^{-31} \text{ kg}$ ;  $\epsilon_0 = 8.85 \cdot 10^{-12} \text{ F/m}$ ,  $e = 1.6 \cdot 10^{-19} \text{ C}$ ). The porosity of the film of meso-PC is considered equal to 60 %.

- 3) How will the concentration of FCC change during thermal oxidation of mesoporous silicon and why? **(2 points)**
- 4) Is there FCC in microporous silicon? **(2 points)**

### 9b9 (9 points)

One of the main microscopic methods used in nanoworld research is atomic-force microscopy. The design of the scanning head of such a device is shown in the figure. The scanning element of microscope (probe) is silicon cantilever. The image quality and accuracy of the measured values depends on the correct setting of the microscope. Typical cantilevers have the form of a rectangular beam of length  $L = 200 \text{ }\mu\text{m}$ , thickness  $t = 0.5 \text{ }\mu\text{m}$  and width  $w = 40 \text{ }\mu\text{m}$ , Young's modulus of silicon  $E = 2 \cdot 10^{10} \text{ Pa}$ .

Before starting the work, the experimenter sets up a laser system for detecting the bending of the cantilever. In this case the laser beam must be set on the edge of the beam - just above the spot where the needle is located. In a free state cantilever is located horizontally, and during the scan, force on the part of the sample affects and it bends. Bending is fixed by laser system: a laser beam reflects from the beam and falls on the four-section photo diode. While the cantilever is in a free state, the beam hits the very center of the photodiode, when the cantilever is bent, the beam is displaced (signal deviation or Deflection) and this shift is proportional to the bending of the cantilever. Beam displacement as per photodiode in turn is converted into the height, and the methods of conversion may be different.

1. Estimate the value of relative error of measurements (in%), which occurs in the signal Deflection, if the laser beam is substantially removed from the edge of the cantilever (located in the middle of the beam or even farther). **(5 points)**
2. What causes this error on the topographical image (signal height)? **(2 points)**
3. Is this error essential? In the study of what objects, this effect must be stronger? **(1 point)**
4. For what geometric parameters of cantilever the error will be minimized? **(1 point)**

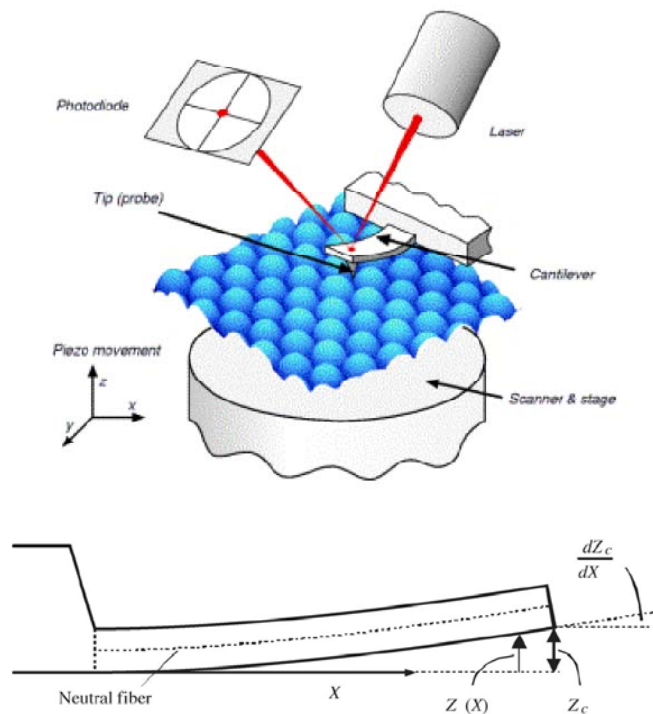


Figure. Schematic diagram of the scanning head of an atomic-force microscope.

**9b10 (5 points)**

Among the number of chemical synthesis methods (and not only) it is required to convert water or water solutions into an aerosol - a suspension of tiny drops in a gaseous environment. Consider the following method of obtaining water "nano-drop": parallel stream of water with velocity  $V$  crashes from vertical streams, which strikes perpendicular to the solid surface and breaks into small drops of different diameters. Estimate the speed of the stream of water  $V$ , required to obtain a drop size of about 100 nm, 10 nm (the surface tension of water = 0.07 H / m). Can the stream be heard?

**ANSWERS TO TEST QUESTIONS  
AND  
SOLUTIONS OF PROBLEMS**





## 1. DIGITAL INTEGRATED CIRCUITS

### a) Test questions

|       |   |       |   |        |   |
|-------|---|-------|---|--------|---|
| 1a1.  | B | 1a43. | D | 1a85.  | A |
| 1a2.  | D | 1a44. | B | 1a86.  | B |
| 1a3.  | E | 1a45. | E | 1a87.  | D |
| 1a4.  | B | 1a46. | E | 1a88.  | D |
| 1a5.  | A | 1a47. | B | 1a89.  | D |
| 1a6.  | C | 1a48. | C | 1a90.  | C |
| 1a7.  | B | 1a49. | B | 1a91.  | C |
| 1a8.  | B | 1a50. | E | 1a92.  | A |
| 1a9.  | D | 1a51. | A | 1a93.  | D |
| 1a10. | A | 1a52. | C | 1a94.  | B |
| 1a11. | A | 1a53. | C | 1a95.  | D |
| 1a12. | D | 1a54. | E | 1a96.  | C |
| 1a13. | C | 1a55. | B | 1a97.  | D |
| 1a14. | B | 1a56. | B | 1a98.  | B |
| 1a15. | E | 1a57. | D | 1a99.  | C |
| 1a16. | A | 1a58. | C | 1a100. | A |
| 1a17. | C | 1a59. | C | 1a101. | B |
| 1a18. | E | 1a60. | C | 1a102. | B |
| 1a19. | A | 1a61. | B | 1a103. | A |
| 1a20. | C | 1a62. | D | 1a104. | C |
| 1a21. | D | 1a63. | C | 1a105. | B |
| 1a22. | B | 1a64. | D | 1a106. | E |
| 1a23. | D | 1a65. | C | 1a107. | E |
| 1a24. | E | 1a66. | B |        |   |
| 1a25. | E | 1a67. | B |        |   |
| 1a26. | B | 1a68. | B |        |   |
| 1a27. | B | 1a69. | C |        |   |
| 1a28. | B | 1a70. | D |        |   |
| 1a29. | E | 1a71. | A |        |   |
| 1a30. | C | 1a72. | B |        |   |
| 1a31. | D | 1a73. | D |        |   |
| 1a32. | B | 1a74. | B |        |   |
| 1a33. | D | 1a75. | A |        |   |
| 1a34. | D | 1a76. | A |        |   |
| 1a35. | C | 1a77. | B |        |   |
| 1a36. | D | 1a78. | C |        |   |
| 1a37. | C | 1a79. | B |        |   |
| 1a38. | C | 1a80. | C |        |   |
| 1a39. | A | 1a81. | B |        |   |
| 1a40. | D | 1a82. | B |        |   |
| 1a41. | A | 1a83. | D |        |   |
| 1a42. | D | 1a84. | C |        |   |

**b) Problems**

**1b1.**

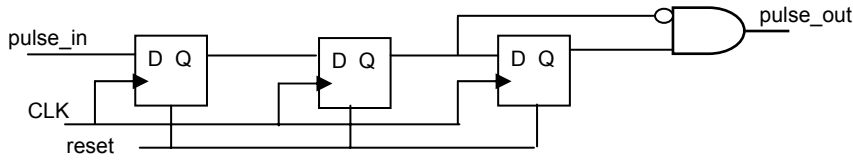
$$t_{suX} = t_{br} + t_{nor} + t_{su} - t_{clbfr} = 2 + 7 + 4 - 3 = 11 \text{ ns}$$

$$t_{hdX} = t_{clbfr} + t_{hd} - t_{br} - t_{nor} = 3 + 5 - 2 - 7 = -1 \text{ ns}$$

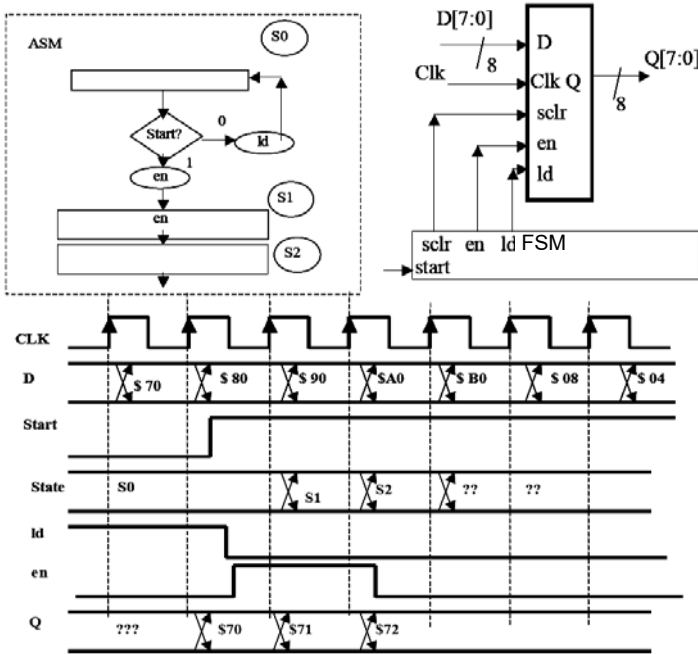
$$t_L = t_{clbfr} + t_{CQ} + t_{or} = 3 + 6 + 8 = 17 \text{ ns}$$

$$t_{cycle} = t_{CQ} + t_{nor} + t_{su} = 6 + 7 + 4 = 17 \text{ ns}$$

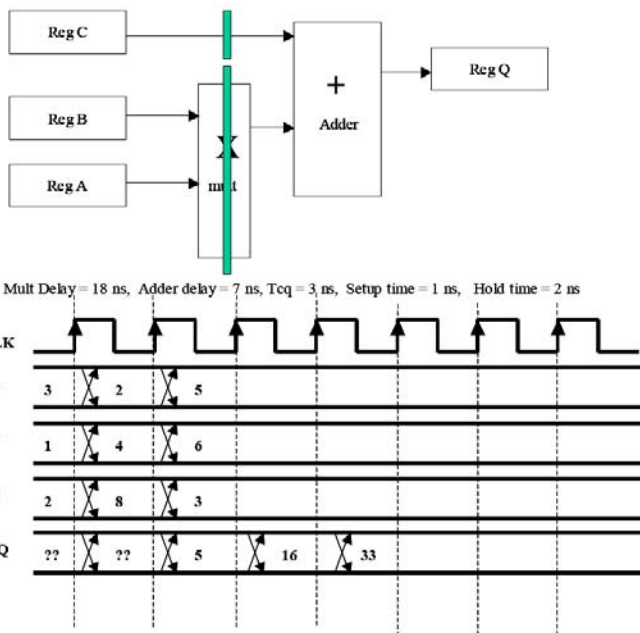
**1b2.**



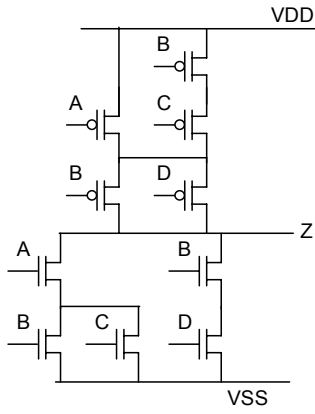
**1b3.**



**1b4.**

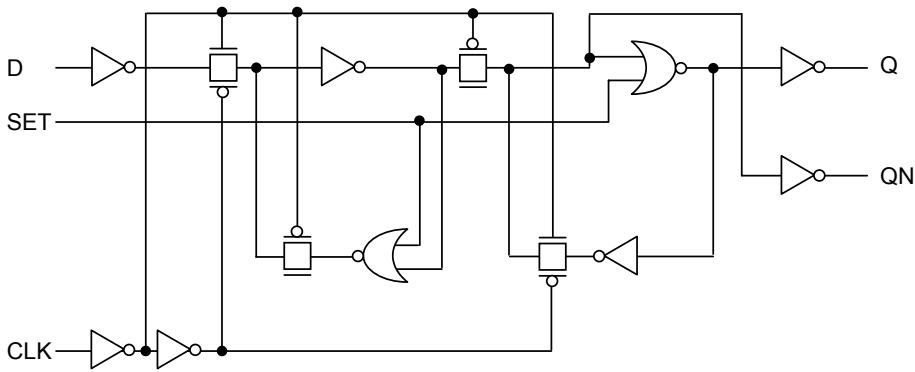


1b5.

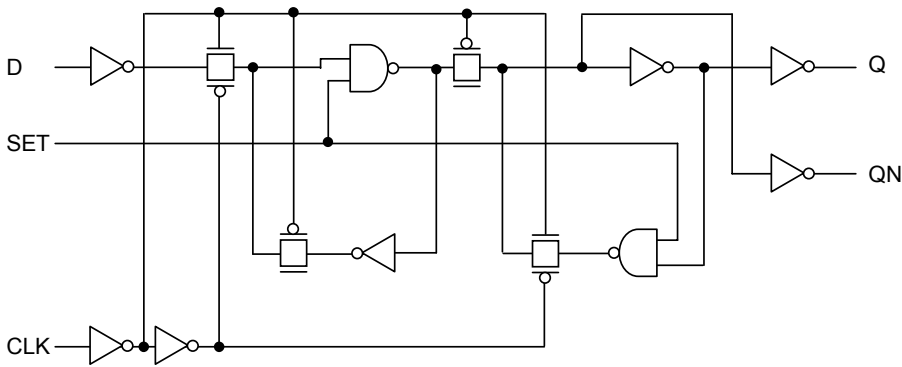


1b6.

D Flip Flop with High Active Set, v.1

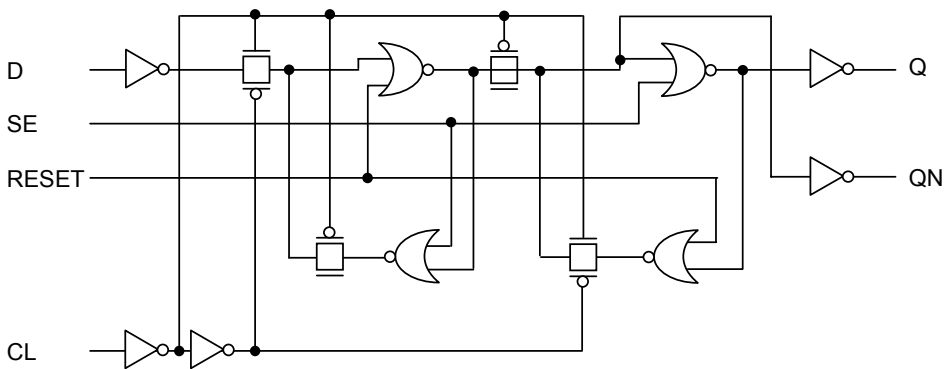


D Flip Flop with High Active Set, v.2

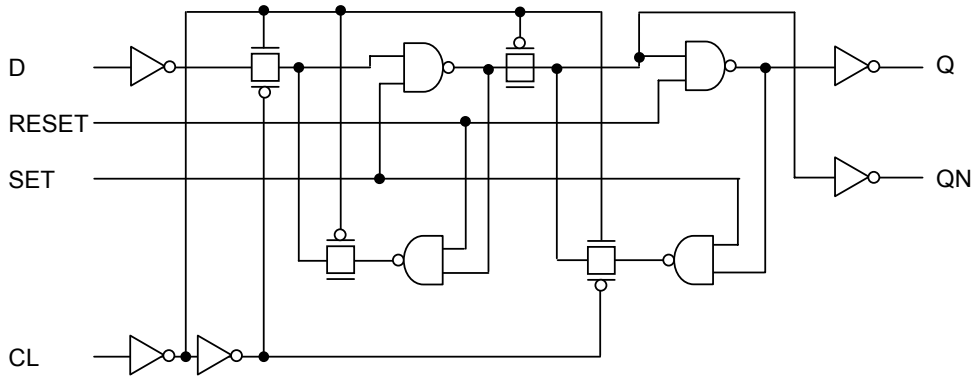


1b7.

D Flip Flop with High Active Set and Reset, v.1



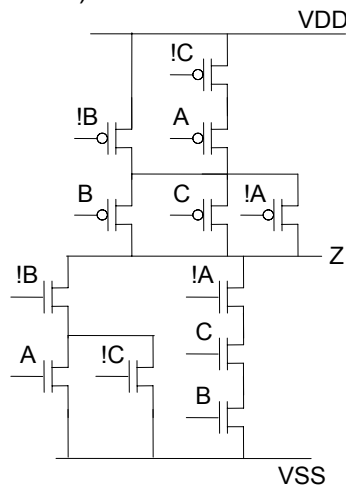
D Flip Flop with High Active Set and Reset, v.2



1b8.

| A | B | C | Z | !Z |
|---|---|---|---|----|
| 0 | 0 | 0 | 0 | 1  |
| 0 | 0 | 1 | 1 | 0  |
| 0 | 1 | 0 | 1 | 0  |
| 0 | 1 | 1 | 0 | 1  |
| 1 | 0 | 0 | 0 | 1  |
| 1 | 0 | 1 | 0 | 1  |
| 1 | 1 | 0 | 1 | 0  |
| 1 | 1 | 1 | 1 | 0  |

$$Z = A \& B + !C \& B + !A \& !B \& C = !(B(A + !C) + !A \& B \& C)$$



1b9.

$$V_{SP} = \frac{\sqrt{\frac{\beta_n}{\beta_p}} \cdot V_{THN} + (V_{DD} - V_{THP})}{1 + \sqrt{\frac{\beta_n}{\beta_p}}}, \quad \beta = kp \frac{W}{L}$$

$$V_{SP} = \frac{\sqrt{\frac{\frac{W_n}{kpn} \frac{L_n}{kpp} \frac{W_p}{L_p}}{\frac{W_n}{kpn} \frac{L_n}{kpp} \frac{W_p}{L_p}}} \cdot V_{THN} + (V_{DD} - V_{THP})}{1 + \sqrt{\frac{\frac{W_n}{kpn} \frac{L_n}{kpp} \frac{W_p}{L_p}}{\frac{W_n}{kpn} \frac{L_n}{kpp} \frac{W_p}{L_p}}}} = \frac{\sqrt{\frac{kpn}{kpp}} \cdot \sqrt{\frac{W_n}{W_p}} \cdot V_{THN} + (V_{DD} - V_{THP})}{1 + \sqrt{\frac{kpn}{kpp}} \cdot \sqrt{\frac{W_n}{W_p}}}$$

$$= \frac{\sqrt{\frac{k_{pn}}{k_{pp}}} \cdot \sqrt{\frac{W_n}{W_p}} \cdot 0.6 + (5 - 0.8)}{1 + \sqrt{\frac{k_{pn}}{k_{pp}}} \cdot \sqrt{\frac{W_n}{W_p}}} \approx \frac{\sqrt{\frac{1}{3}} \cdot 0.6 \cdot \sqrt{\frac{W_n}{W_p}} + 4.8}{1 + \sqrt{\frac{1}{3}} \cdot \sqrt{\frac{W_n}{W_p}}} \approx \frac{4.8}{1 + \sqrt{\frac{1}{3}} \cdot \sqrt{\frac{W_n}{W_p}}}$$

1.  $W_n=3 \mu\text{m}$

$$V_{SP3} = \frac{4.8}{1 + \sqrt{\frac{1}{3}} \cdot \sqrt{\frac{3}{10}}} = 3.64 \text{ V}$$

2.  $W_n=12 \mu\text{m}$

$$V_{SP12} = \frac{4.8}{1 + \sqrt{\frac{1}{3}} \cdot \sqrt{\frac{12}{10}}} = 3.93 \text{ V}$$

$$V_{SP12} < V_{SP3}$$

$$\frac{W_p}{W_n} = \frac{10}{12} \quad \frac{W_p}{W_n} = \frac{10}{3}$$

Therefore, the inverter corresponds to the left curve for which  $W_n=12 \mu\text{m}$ .

### 1b10.

For both circuits the best case delay is the delay from input C to output Y.

For the shown NAND cell the delay from A input to Y output is smaller than from B input. Therefore, the best case delay of the 2<sup>nd</sup> circuit is smaller.

### 1b11.

The frequency of the ring oscillator is defined by  $f = \frac{1}{(t_{PHL1} + t_{PLH1}) + (t_{PHL2} + t_{PLH2}) + \dots + (t_{PHLn} + t_{PLHn})}$ ,

where  $t_{PHLi}$  and  $t_{PLHi}$  are the delays of i-th inverter's output and fall accordingly.

The number of inverters in a ring oscillator must be odd, so the 1<sup>st</sup> and the 2<sup>nd</sup> circuits are not included in the ring oscillator (in operation, logic "1" is given to set input and the output NAND cell operates as an inverter).

The delay from "a" input to the output of the used NAND cell is smaller than the delay from "b" input to the output, so the 4<sup>th</sup> ring oscillator is the fastest.

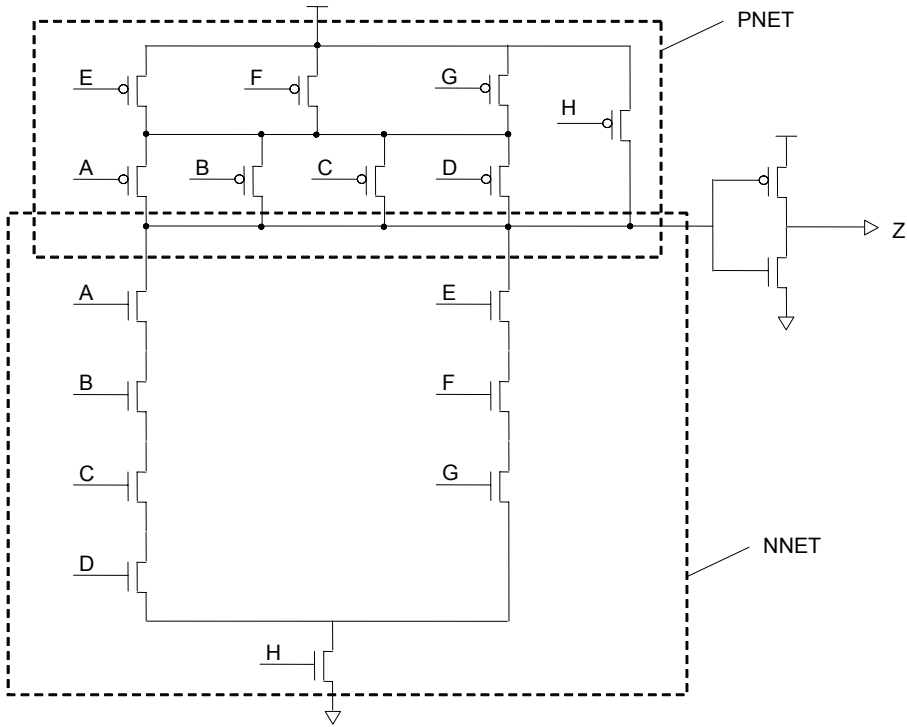
### 1b12.

a. In order to get the signal from I12 in Q and QN, it is necessary to have the correct logic state in I22 and I23 feedback until CK fall transition. To say it otherwise, it is necessary to keep CK constant until the signal "passes" I3-I17-I23-I22 path. The signal level in I22 and I23 cells' outputs must be either 0.9 VDD or 0.1 VDD (VDD is supply voltage).

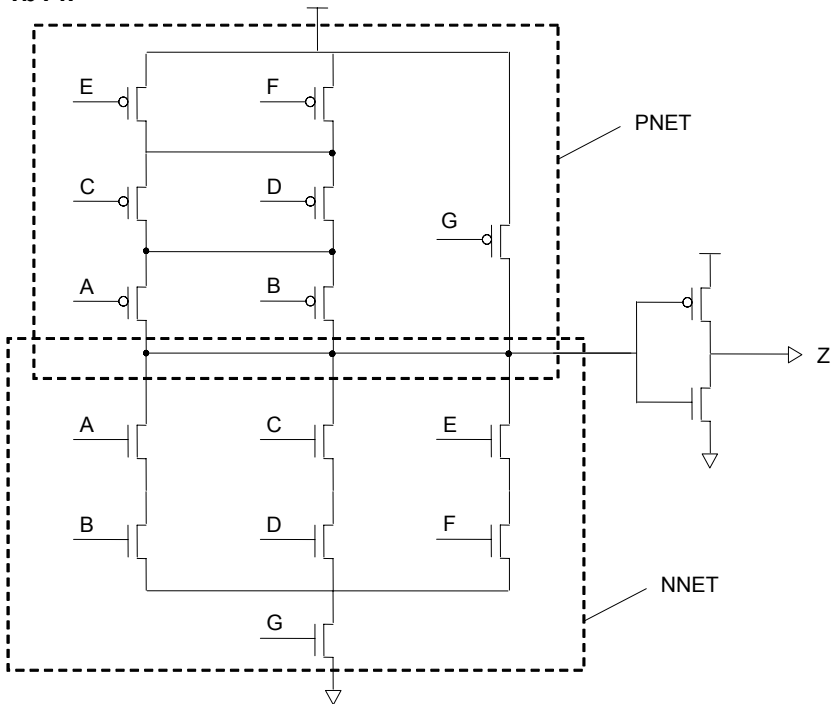
Thus, CK mustn't switch at  $\max(t_{PHL}I3 + t_{PLH}I17, t_{PLH}I3 + t_{PHL}I17) + \max(t_{rise}I23 + t_{fall}I22, t_{fall}I23 + t_{rise}I22) = 1650 \text{ ps}$ .

b. As explained in item a., in this case D also must remain constant at  $\max(t_{PHL}I1 + t_{PLH}I11, t_{PLH}I1 + t_{PHL}I11) + \max(t_{rise}I13 + t_{fall}I12, t_{fall}I13 + t_{rise}I12) - t_{PLH}I12 = 1250 \text{ ps}$ .

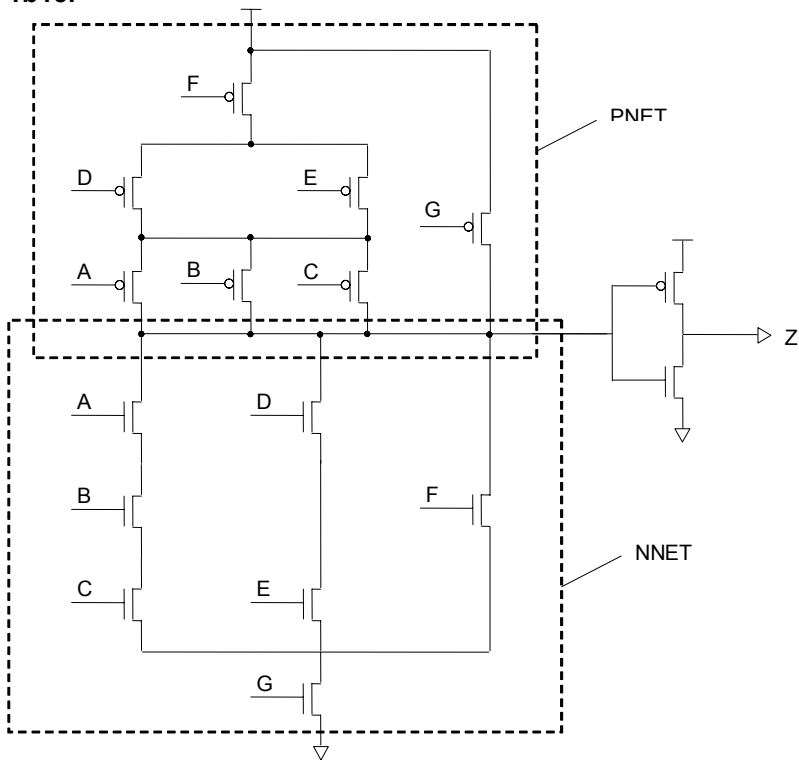
1b13.



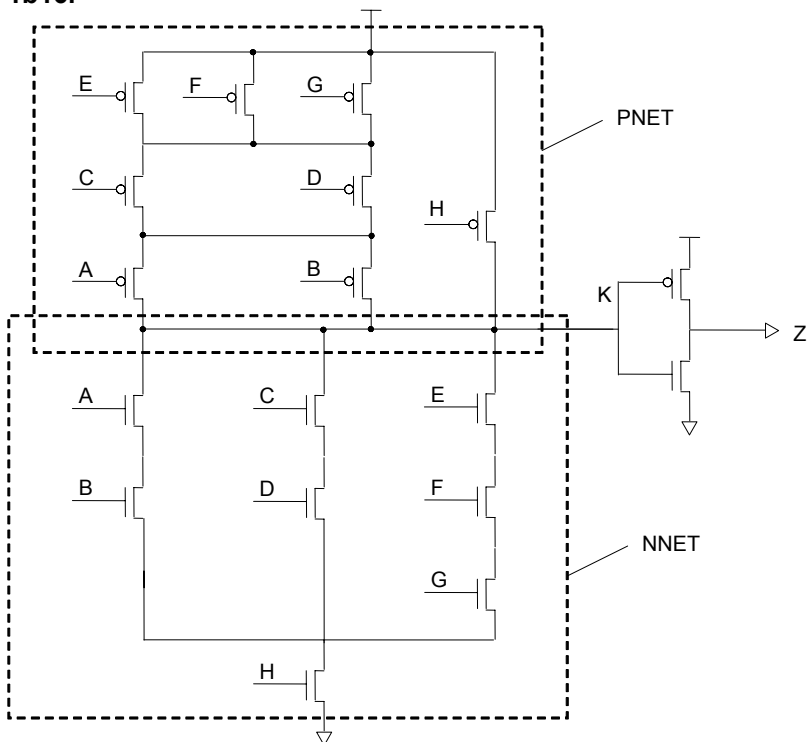
1b14.



1b15.



1b16.



P and N parts of the given circuit do not have common Euler path. For P net, Euler path is the following: H, E, F, G, C, A, B, D. For N net - H, B, A, C, D, E, F, G.

1b17.

The total charge of  $C_s$  and  $C_{BL}$  before M1 is on.

$$Q = C_s V_s + C_{BL} V_{DD} / 2$$

After M1 is on, the voltages across  $C_s$  and  $C_{BL}$  are equalized to  $V_{BL}$ .

$$V_{BL}(C_s + C_{BL}) = C_s V_s + C_{BL} V_{DD} / 2$$

When reading "1",  $V_s = V_{DD} - V_t = 1.2 - 0.3 = 0.9V$

$$V_{BL} = \frac{C_s V_s + C_{BL} V_{DD} / 2}{C_s + C_{BL}} = (0.9C_s + 0.6 \cdot 10C_s) / (C_s + 10C_s) = 6.9/11 = 0.63V$$

When reading "0",  $V_s = 0$

$$V_{BL} = 0.6 \cdot 10C_s / 11C_s = 0.55V$$

### 1b18.

$$R = 2V_{DD} \cdot L / W \cdot k (V_{DD} - V_t)^2; R_1 = 2 \cdot 2.5 \cdot 0.25 / 4 \cdot 115 \cdot 4 = 0.68 \text{ kOhm}$$

$$R_2 = 2 \cdot 2.5 \cdot 0.25 / 0.5 \cdot 30 \cdot 2 \cdot 1^2 = 19 \text{ kOhm}$$

a)  $V_{OH} = V_{DD} = 2.5V$ ,  $V_{OL} = R_1 / (R_1 + R_2) \cdot 2.5 = 0.68 / (0.68 + 19) \cdot 2.5 = 0.87 \text{ mV}$

b) 1.  $V_{in} = \text{Low}$ ,  $I_n = 0$ ,  $P_{diss} = 0$ ,

2.  $V_{in} = \text{High} = 2.5V$ ,  $I_n = I_p = (W/L)_p \cdot k_p (V_{DD} - |V_{tp}|)^2 / 2 = 133.2 \mu A$ .  $P_{static} = V_{DD} \cdot I_p = 2.5 \cdot 133.2 = 333 \mu W$ .

c)  $t_{pLH} = 0.7R_p C_L = 0.7 \cdot 19 \cdot 1 = 13.3 \text{ ns}$ ;  $t_{pHL} = 0.7R_n C_L = 0.7 \cdot 0.68 \cdot 1 = 0.476 \text{ ns}$

### 1b19.

a)  $Y = !(CD(A+B))$

NMOS  $(W/L)_A = (W/L)_B = (W/L)_C / 2 = (W/L)_D / 2$ ;  $(W/L)_C = (W/L)_D = 4 \cdot 3 = 12$ ;  $(W/L)_A = (W/L)_B = 6$

PMOS  $(W/L)_A = (W/L)_B = 2(W/L)_C = 2(W/L)_D$ ;  $(W/L)_C = (W/L)_D = 8/3$ ;  $(W/L)_A = (W/L)_B = 16/3$

b)  $t_{pHL}$ : ABCD=1010 -> 1011 or 1010 -> 0111; transistor discharges the caps in all nodes of the pull-down network, before transition all caps are charged. If D were on before transition then D's drain node cap would be discharged, so  $t_{pHL}$  would be less.

$t_{pLH}$ : ABCD=1111 -> 0011; before transition all caps are charged, AB path discharges these caps.

### 1b20.

$$P(Y=1) = (1 - P(A=1))P(C=1)P(D=1) (1 - P(B=1)P(C=1)P(D=1)) =$$

$$= (1 - 0.5 \cdot 0.3 \cdot 0.8)(1 - 0.2 \cdot 0.3 \cdot 0.8) = 0.88 \cdot 0.952 = 0.83776$$

$$P(Y=0) = 1 - P(Y=1) = 1 - 0.83776 = 0.16224$$

$$P_{0-1} = P_{1-0} = P(Y=0) \cdot P(Y=1) = 0.1359181824 = 0.136$$

$$\alpha_{sw} = P_{0-1}$$

$$P_{sw} = \alpha_{sw} V_{DD}^2 F_{clk} C_{out} = 0.136 \cdot 2.5^2 \cdot 250 \cdot 10^6 \cdot 30 \cdot 10^{-15} = 0.659 \cdot 10^{-5} W = 6.59 \mu W$$

### 1b21.

#### Verilog Description

```

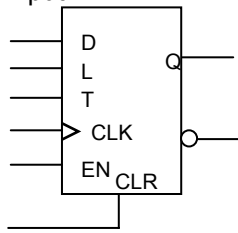
module counter_rev(clk, ce, clr, load, d, up, q, tc);
input clk, ce, clr, load, up;
input[7:0] d;
output tc;
reg tc;
output[7:0] q;
reg[7:0] q;
always @(posedge clk or posedge clr)
begin
if(clr==1) q<=8'b0;
else if(load) q<=d;
else if(ce==0)
q<=q;
else if(up==1)
q<=q+1;
else q<=q-1;
end
always @(q or up)
begin if((q==8'd255) && (up==1))
tc=1;
else if((q==8'b0) && (up==0))
tc=1;
else tc=0; end
endmodule

```



Constructing a counter circuit diagram based on T Flip-Flop

To construct a circuit diagram use T flip-flops with asynchronous reset, synchronous loading and enable input.



- T- toggling enable input
- L – load enable
- D – data input
- CCLK– clock input
- EN - clock enable input
- CLR – asynchronous reset

Truth table of FF:

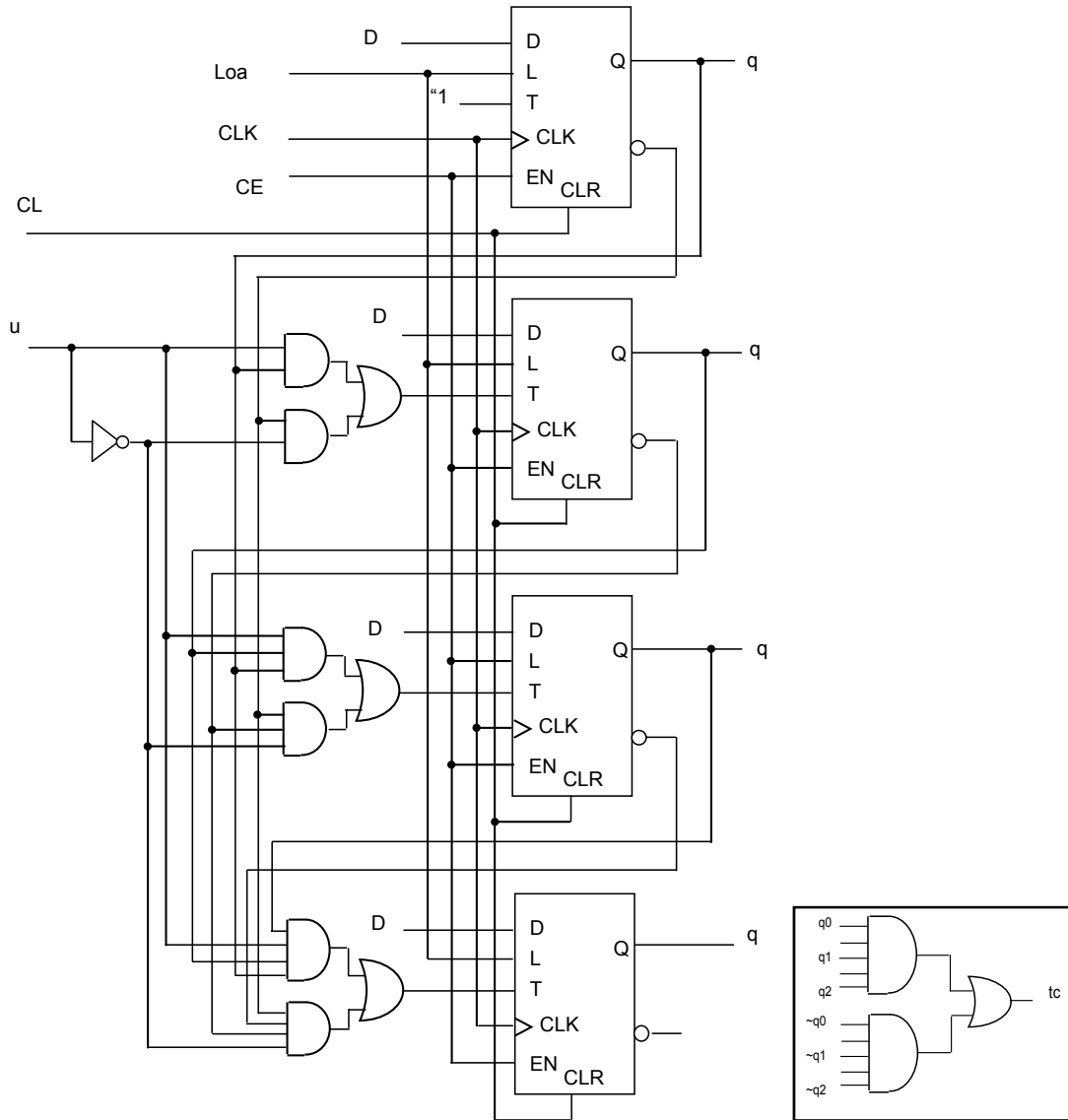
| CLR | EN | L | D | T | CLK | Q                  | Mode             |
|-----|----|---|---|---|-----|--------------------|------------------|
| 1   | x  | x | x | x | x   | 0                  | Reset            |
| 0   | x  | 1 | 0 | x | ↑   | 0                  | Syn. „0” writing |
| 0   | x  | 1 | 1 | x | ↑   | 1                  | Syn. „0” writing |
| 0   | 0  | 0 | x | x | x   | Q <sub>last</sub>  | Hold             |
| 0   | 1  | 0 | x | 0 | ↑   | Q <sub>last</sub>  | Hold             |
| 0   | 1  | 0 | x | 0 | x   | Q <sub>last</sub>  | Hold             |
| 0   | 1  | 0 | x | 1 | ↑   | ~Q <sub>last</sub> | Switching        |

Definition of reversible counter's excitation function

| Present_state |    |    |    | Input | Next_state |    |    |    | Excitation Function |    |    |    |
|---------------|----|----|----|-------|------------|----|----|----|---------------------|----|----|----|
| q3            | q2 | q1 | q0 | up    | q3         | q2 | q1 | q0 | t3                  | t2 | t1 | t0 |
| 0             | 0  | 0  | 0  | 0     | 1          | 1  | 1  | 1  | 1                   | 1  | 1  | 1  |
|               |    |    |    | 1     | 0          | 0  | 0  | 1  | 0                   | 0  | 0  | 1  |
| 0             | 0  | 0  | 1  | 0     | 0          | 0  | 0  | 0  | 0                   | 0  | 0  | 1  |
|               |    |    |    | 1     | 0          | 0  | 1  | 0  | 0                   | 0  | 1  | 1  |
| 0             | 0  | 1  | 0  | 0     | 0          | 0  | 0  | 1  | 0                   | 0  | 1  | 1  |
|               |    |    |    | 1     | 0          | 0  | 1  | 1  | 0                   | 0  | 0  | 1  |
| 0             | 0  | 1  | 1  | 0     | 0          | 0  | 1  | 0  | 0                   | 0  | 0  | 1  |
|               |    |    |    | 1     | 0          | 1  | 0  | 0  | 0                   | 1  | 1  | 1  |
| 0             | 1  | 0  | 0  | 0     | 0          | 0  | 1  | 1  | 0                   | 1  | 1  | 1  |
|               |    |    |    | 1     | 0          | 1  | 0  | 1  | 0                   | 0  | 0  | 0  |
| 0             | 1  | 0  | 1  | 0     | 0          | 1  | 0  | 0  | 0                   | 0  | 0  | 1  |
|               |    |    |    | 1     | 0          | 1  | 1  | 0  | 0                   | 0  | 1  | 1  |
| 0             | 1  | 1  | 0  | 0     | 0          | 1  | 0  | 1  | 0                   | 0  | 1  | 1  |
|               |    |    |    | 1     | 0          | 1  | 1  | 1  | 0                   | 0  | 0  | 0  |
| 0             | 1  | 1  | 1  | 0     | 0          | 1  | 1  | 0  | 1                   | 0  | 0  | 1  |
|               |    |    |    | 1     | 1          | 0  | 0  | 0  | 0                   | 0  | 1  | 1  |
| 1             | 0  | 0  | 0  | 0     | 1          | 1  | 0  | 1  | 1                   | 1  | 1  | 1  |
|               |    |    |    | 1     | 1          | 0  | 0  | 1  | 0                   | 0  | 0  | 0  |
| 1             | 0  | 0  | 1  | 0     | 1          | 0  | 0  | 0  | 0                   | 0  | 0  | 1  |
|               |    |    |    | 1     | 1          | 0  | 1  | 0  | 0                   | 0  | 1  | 1  |
| 1             | 0  | 1  | 0  | 0     | 1          | 0  | 1  | 0  | 0                   | 0  | 0  | 1  |
|               |    |    |    | 1     | 1          | 1  | 0  | 0  | 0                   | 1  | 1  | 1  |
| 1             | 0  | 1  | 1  | 0     | 1          | 0  | 1  | 1  | 0                   | 0  | 0  | 1  |
|               |    |    |    | 1     | 1          | 1  | 0  | 0  | 0                   | 0  | 1  | 1  |
| 1             | 1  | 0  | 0  | 0     | 1          | 1  | 0  | 0  | 0                   | 0  | 0  | 1  |
|               |    |    |    | 1     | 1          | 1  | 1  | 0  | 0                   | 0  | 1  | 1  |
| 1             | 1  | 1  | 0  | 0     | 1          | 1  | 0  | 1  | 0                   | 0  | 0  | 1  |
|               |    |    |    | 1     | 1          | 1  | 1  | 1  | 0                   | 0  | 0  | 0  |
| 1             | 1  | 1  | 1  | 0     | 1          | 1  | 1  | 1  | 0                   | 0  | 0  | 1  |
|               |    |    |    | 1     | 1          | 1  | 1  | 1  | 0                   | 0  | 0  | 0  |
| 1             | 1  | 1  | 1  | 0     | 1          | 1  | 1  | 0  | 0                   | 0  | 0  | 1  |
|               |    |    |    | 1     | 0          | 0  | 0  | 0  | 0                   | 0  | 1  | 1  |

$t_0 = 1;$   $t_1 = q_0 \cdot up + \sim q_0 \cdot \sim up;$   
 $t_2 = q_0 \cdot q_1 \cdot up + \sim q_0 \cdot \sim q_1 \cdot \sim up;$   
 $t_3 = q_0 \cdot q_1 \cdot q_2 \cdot up + \sim q_0 \cdot \sim q_1 \cdot \sim q_2 \cdot \sim up;$   
 $tc = q_0 \cdot q_1 \cdot q_2 \cdot q_3 \cdot up + \sim q_0 \cdot \sim q_1 \cdot \sim q_2 \cdot \sim q_3 \cdot \sim up;$

Based on the obtained results, construct the counter circuit.



**1b22.**

**Verilog Description**

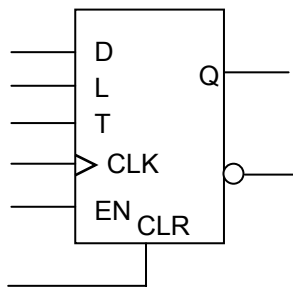
```

module counter_dec(clk,ce,reset,load,d,q,tc);
input clk, ce, reset,load;
input[3:0] d;
output tc;
reg tc;
output[3:0] q;
reg[3:0] q;
always @(posedge clk)
begin
if(reset==1) q<=4'b0;
else if(load) begin if (d>= 4'd10) q<=4'b0;
else q<=d; end
else if ((ce==1)&&(q==4'd9)) q <= 4'b0;
else if(ce==1) q<=q+1;
else q<=q;
end
always @(q)
if(q==4'd9) tc=1;
else tc=0;
endmodule

```

### Constructing a counter circuit diagram based on T Flip-Flop

To construct a circuit diagram use T flip-flops with asynchronous reset, synchronous loading and enable input.



- T- toggling enable input
- L – load enable
- D – data input
- CLK– clock input
- EN - clock enable input
- CLR – synchronous reset

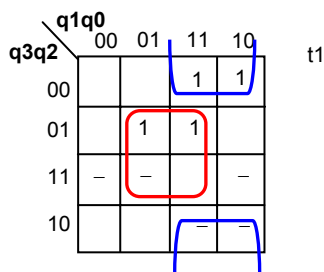
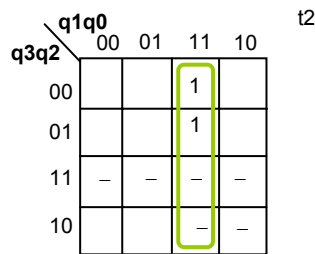
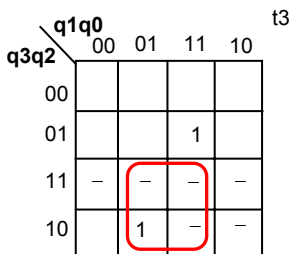
Truth table of FF:

| CLR | EN | L | D | T | CLK | Q               | Mode             |
|-----|----|---|---|---|-----|-----------------|------------------|
| 1   | x  | x | x | x | ↑   | 0               | Reset            |
| 1   | x  | 1 | 0 | x | ↑   | 0               | Syn. „0” writing |
| 1   | x  | 1 | 1 | x | ↑   | 1               | Syn. „1” writing |
| 1   | 0  | 0 | x | x | x   | $Q_{last}$      | Hold             |
| 1   | 1  | 0 | x | 0 | ↑   | $Q_{last}$      | Hold             |
| 1   | 1  | 0 | x | 0 | x   | $Q_{last}$      | Hold             |
| 1   | 1  | 0 | x | 1 | ↑   | $\sim Q_{last}$ | Switching        |

Definition of binary-coded decimal counter's excitation function:

| q3 | q2 | q1 | q0 | t3 | t2 | t1 | t0 |
|----|----|----|----|----|----|----|----|
| 0  | 0  | 0  | 0  | 0  | 0  | 0  | 1  |
| 0  | 0  | 0  | 1  | 0  | 0  | 0  | 1  |
| 0  | 0  | 1  | 0  | 0  | 0  | 1  | 1  |
| 0  | 0  | 1  | 1  | 0  | 1  | 1  | 1  |
| 0  | 1  | 0  | 0  | 0  | 0  | 0  | 1  |
| 0  | 1  | 0  | 1  | 0  | 0  | 1  | 1  |
| 0  | 1  | 1  | 0  | 0  | 0  | 0  | 1  |
| 0  | 1  | 1  | 1  | 1  | 1  | 1  | 1  |
| 1  | 0  | 0  | 0  | 0  | 0  | 0  | 1  |
| 1  | 0  | 0  | 1  | 1  | 0  | 0  | 1  |

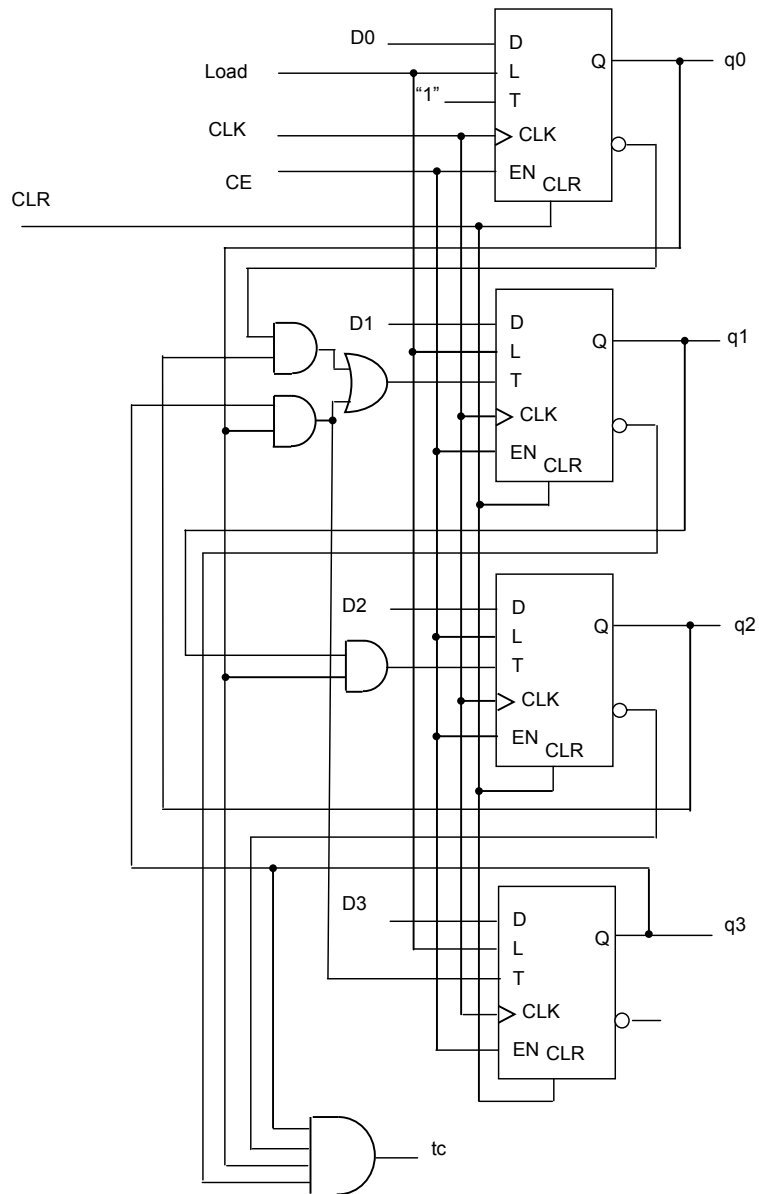
Excitation Function's Minimization using Karnaugh Maps



$$\begin{aligned}
 t0 &= 1 \\
 t1 &= q0 \cdot q3 + \sim q0 \cdot q3 \\
 t2 &= q0 \cdot q1 \\
 t3 &= q0 \cdot q3
 \end{aligned}$$

$$\begin{aligned}
 t0 &= 1 \\
 t1 &= q0 \cdot q3 + \sim q0 \cdot q3 \\
 t2 &= q0 \cdot q1 \\
 t3 &= q0 \cdot q3
 \end{aligned}$$

Circuit diagram is constructed based on the obtained results.



**1b23.**

Creation of State Transition Graph of FSM

The digits of entering decimal number are given on the entries of FSM sequentially. The remainder of division of a decimal number by 3 is equal to the remainder of division of the sum of decimal digits of this number by 3.

FSM is given by means of the following five sets:  $A = \{ X, Y, S, \delta, \lambda \}$ , where:

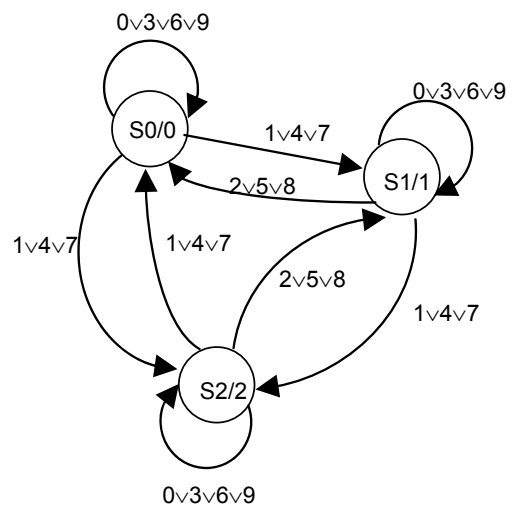
- $X = \{X_1, X_2, \dots, X_M\}$  – set of input symbols;
- $Y = \{Y_1, Y_2, \dots, Y_N\}$  – set of output symbols;
- $S = \{S_0, S_1, \dots, S_{K-1}\}$  – set of internal states of FSM;
- $\delta$  – next state (transition) function,
- $\lambda$  - output function.

The considered FSM has 3 states  $S = \{S_0, S_1, S_2\}$ , where,  $S_0$  – the remainder equal to 0,  $S_1$  – the remainder equal to 1,  $S_2$  – the remainder equal to 2.

$X = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$Y = \{0, 1, 2\}$

$S = \{S_0, S_1, S_2\}$



### Verilog Description of FSM

```

module fsm_remainder(in,clk,reset,out);
input[3:0] in;
input clk, reset;
output[1:0] out;
reg [1:0] out;
reg[1:0] state, next_state;
parameter S0=2'b00, S1=2'b01, S2=2'b10;

always @(posedge clk or negedge reset)
 if(!reset) state <= S0;
 else state<=next_state;

always @(in or state)
begin
case(state)
S0: case(in)
 4'd0,4'd3,4'd6,4'd9: next_state=S0;
 4'd1, 4'd4, 4'd7: next_state=S1;
 4'd2, 4'd5, 4'd8: next_state=S2;
 default: next_state=S0;
 endcase
S1: case(in)
 4'd0,4'd3,4'd6,4'd9: next_state=S1;
 4'd1, 4'd4, 4'd7: next_state=S2;
 4'd2, 4'd5, 4'd8: next_state=S0;
 default: next_state=S0;
 endcase
S2: case(in)
 4'd0,4'd3,4'd6,4'd9: next_state=S2;
 4'd1, 4'd4, 4'd7: next_state=S0;
 4'd2, 4'd5, 4'd8: next_state=S1;
 default: next_state=S0;
 endcase
 default: next_state=S0;
endcase
endcase
end

always @(state)
if(state==S0) out=2'b00;
else if (state==S1) out = 2'b01;
else if (state ==S2) out = 2'b10;
else out=2'bxx;

endmodule

```

### Synthesis of FSM Using JK Flip-Flops

#### 1. Definition of input and output signals

To code decimal digits  $n = \lceil \log_2 10 \rceil = 4$  input variables are required, as  $x_3, x_2, x_1, x_0$ .

Binary-coded decimal digits:  $Y = \{0, 1, 2\}$ .

| Decimal digit | x3 | x2 | x1 | x0 |
|---------------|----|----|----|----|
| "0"           | 0  | 0  | 0  | 0  |
| "1"           | 0  | 0  | 0  | 1  |
| "2"           | 0  | 0  | 1  | 0  |
| "3"           | 0  | 0  | 1  | 1  |
| "4"           | 0  | 1  | 0  | 0  |
| "5"           | 0  | 1  | 0  | 1  |
| "6"           | 0  | 1  | 1  | 0  |
| "7"           | 0  | 1  | 1  | 1  |
| "8"           | 1  | 0  | 0  | 0  |
| "9"           | 1  | 0  | 0  | 1  |

To code output signals two variables are required, as  $y_1, y_0$ .

| Output | y1 | y0 |
|--------|----|----|
| "0"    | 0  | 0  |
| "1"    | 0  | 1  |
| "2"    | 1  | 0  |

So, FSM has 4 inputs and 2 outputs.

2. State assignment

The number of required state variables  $k$  is defined as follows:

$$3 \leq k \leq \log_2 3. \text{ Accept } k=2.$$

S0 – 00

S1 – 01

S2 – 10

Unused state – 00 (solution of minimal-value)

Definition of excitation function

| Transition | J | K |
|------------|---|---|
| 0 - 0      | 0 | x |
| 0 - 1      | 1 | x |
| 1 - 0      | x | 1 |
| 1 - 1      | x | 0 |

State and output table

| Present state<br>q1 q0 | Output<br>y1 y0 | Input<br>X | Next State<br>q1 q0 | Excitation Functions |    |    |    |
|------------------------|-----------------|------------|---------------------|----------------------|----|----|----|
|                        |                 |            |                     | j1                   | k1 | j0 | k0 |
| 0 0                    | 0 0             | 0v3v6i9    | 0 0                 | 0                    | -  | 0  | -  |
|                        |                 | 1i4i7      | 0 1                 | 0                    | -  | 1  | -  |
|                        |                 | 2i5i8      | 1 0                 | 1                    | -  | 0  | -  |
| 0 1                    | 0 1             | 0v3v6v9    | 0 1                 | 0                    | -  | -  | 0  |
|                        |                 | 1v4v7      | 1 0                 | 1                    | -  | -  | 1  |
|                        |                 | 2v5v8      | 0 0                 | 0                    | -  | -  | 1  |
| 1 0                    | 1 0             | 0v3v6v9    | 1 0                 | -                    | 0  | 0  | -  |
|                        |                 | 1v4v7      | 0 0                 | -                    | 1  | 0  | -  |
|                        |                 | 2v5v8      | 0 1                 | -                    | 1  | 1  | -  |

Excitation functions depend on 6 variables:  $q1, q2, x0, x1, x2, x3$ .

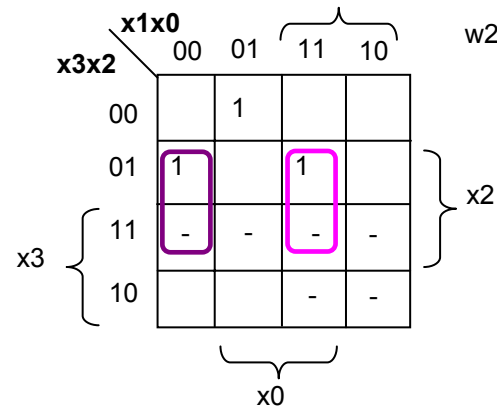
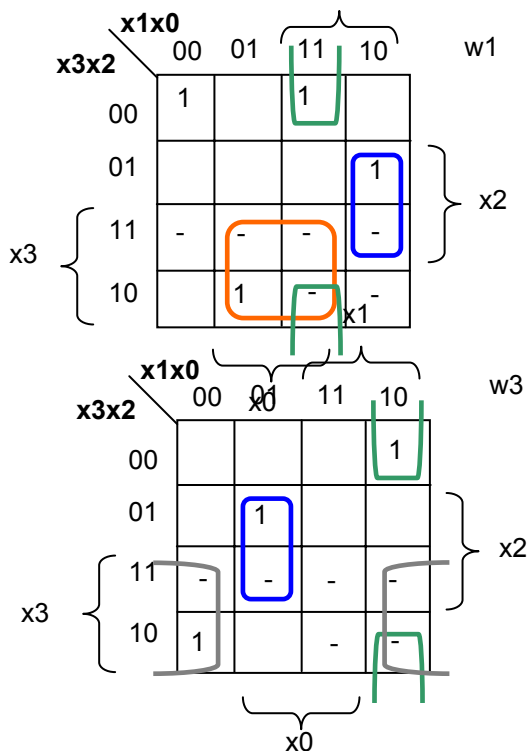
To simplify the process of designing  $w1, w2, w3$  extra variables are inserted:

0v3v6v9 -  $w1$ ;

1v4v7 -  $w2$ ;

2v5v8 -  $w3$ ;

Karnaugh maps for  $w1, w2, w3$ .



$$w1 = \sim x3 \cdot \sim x2 \cdot \sim x1 \cdot \sim x0 + \sim x2 \cdot x1 \cdot x0 + x2 \cdot x1 \cdot \sim x0 + x3 \cdot x0$$

$$w2 = \sim x3 \cdot \sim x2 \cdot \sim x1 \cdot x0 + x2 \cdot \sim x1 \cdot \sim x0 + x2 \cdot x1 \cdot x0$$

$$w3 = x3 \cdot \sim x0 + x2 \cdot \sim x1 \cdot x0 + \sim x2 \cdot x1 \cdot x0$$

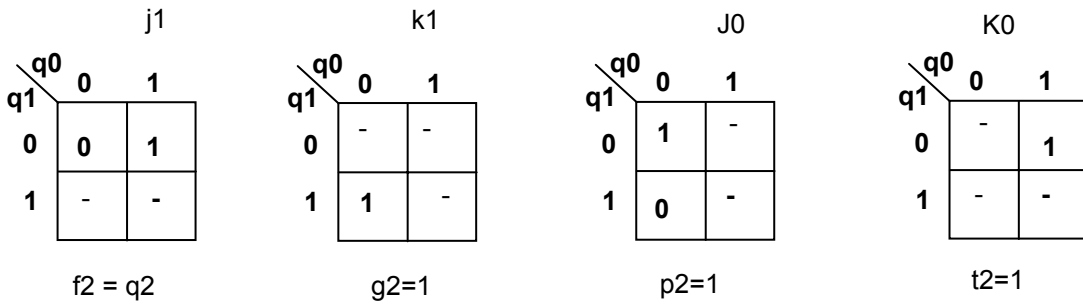
Definition of  $j1, k1, j2, k2$  excitation functions as function of  $w1, w2, w3, q1, q0$ .

$$j1 = w1f1 + w2f2 + w3f3; k1 = w1g1 + w2g2 + w3g3$$

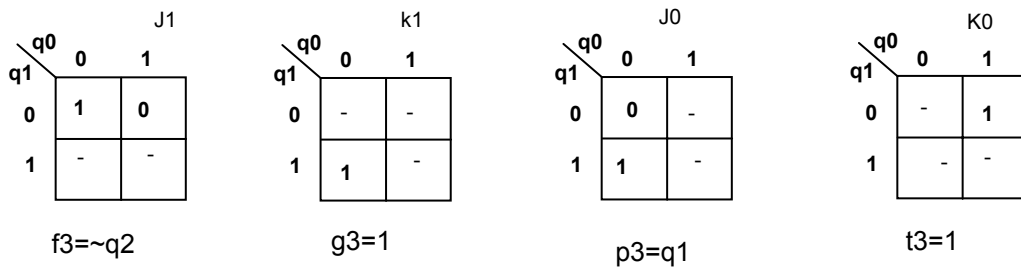
$j0 = w1p1 + w2p2 + w3p3, k0 = w1t1 + w2t2 + w3t3$ , where  $f1...f3, g1,...g3, p1...p3, t1... t3$  depend on  $q1, q2$  variables.

As follows from the above table, f1 and g1 are equal to 0.

Maps for definition f2,g2,p2,t2  
(factors for w2)



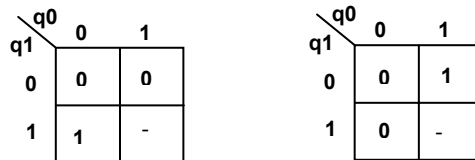
Maps for definition f3,g3,p3,t3  
(factors for w3)



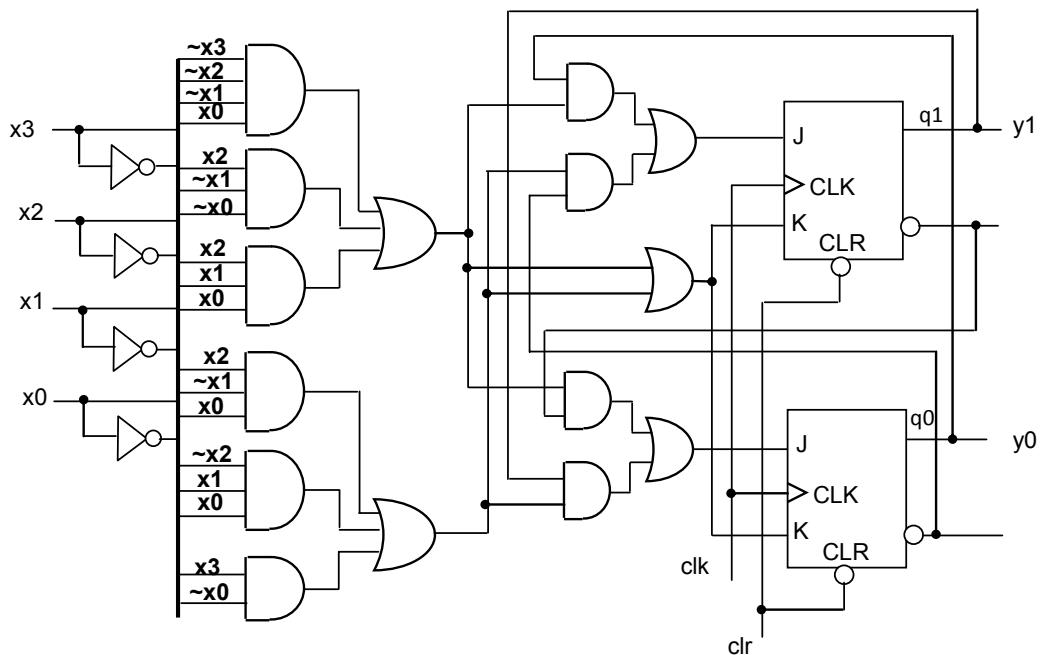
So,  
 $j1 = w2q2 + w3\sim q2$ ;  
 $k1 = w2 + w3$ ;  
 $j0 = w2\sim q1 + w3q1$ ;  
 $k0 = w2 + w3$ ;

Maps for defining y1, y0

$y1 = q1$ ;  $y0 = q0$ ;



FSM circuit diagram



### 1b24.

#### Creation of State Transition Graph of FSM

FSM is given by means of the following five sets:  $A = \{X, Y, S, \delta, \lambda\}$ , where  $X = \{X_1, X_2, \dots, X_M\}$  – set of input symbols;

$Y = \{Y_1, Y_2, \dots, Y_N\}$  – set of output symbols;

$S = \{S_0, S_1, \dots, S_{K-1}\}$  – set of internal states of FSM;

$\delta$  – next state (transition) function,

$\lambda$  – output function

$X = \{00, 01, 10, 11\}$

$Y = \{0, 1\}$

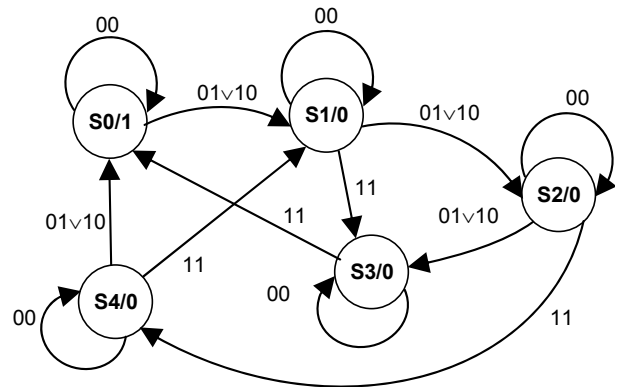
S0 – got zero 1s (modulo 5)

S1 – got one 1 (modulo 5)

S2 – got two 1s (modulo 5)

S3 – got three 1s (modulo 5)

S4 – got four 1s (modulo 5)



#### Verilog Description of FSM

```
module moore_countmod5(data, clock, reset, out);
 input reset, clock;
 input [1:0] data;
 output out;
 reg out;
 reg [2:0] state, next_state;
 parameter st0 = 3'b000, st1 = 3'b001, st2 = 3'b010, st3=3'b011, st4=3'b100;
 //FSM register
 always @(posedge clock or negedge reset)
 begin: statereg
 if(!reset) //asynchronous reset
 state <= st0;
 else state <= next_state;
 end //statereg

 //FSM next_state logic
 always @(state or data)
 begin: fsm
 case (state)
 st0: case (data)
 2'b00: next_state = st0;
 2'b01, 2'b10: next_state = st1;
 2'b11: next_state = st2;
 default: next_state = st0;
 endcase
 st1: case (data)
 2'b00: next_state = st1;
 2'b01, 2'b10: next_state = st2;
 2'b11: next_state = st3;
 default: next_state = st0;
 endcase
 st2: case (data)
 2'b00: next_state = st2;
 2'b01, 2'b10: next_state = st3;
 2'b11: next_state = st4;
 default: next_state = st0;
 endcase
 st3: case (data)
 2'b00: next_state = st3;
 2'b01, 2'b10: next_state = st4;
 2'b11: next_state = st0;
 default: next_state = st0;
 endcase
 st4: case (data)
 2'b00: next_state = st4;
 2'b01, 2'b10: next_state = st0;
 2'b11: next_state = st1;
 default: next_state = st0;
 endcase
 endcase
 end
endmodule
```



```

 default: next_state = st0;
 endcase
end//fsm
//Moore output definition using pres_state only
always @(state)
begin: def_out
if (state == st0)
out = 1'b1;
else out = 1'b0;
end//def_out
endmodule

```

**Synthesis of FSM using D flip-flops**

**1. State Assignment (first example)**

Number of flip-flops:  $n = \lceil \log_2 5 \rceil = 3$ .

S0 – 000

S1 – 001

S2 – 010

S3 – 101

S4 – 100

FSM inputs are designated as x1, x2.

FSM output is designated as y.

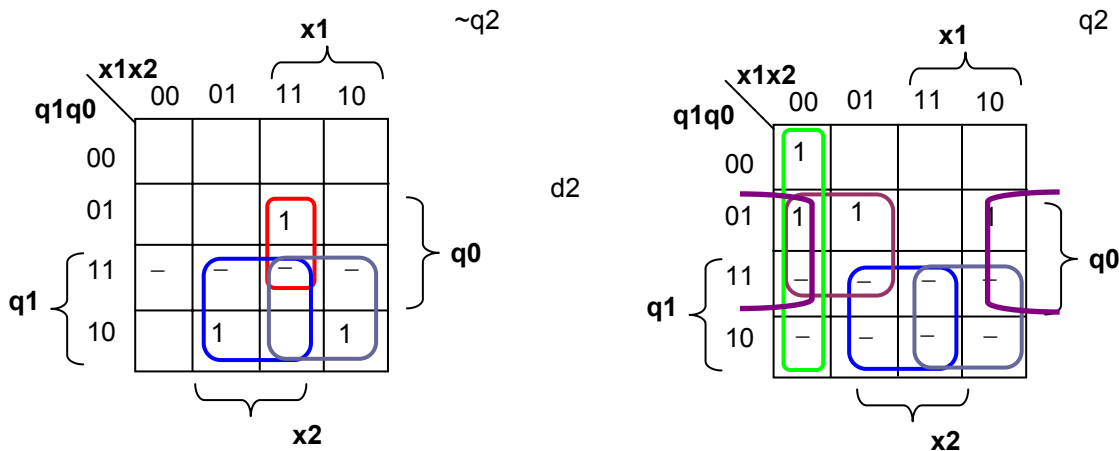
**2. Transition and output table**

| Present state |    |    | Input | Next state |    |    | Excitation Functions |    |    |    |    |    |
|---------------|----|----|-------|------------|----|----|----------------------|----|----|----|----|----|
| q2            | q1 | q0 |       | y          | x1 | x2 | q2                   | q1 | q0 | d2 | d1 | d0 |
| 0             | 0  | 0  | 1     | 00         |    |    | 0                    | 0  | 0  | 0  | 0  | 0  |
|               |    |    |       | 01v10      |    |    | 0                    | 0  | 1  | 0  | 0  | 1  |
|               |    |    |       | 11         |    |    | 0                    | 1  | 0  | 0  | 1  | 0  |
| 0             | 0  | 1  | 0     | 00         |    |    | 0                    | 0  | 1  | 0  | 0  | 1  |
|               |    |    |       | 01v10      |    |    | 0                    | 1  | 0  | 0  | 1  | 0  |
|               |    |    |       | 11         |    |    | 1                    | 0  | 1  | 1  | 0  | 1  |
| 0             | 1  | 0  | 0     | 00         |    |    | 0                    | 1  | 0  | 0  | 1  | 0  |
|               |    |    |       | 01v10      |    |    | 1                    | 0  | 1  | 1  | 0  | 1  |
|               |    |    |       | 11         |    |    | 1                    | 0  | 0  | 1  | 0  | 0  |
| 1             | 0  | 1  | 0     | 00         |    |    | 1                    | 0  | 1  | 1  | 0  | 1  |
|               |    |    |       | 01v10      |    |    | 1                    | 0  | 0  | 1  | 0  | 0  |
|               |    |    |       | 11         |    |    | 0                    | 0  | 0  | 0  | 0  | 0  |
| 1             | 0  | 0  | 0     | 00         |    |    | 1                    | 0  | 0  | 1  | 0  | 0  |
|               |    |    |       | 01v10      |    |    | 0                    | 0  | 0  | 0  | 0  | 0  |
|               |    |    |       | 11         |    |    | 0                    | 0  | 1  | 0  | 0  | 1  |

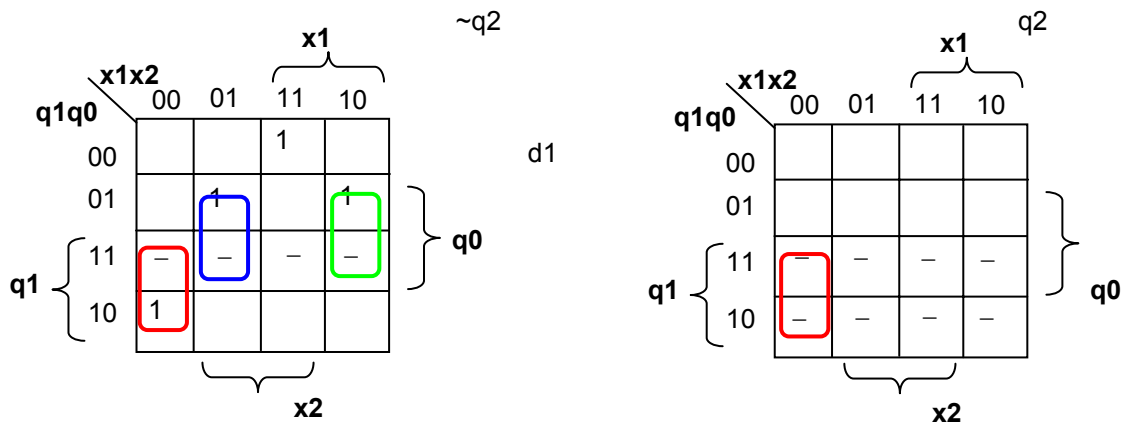
**3. Minimization of Excitation Functions**

d2, d1, d0 depend on 5 variables: q2,q1,q0,x1,x2.

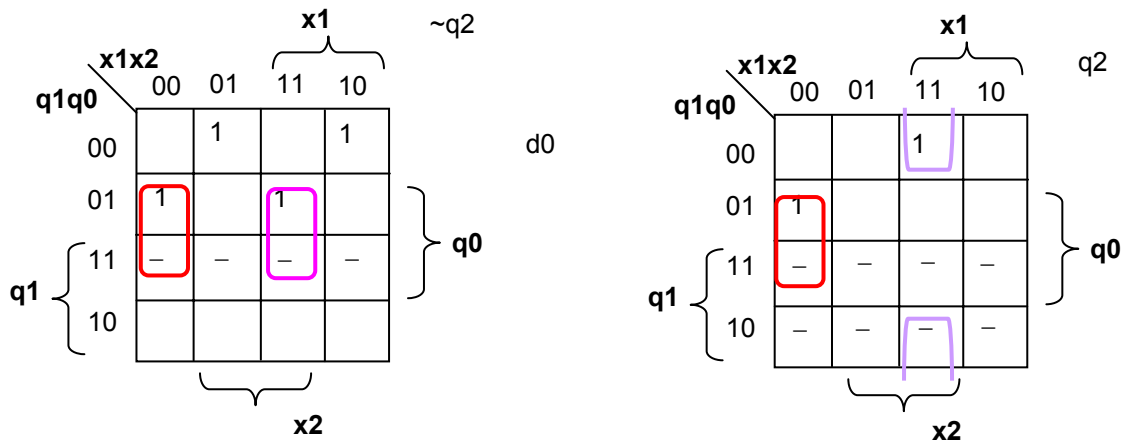
Unused states: 011, 110, 111. (solution of minimal-value)



$$d2 = q1 \cdot x2 + q1 \cdot x1 + \sim q2 q0 x1 x2 + q2 \sim x1 \sim x2 + q0 q2 \sim x1 + q2 q0 \sim x2$$



$$d1 = q1 \sim x1 \sim x2 + q0 \sim q2 \sim x1 x2 + q0 \sim q2 x1 \sim x2 + \sim q2 \sim q1 \sim q0 x1 x2$$



$$d0 = q0 \sim x1 \sim x2 + q0 \sim q2 x1 x2 + \sim q0 \sim q1 \sim q2 \sim x1 x2 + \sim q0 \sim q1 \sim q2 x1 \sim x2 + \sim q0 q2 x1 x2$$

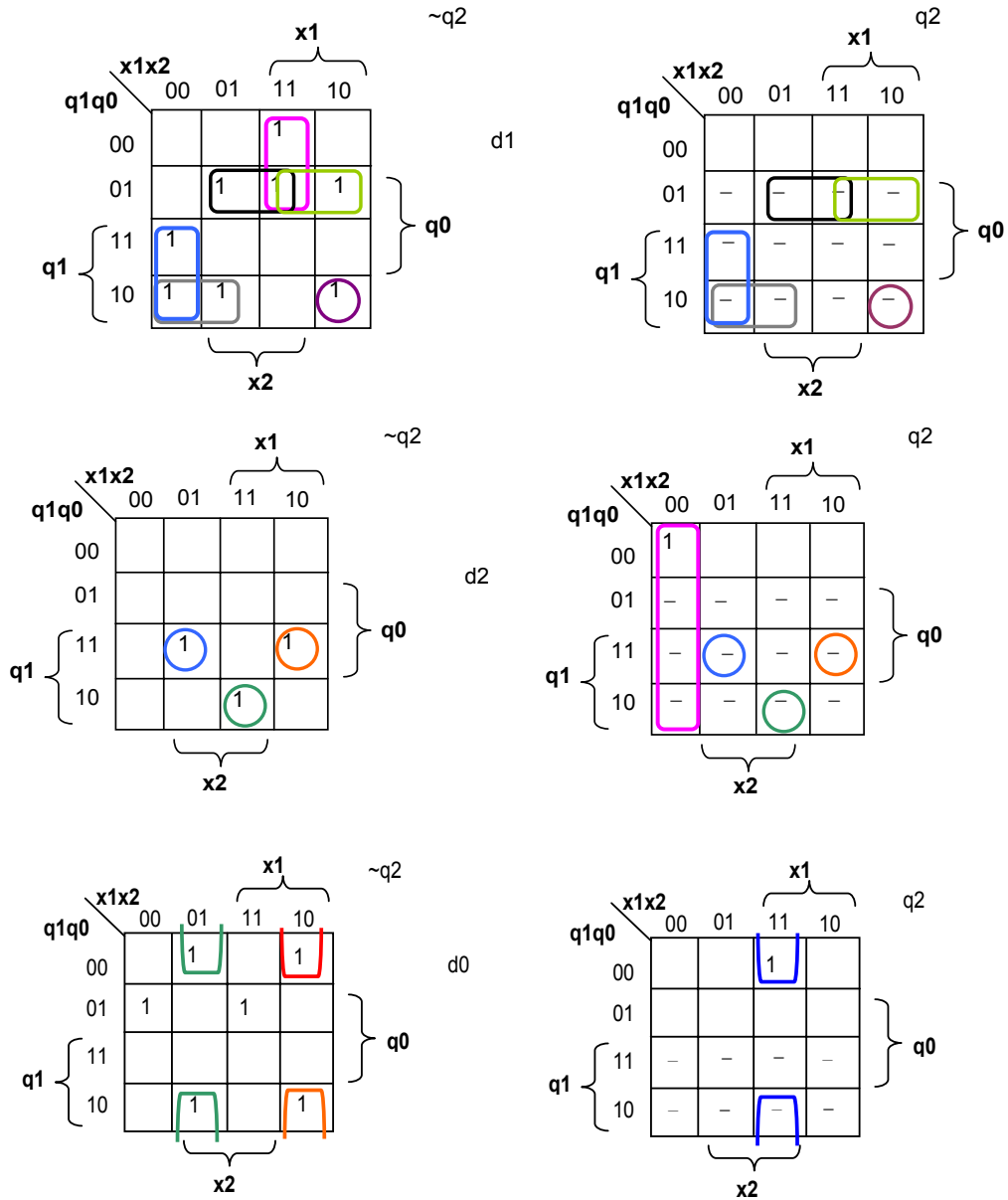
The number of inputs of gates is equal to 69.

4. The second example of states assignment using D flip-flops  
 S0 – 000, S1 – 001, S2 – 010, S3 – 011, S4 – 100.

Unused states. 101, 110, 111 (minimal-value solution)

| Present state |       | Input | Next state | Excitation Functions |    |    | Excitation Functions |    |    |    |    |    |    |    |    |
|---------------|-------|-------|------------|----------------------|----|----|----------------------|----|----|----|----|----|----|----|----|
| q2            | q1 q0 |       |            | y                    | x1 | x2 | q2                   | q1 | q0 | t2 | t1 | t0 | d2 | d1 | d0 |
| 0             | 0     | 1     | 00         | 0                    | 0  | 0  | 0                    | 0  | 0  | 0  | 0  | 0  | 0  | 0  |    |
|               |       |       | 01v10      | 0                    | 0  | 1  | 0                    | 0  | 1  | 0  | 0  | 1  | 0  | 0  | 1  |
|               |       |       | 11         | 0                    | 1  | 0  | 0                    | 1  | 0  | 0  | 1  | 0  | 0  | 1  | 0  |
| 0             | 0     | 0     | 00         | 0                    | 0  | 1  | 0                    | 0  | 0  | 0  | 0  | 0  | 0  | 1  |    |
|               |       |       | 01v10      | 0                    | 1  | 0  | 0                    | 1  | 1  | 0  | 1  | 1  | 0  | 1  | 0  |
|               |       |       | 11         | 0                    | 1  | 1  | 0                    | 1  | 0  | 0  | 1  | 0  | 0  | 1  | 1  |
| 0             | 1     | 0     | 00         | 0                    | 1  | 0  | 0                    | 0  | 0  | 0  | 0  | 0  | 1  | 0  |    |
|               |       |       | 01v10      | 0                    | 1  | 1  | 0                    | 1  | 1  | 0  | 0  | 1  | 0  | 1  | 1  |
|               |       |       | 11         | 1                    | 0  | 0  | 1                    | 1  | 0  | 0  | 1  | 1  | 0  | 0  | 0  |
| 1             | 0     | 0     | 00         | 0                    | 1  | 1  | 0                    | 0  | 0  | 0  | 0  | 0  | 1  | 1  |    |
|               |       |       | 01v10      | 1                    | 0  | 0  | 0                    | 1  | 0  | 0  | 1  | 0  | 0  | 1  | 0  |
|               |       |       | 11         | 0                    | 0  | 0  | 0                    | 1  | 1  | 0  | 0  | 1  | 0  | 0  | 0  |
| 1             | 0     | 0     | 00         | 1                    | 0  | 0  | 0                    | 0  | 0  | 0  | 0  | 1  | 0  | 0  |    |
|               |       |       | 01v10      | 0                    | 0  | 0  | 0                    | 1  | 0  | 0  | 1  | 0  | 0  | 0  | 0  |
|               |       |       | 11         | 0                    | 0  | 1  | 0                    | 1  | 0  | 1  | 0  | 1  | 0  | 0  | 1  |

### Minimization of Excitation Functions



$$d_2 = q_1 \cdot q_0 \cdot \sim x_1 \cdot x_2 + q_1 \cdot \sim q_0 \cdot x_1 \cdot x_2 + q_2 \cdot x_1 \cdot x_2 + q_1 \cdot q_0 \cdot x_1 \cdot \sim x_2$$

$$d_1 = q_1 \cdot \sim x_1 \cdot \sim x_2 + q_0 \cdot \sim q_1 \cdot x_2 + q_1 \cdot \sim q_0 \cdot \sim x_1 + \sim q_1 \cdot \sim q_2 \cdot x_1 \cdot x_2 + q_0 \cdot \sim q_1 \cdot x_1 + q_1 \cdot \sim q_0 \cdot x_1 \cdot \sim x_2$$

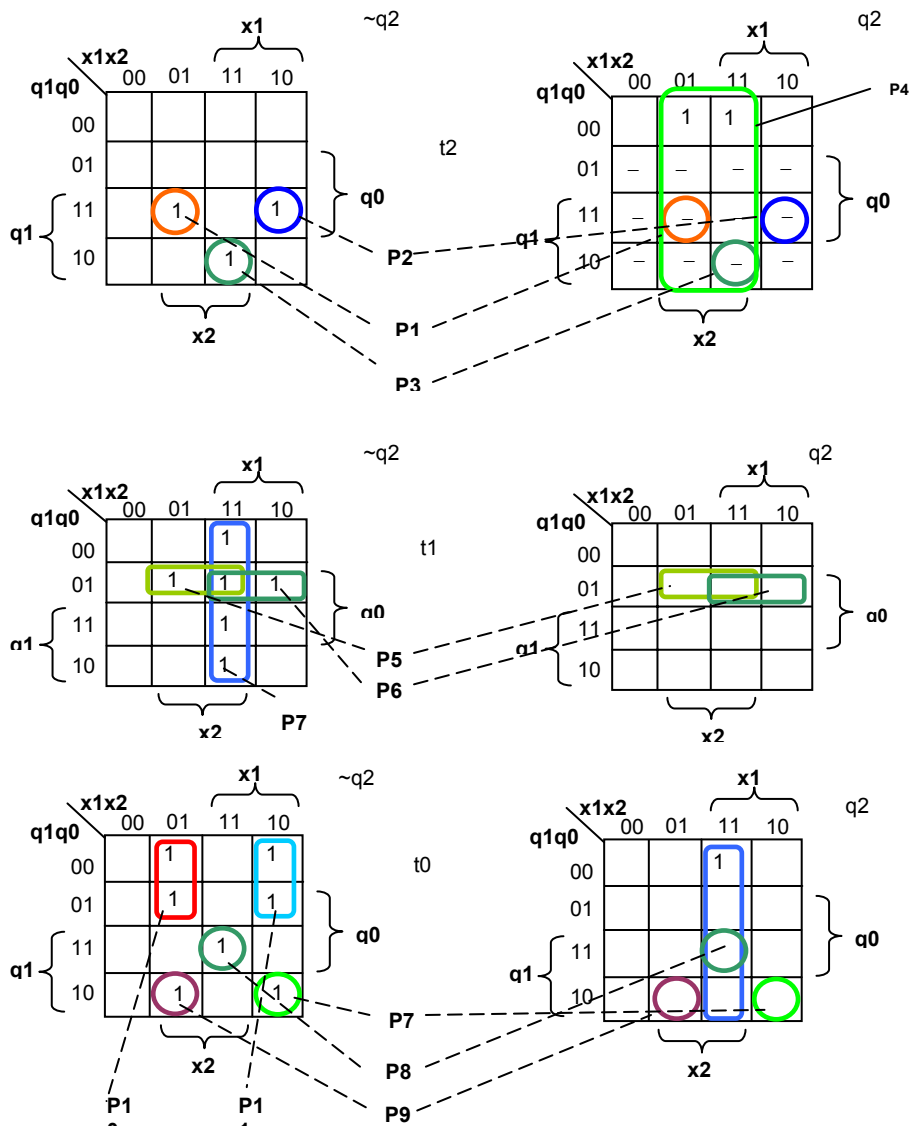
$$d_0 = \sim q_2 \cdot \sim q_0 \cdot \sim x_1 \cdot x_2 + \sim q_2 \cdot \sim q_0 \cdot x_1 \cdot x_2 + \sim q_0 \cdot q_2 \cdot x_1 \cdot x_2 + \sim q_1 \cdot \sim q_2 \cdot q_0 \cdot \sim x_1 \cdot x_2 + \sim q_2 \cdot \sim q_1 \cdot q_0 \cdot x_1 \cdot x_2$$

The number of gates' inputs is equal to 72.

The values of both examples are approximately the same.

Synthesis of FSM using T flip-flops

Excitation functions are shown in the table above.



$$t_2 = q_1 q_0 \sim x_1 x_2 + q_1 \sim q_0 x_1 x_2 + q_1 q_0 x_1 \sim x_2 + q_2 x_2;$$

$$t_1 = \sim q_1 q_0 x_2 + \sim q_1 q_0 x_1 + \sim q_2 x_1 x_2$$

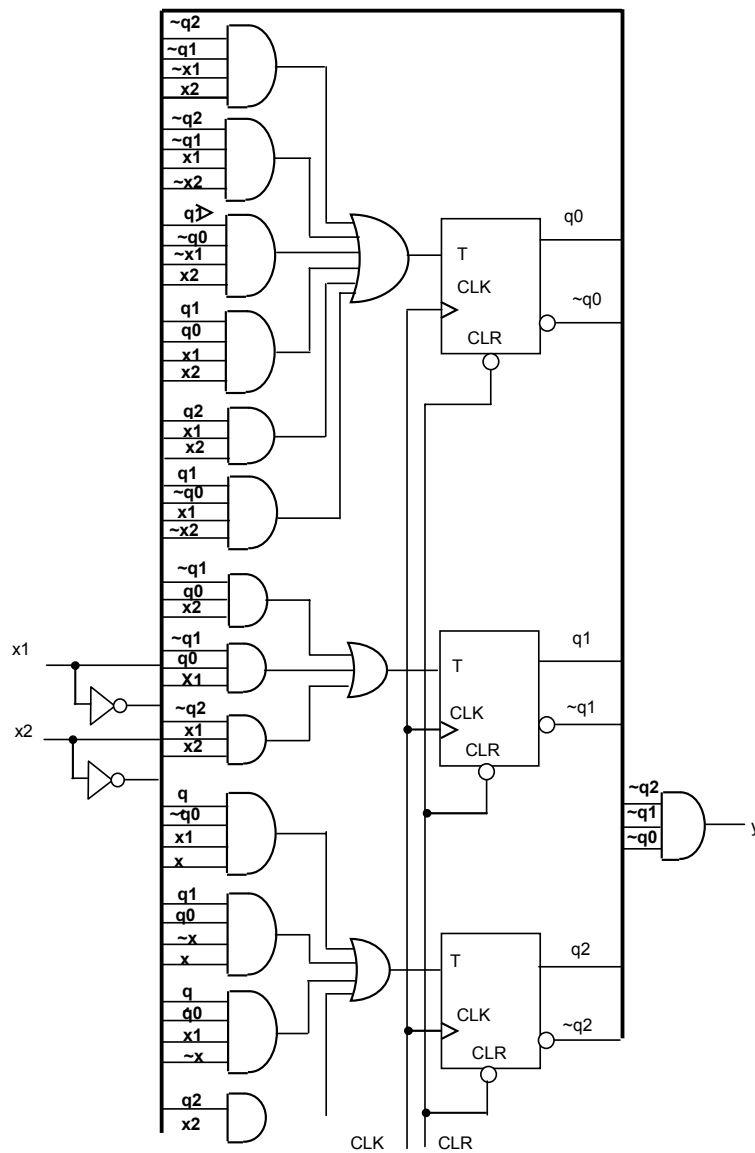
$$t_0 = \sim q_2 \sim q_1 \sim x_1 x_2 + \sim q_2 \sim q_1 x_1 \sim x_2 + q_2 x_1 x_2 + q_1 \sim q_0 \sim x_1 x_2 + q_1 q_0 x_1 x_2 + q_1 \sim q_0 x_1 \sim x_2$$

The number of gates' inputs is equal to 63.

Definition of  $y = f(q_2, q_1, q_0)$

$$y = \sim q_2 \sim q_1 \sim q_0$$

The last variant of implementation should be chosen.



FSM circuit diagram

**1b25.**

The minimum signal formation time in some  $i$ -th net is defined by the following formula:

$$t_{bi} = \max[t_{b(i-1)} + t_{(i-1, i)}],$$

where  $t_{bi}$  and  $t_{b(i-1)}$  – minimum limit times of  $i$ -th and  $(i-1)$ -th nets correspondingly;  $t_{(i-1, i)}$  – is the delay of the element for which  $(i-1)$ -th net is input and  $i$ -th net – output.

The maximum signal formation time in some  $i$ -th net is defined by the following formula:

$$t_{ri} = \min[t_{r(i+1)} - t_{(i+1, i)}],$$

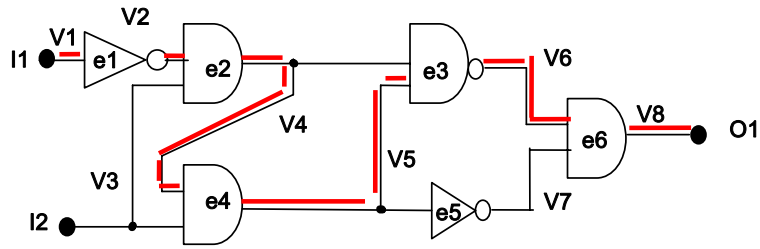
where  $t_{ri}$  and  $t_{r(i+1)}$  – maximum limit times of  $i$ -th and  $(i+1)$ -th nets correspondingly;  $t_{(i+1, i)}$  – is the delay of the element for which  $(i+1)$ -th net is output and  $i$ -th net – input.

Time reserve for  $i$ -th net will be:

$$h_i = t_{maxi} - t_{mini}$$

Calculation results are shown in the table

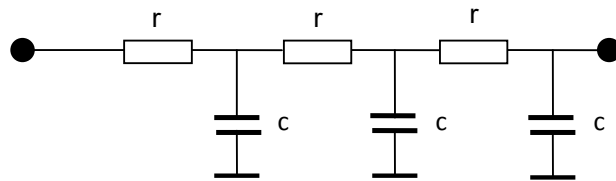
| Net        | V1 | V2 | V3 | V4 | V5 | V6 | V7 | V8 |
|------------|----|----|----|----|----|----|----|----|
| $t_{mini}$ | 0  | 10 | 0  | 30 | 50 | 65 | 60 | 85 |
| $t_{maxi}$ | 0  | 10 | 10 | 30 | 50 | 65 | 65 | 85 |



Critical path passes through all the nets for which  $t_{bi}=t_{ri}$ , i.e. V1, V2, V4, V5, V6, V8. The total delay of that path is 85.

**1b26.**

The equivalent circuit of interconnect's 3-segment, R,C distributed parameters will look like this:

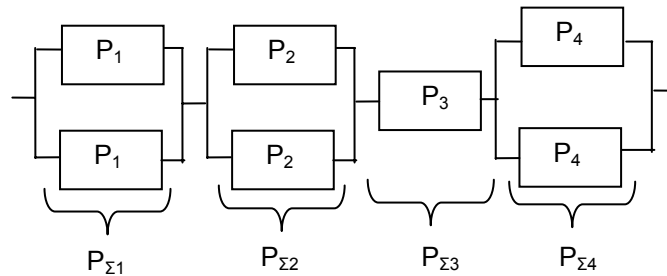


The delay in it will be

$$\begin{aligned} \tau &= rc + 2rc + 3rc = 6rc: \\ r &= [(300:3):0,2] \times 0,1 = 50 \text{ Ohm}, \\ c &= \frac{300}{3} \cdot 0,1 = 10 \text{ fF} \\ c &= (300:3) \times 0,1 = 10 \text{ fF}, \\ \tau &= 6 \times 50 \times 10 = 3000 \text{ OhmfF} = 3 \text{ ns}. \end{aligned}$$

**1b27.**

Present the given circuit in the view of 4 sequentially connected regions:



Designate faultness probabilities of those regions

$$P_{\Sigma 1}, P_{\Sigma 2}, P_{\Sigma 3}, P_{\Sigma 4}.$$

Using probability summing, multiplication and full probability laws, the following can be written:

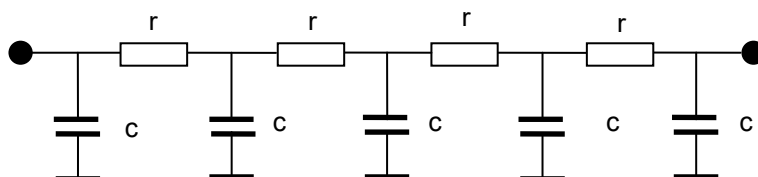
$$\begin{aligned} P_{\Sigma 1} &= P_1 P_1 + 2P_1(1-P_1) = P_1(2-P_1) = 0,5(2-0,5) = 0,75 \\ P_{\Sigma 2} &= P_2(2-P_2) = 0,84 \\ P_{\Sigma 3} &= P_3 = 0,8 \\ P_{\Sigma 4} &= P_4(2-P_4) = 0,64 \end{aligned}$$

The faultness probability of the circuit will be:

$$P = P_{\Sigma 1} \cdot P_{\Sigma 1} \cdot P_{\Sigma 1} \cdot P_{\Sigma 1} = 0,75 \cdot 0,84 \cdot 0,8 \cdot 0,64 = 0,32256.$$

**1b28.**

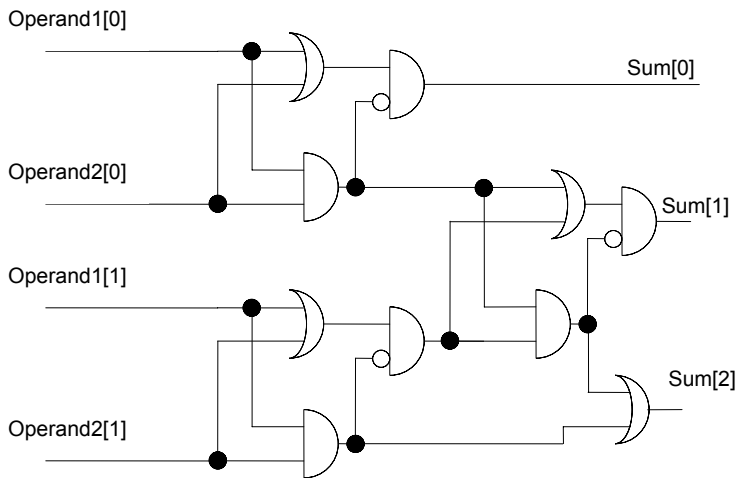
R, C equivalent circuit of interconnect will look as follows:



The delay in the transmission line connecting two contacts will be:

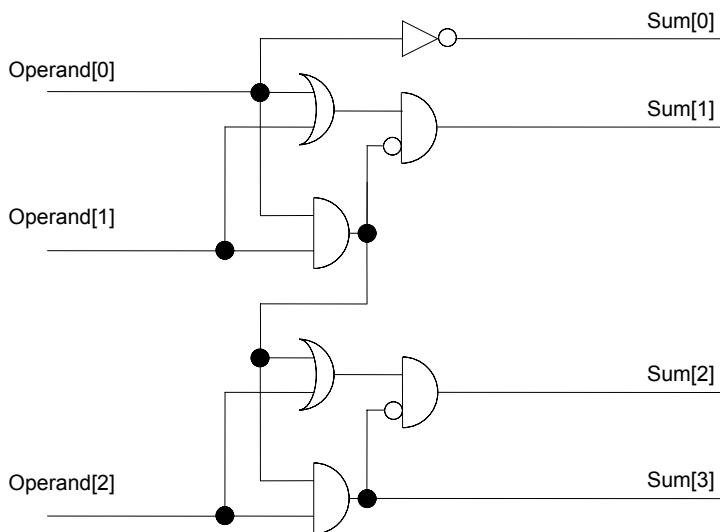
$$\tau = rc + 2rc + 3rc + 4rc = 10rc = 10 \cdot 1 \cdot 100 = 1000 \text{ OhmF} = 10 \text{ ns.}$$

**1b29.**

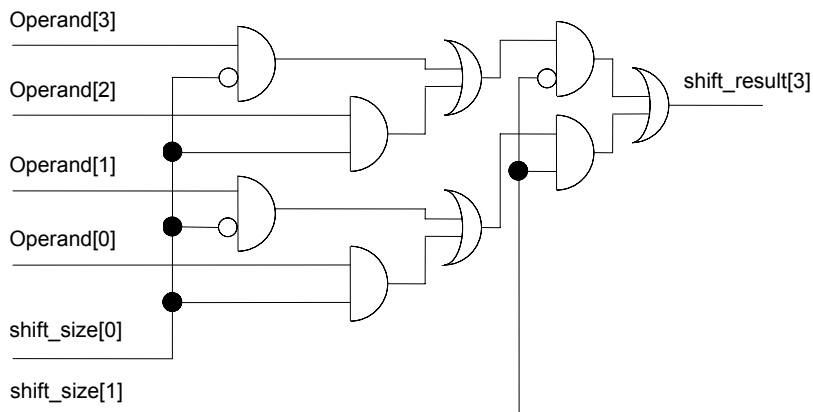


Yes, given logic is a pure combinatorial logic as it doesn't contain any memory elements and any logical feedbacks. Also, the outputs are only and only dependent from the inputs (from the combinations of inputs).

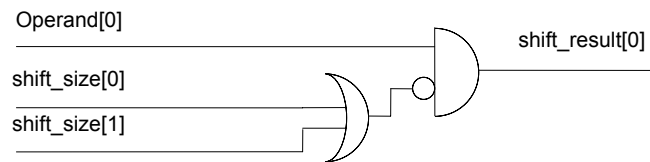
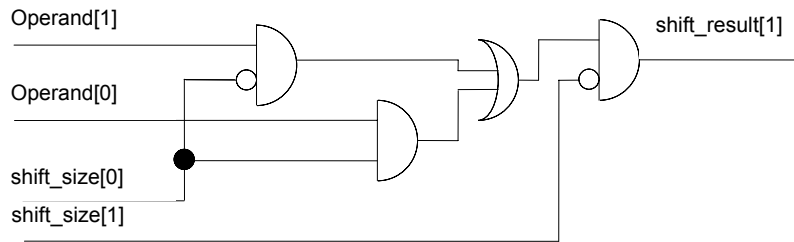
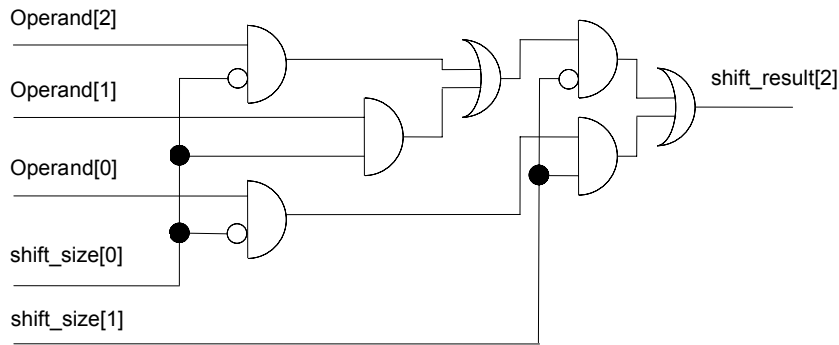
**1b30.**



Yes, given logic is a pure combinatorial logic as it doesn't contain any memory elements and any logical feedbacks. Also, the outputs are only and only dependent from the inputs (from the combinations of inputs).

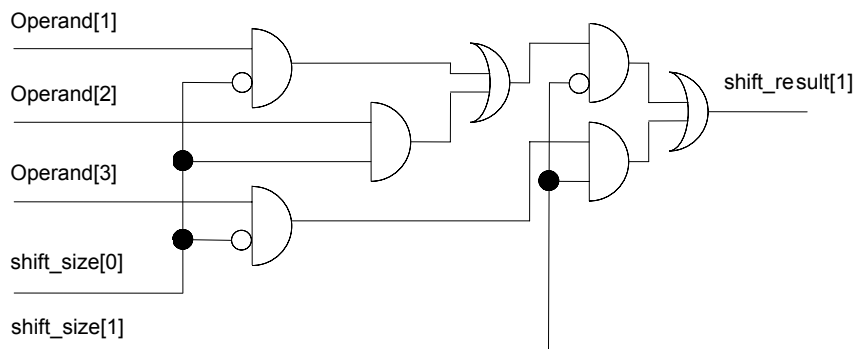
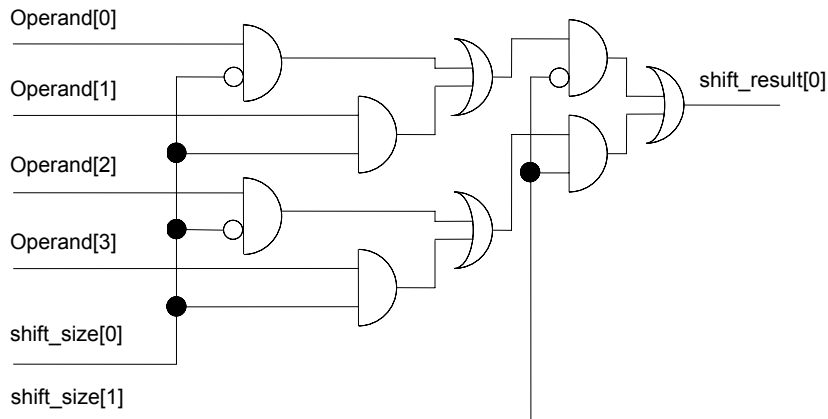


**1b31.**

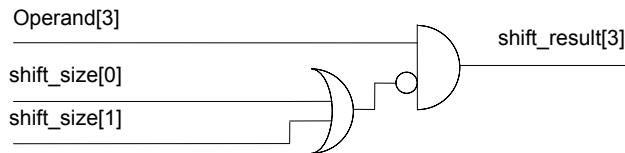
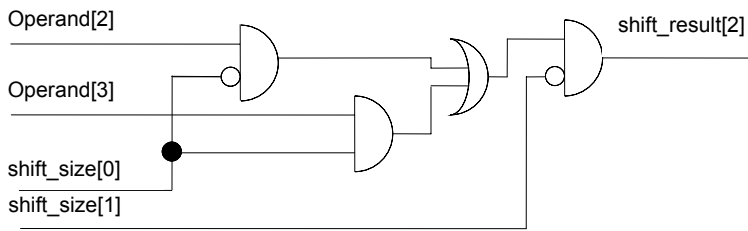


Yes, given logic is a pure combinatorial logic as it doesn't contain any memory elements and any logical feedbacks. Also, the outputs are only and only dependent from the inputs (from the combinations of inputs).

**1b32.**







Yes, given logic is a pure combinatorial logic as it does not contain any memory elements and any logical feedbacks. Also, the outputs are only and only dependent from the inputs (from the combinations of inputs).

**1b33.**

$$t_{d1} = R_1 C_{L1} = R_1 2y C_{in1} = 2y R_1 C_{in1} = 2y R_1 C$$

$$t_{d2} = R_2 C_{L2} = (R_1/y) 3z C_{in1} = 3z R_1 C/y$$

$$t_{d3} = R_3 C_{L3} = (R_1/z) 4.5 C_{in1} = 4.5 R_1 C/z$$

$$t_d = t_{d1} + t_{d2} + t_{d3} = R_1 C (2y + 3z/y + 4.5/z)$$

$$\partial t_d / \partial z = R_1 C (3/y - 4.5/z^2) = 0$$

$$\partial t_d / \partial y = R_1 C (2 - 3z/y^2) = 0$$

From the solution of system  $y=1.275$ ;  $z=1.084$ .

**1b34.**

Answer: 0.29 V

**1b35.**

When  $V_{in} = V_H$ , NMOS is in active mode, and PMOS is in saturated mode.

$$C_{ox} = \frac{\epsilon_{ox} \epsilon_0}{T_{ox}} = \frac{3.9 \cdot 8.854 \cdot 10^{-14}}{10^{-6}} = 3.45 \cdot 10^{-7} \text{ F / cm}^2$$

$$k_p = \mu_p C_{ox} = 241 \cdot 10^{-7} \text{ A/V}^2 = 24.1 \mu\text{A/V}^2; \quad k_n = \mu_n C_{ox} = 93.15 \mu\text{A/V}^2;$$

a.  $V_{OH} = V_{DD} = 1.8 \text{ V}$ ,

$$\beta_p (V_{DD} - V_{TP})^2 / 2 = \beta_n ((V_{DD} - V_{TN}) V_{OL} - V_{OL}^2 / 2)$$

$$V_{OL} = (V_{DD} - V_T) \left( 1 - \sqrt{1 - \frac{\beta_p}{\beta_n}} \right)$$

a.  $\beta_p = k_p (W/L)_p = 669.4 \mu\text{A/V}^2$

$\beta_n = k_n (W/L)_n = 10350 \mu\text{A/V}^2$

$$V_{OL} = (1.8 - 0.5) \left( 1 - \sqrt{1 - 669.4/10350} \right) = 0.043 \text{ V}$$

$V_{OL} = 0.043 \text{ V}$ ;  $V_{OH} = V_{DD} = 1.8 \text{ V}$ ,

b.  $\beta_n = 4k_n (W/L)_n = 41400 \mu\text{A/V}^2$

$$V_{OL} = (1.8 - 0.5) \left( 1 - \sqrt{1 - 669.4/41400} \right) = 0.011 \text{ V}$$

$V_{OL} = 0.011 \text{ V}$ ;  $V_{OH} = V_{DD} = 1.8 \text{ V}$

**1b36.**

Minimum range of clock pulses in digital circuits is defined from the condition of not violating setup time of flip-flop:

$$T_{CLKmin} = t_{c2q} + t_{pinv} + t_{su} = 100 + 30 + 20 = 150 \text{ ps}$$

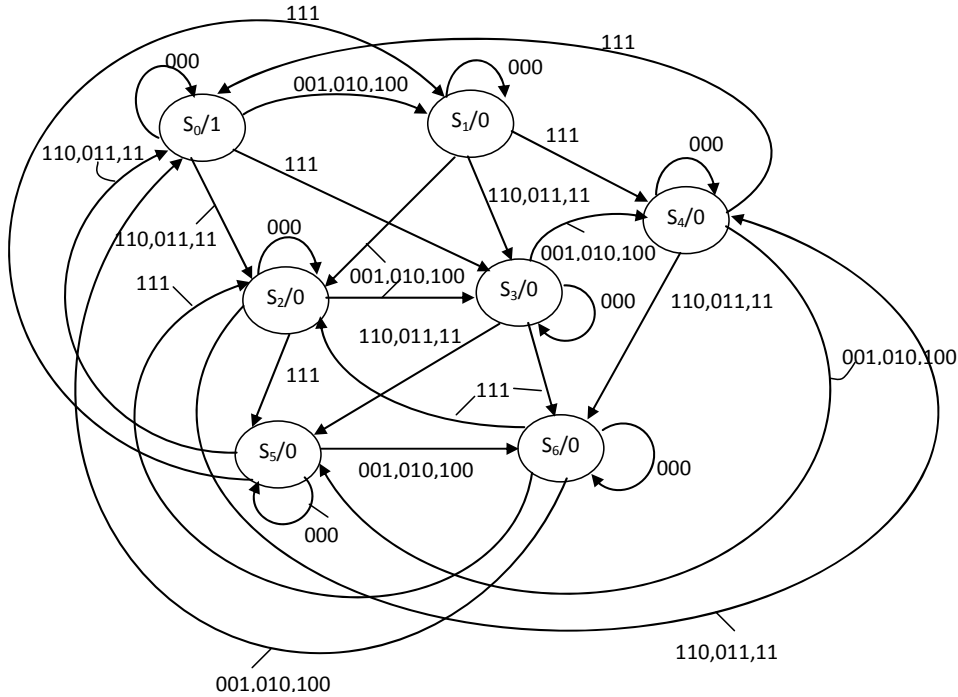
1b37.

$$a. \Delta V_V = \frac{C_{AV}}{C_V + C_{AV}} \Delta V_A = \frac{100}{60 + 100} \cdot 1 = 0.625V$$

$$b. C_{Vsw} = C_V + 2C_{AV} = 60 + 2 \cdot 100 = 260 \text{ fF}$$

1b38.

Design of Moore's FSM:



Description of FSM by Verilog

```

module fsm(in,out,clk,reset);
input clk,reset;
input[2:0] in;
output out;
reg out;
reg[2:0] state, next_state;
parameter s0=3'b000, s1=3'b001, s2=3'b010, s3=3'b011, s4=3'b100,s5=3'b101,s6=3'b110;
//state register
always @(posedge clk or negedge reset)
begin
if (!reset) state=s0;
else state=next_state;
end

//next state logic
always @(state or in)
begin
case (state)
s0: case (in)
3'b000: next_state=state;
3'b001,3'b010,3'b100: next_state=s1;
3'b011,3'b101,3'b110: next_state=s2;
3'b111: next_state=s3;
default: next_state=s0;
endcase
s1: case (in)
3'b000: next_state=state;
3'b001,3'b010,3'b100: next_state=s2;
3'b011,3'b101,3'b110: next_state=s3;
3'b111: next_state=s4;
default: next_state=s0;

```

```

 endcase
s2: case (in)
 3'b000: next_state=state;
 3'b001,3'b010,3'b100: next_state=s3;
 3'b011,3'b101,3'b110: next_state=s4;
 3'b111: next_state=s5;
 default: next_state=s0;
 endcase
s3: case (in)
 3'b000: next_state=state;
 3'b001,3'b010,3'b100: next_state=s4;
 3'b011,3'b101,3'b110: next_state=s5;
 3'b111: next_state=s6;
 default: next_state=s0;
 endcase
s4: case (in)
 3'b000: next_state=state;
 3'b001,3'b010,3'b100: next_state=s5;
 3'b011,3'b101,3'b110: next_state=s6;
 3'b111: next_state=s0;
 default: next_state=s0;
 endcase
s5: case (in)
 3'b000: next_state=state;
 3'b001,3'b010,3'b100: next_state=s6;
 3'b011,3'b101,3'b110: next_state=s0;
 3'b111: next_state=s1;
 default: next_state=s0;
 endcase
s6: case (in)
 3'b000: next_state=state;
 3'b001,3'b010,3'b100: next_state=s0;
 3'b011,3'b101,3'b110: next_state=s1;
 3'b111: next_state=s2;
 default: next_state=s0;
 endcase
default: next_state=s0;
endcase
end
//output logic
always @(state)
 if (state==s0)
 out=1'b1;
 else out=1'b0;
endmodule

```

### Testbench

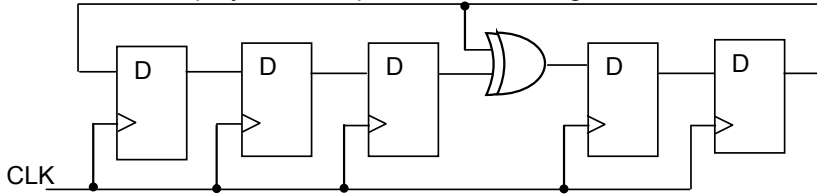
```

module stimulus;
reg[2:0] in;
reg clk, reset;
fsm test(in,out,clk,reset);
initial begin
 clk=0;
 reset=1;
 #10 reset = 0;
 in=0;
 #13 reset = 1;
 #10 in = 3'd0;
 #10 in = 3'd2;
 #10 in = 3'd1;
 #10 in = 3'd3;
 #10 in = 3'd5;
 #10 in = 3'd0;
 #10 in = 3'd4;
 #10 in = 3'd7;
 #10 in = 3'd1;
 #10 in = 3'd7;
 #10 in= 3'd0;
 #10 $finish;
end
always #5 clk = ~clk;endmodule

```

**1b39.**

The circuit of the polynomial is presented in the figure.



**The description of the counter by Verilog**

```

module polynomial_counter (clk, preset, q);
input clk, preset;
output[0:4] q;
reg [0:4] q;
always @(posedge clk or negedge preset)
 if (!preset)
 q= 5'b10000;
 else q={q[4], q[0], q[1],q[2]^q[4], q[3]};
endmodule

```

**Testbench**

```

module stimulus;
reg clk, preset;
wire[0:4] q;
polynomial_counter test(clk,preset,q);
 initial begin
 clk=0;
 preset=1;
 #3 preset=0;
 #20 preset = 1;
 #200 $finish;
 end
 end
always #5 clk=~clk;
endmodule

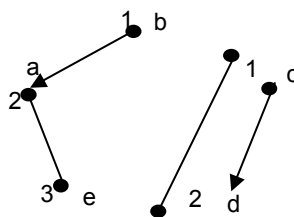
```

**1b40.**

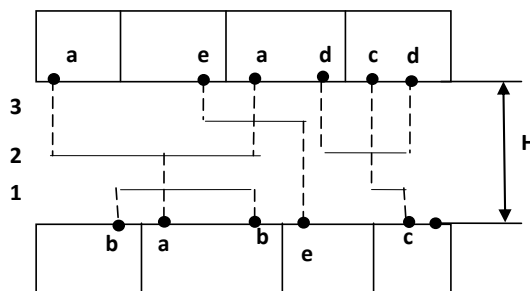
The problem is solved in two phases:

- Definition of minimum number of horizontal channels;
- Calculation of minimum H distance between two rows of cells.

Construct horizontal and vertical limitations graph of a, b, c, d, e nets and heuristically define its chromatic number, as shown in the figure below.



As seen from the figure, chromatic number of the graph equals 3. Hence it follows that for two-layer reciprocally perpendicular routing of the given nets, at least 3 horizontal channels are needed, as shown in the figure.



Thus, the minimum H distance between two rows of cells will be defined by the formula, shown below.  
 $H = 3 \cdot 0,1 + 2 \cdot 0,1 + 2 \cdot 0,2 = 0,9 \text{ um.}$

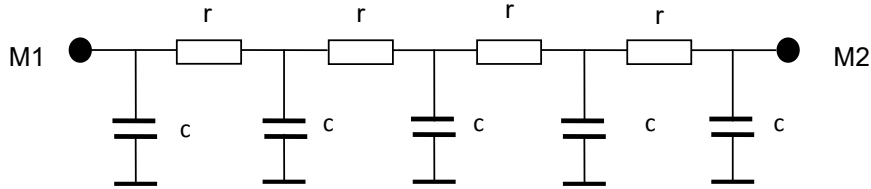
**1b41.**

The solution of the problem is based on the fact that N-MOS transistor is open, if "1" logic level signal is given to its gate, and P-MOS transistor - "0" logic level signal. Therefore the solution of the problem is the following:

- a) T1, T2, T3, T4, T5, T6 → "0";
- b) T1, T3 → "0"; T2, T4, T5, T6 → "1".

**1b42.**

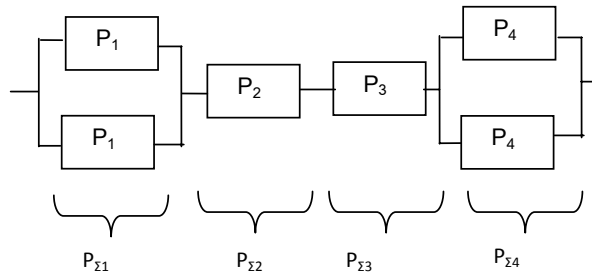
R, C equivalent circuit of M1-M2 transmission line will look as follows:



The delay in the transmission line, connecting two modules, will be:  
 $\tau = rc + 2rc + 3rc + 4rc = 10rc = 10 \times 10 \times 100 = 10000 \text{ OhmF} = 10 \text{ ns}.$

**1b43.**

Present the given circuit in the form of 4 sequentially connected regions:



Designate the probabilities of faultless operation of those regions  $P_{\Sigma 1}, P_{\Sigma 2}, P_{\Sigma 3}, P_{\Sigma 4}.$

Using the rules of addition, multiplication of probabilities and full probabilities, the following can be written:

$$P_{\Sigma 1} = P_1 P_1 + 2P_1(1-P_1) = P_1(2-P_1) = 0,5(2-0,5) = 0,75$$

$$P_{\Sigma 2} = P_2 = 0,6$$

$$P_{\Sigma 3} = P_3 = 0,8$$

$$P_{\Sigma 4} = P_4(2-P_4) = 0,64$$

The probability of faultless operation of the circuit will be

$$P = P_{\Sigma 1} \times P_{\Sigma 2} \times P_{\Sigma 3} \times P_{\Sigma 4} = 0,75 \times 0,6 \times 0,8 \times 0,64 = 0,2304.$$

**1b44.**

The same current flows through both transistors.  $V_x < 4V$ , the transistor below is not saturated  $V_{DS} < V_{GS} - V_T$ , and the one above is saturated  $V_{DS} = V_{GS} - 5 - V_x$ .

$$\frac{1}{2} [V_{GS} - V_t]^2 = [V_{GS} - V_{t0} - \frac{V_{DS2}}{2}] V_{DS2}$$

$$V_t = V_{t0} + \gamma(\sqrt{2|\Phi_F| + V_{SB}} - \sqrt{2|\Phi_F|}) = 1 + 0.39(\sqrt{1.2 + V_x} - \sqrt{1.2})$$

$$\frac{1}{2} [5 - V_x - (1 + 0.39(\sqrt{1.2 + V_x} - \sqrt{1.2}))]^2 = [5 - 1 - \frac{V_x}{2}] V_x$$

Solving these equations, for example graphically, the following will be obtained  $V_x = 1.09V$ .

Taking  $W/L = 1$ ,

$$I_D = K_P [5 - 1 - \frac{V_x}{2}] V_x = 25 \cdot (4 - 0.24) \cdot 0.48 = 45.12 \mu A$$

**1b45.**

a.  $A = (C_L / C_{in1})^{1/N}$

$$A = (20 \cdot 10^3 / 10)^{1/3} = 12.6$$

$$(W/L)_1 = 1, (W/L)_2 = 12.6, (W/L)_2 = 158.7$$

$$t_d = 0.7N(R_{n1} + R_{p1})(C_{out1} + AC_{in1}) = 2.1R_1(C_{out1} + 12.6C_{in1}) \approx 26.5R_1C_{in1}$$

b.  $N = \ln(C_L/C_{in1}) = \ln(20 \cdot 10^3/10) = 7.6$

$N = 7$

$A = (20 \cdot 10^3/10)^{1/7} = 2.96$

$(W/L)_1 = 1, (W/L)_2 = A^2, \dots, (W/L)_7 = A^7$

$t_d = 0.7N(R_{n1} + R_{p1})(C_{out1} + AC_{in1}) = 4.9R_1(C_{out1} + 2.96C_{in1}) \approx 14.5R_1C_{in1}$

c.  $P = V_{DD}^2 F C_{in1} \Sigma(A + A^2 + \dots + A^7)$

**1b46.**

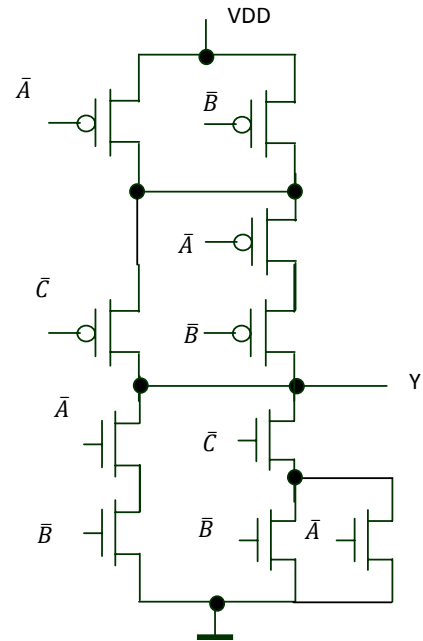
$F = \overline{A\overline{B}} + \overline{A\overline{C}} + \overline{B\overline{C}} = \overline{A\overline{B}} + \overline{C(A + B)}$

$R_{Pnetmin} = 7R_p/6$ , when  $ABC = 111$ ;  $R_{Pnetmax} = 2.5R_p$ , when

$ABC = 110$ ,

$R_{Nnetmin} = 6R_n/7$ , when  $ABC = 000$ ;  $R_{Nnetmax} = 2R_n$ , when  $ABC = 001$

$R_{Pnetmax} = R_{Nnetmax} \Rightarrow R_p = 4R_n/5 \Rightarrow W_p = 8W_n/5$



**1b47.**

a.

| A | B | Out |
|---|---|-----|
| 0 | 0 | 1   |
| 0 | 1 | 0   |
| 1 | 0 | 0   |
| 1 | 1 | 1   |

$Out = A \oplus B$

b.  $Out = V_{OL}$ , when NMOS conducts  $V_{DSn} = 0.3V$ ,  $V_{GSn} = 2.5V$ , NMOS is in linear mode.

PMOS is saturated  $V_{DSp} = 2.5 - 0.3 = 2.2V > V_{GSp} - V_t = 2.5 - 0.5 = 2V$ .

$I_D = 0.5\beta_p(V_{GSp} - V_t)^2 = \beta_n((V_{GSn} - V_t)V_{DSn} - 0.5V_{DSn}^2)$

$0.5W_p k_p (2.5 - 0.5)^2 = W_n k_n ((2.5 - 0.5) \cdot 0.3 - 0.5 \cdot 0.3^2)$ ,  $k_n = 2k_p$

$W_p = 0.6W_n = 0.9\mu m$ .

**1b48.**

a.  $V_x$  changes from 2.5V to 0V. The time of  $V_x$  charge up to 1.25V will be  $t = CR_{Mn} \ln(2.5/2)$ .

b. The final voltage of x node will be  $(2.5 - V_{Tn}) = 2.07V$ , the initial voltage is 0V. The change time of  $V_x$  will be  $t = CR_{Mn} \ln(2.07/(2.07 - 1.25))$

c.  $V_{Bmin} = 1.25 - V_{Tn} = 0.82V$ .

**1b49.**

a.  $V_{SBM3} = V_x$ ,  $V_x = V_{DD} - V_t = V_{DD} - V_{t0} - \gamma((2|\Phi_F| + V_{SB})^{0.5} - (2|\Phi_F|)^{0.5})$

$V_x = 2.5 - 0.5 - 0.5((0.6 + V_x)^{0.5} - (0.6)^{0.5})$

$V_x = 1.61 - 0.5(0.6 + V_x)^{0.5}$

b. When  $V_{in} = V_L$ ,  $V_{DS2} = V_{DD} - (V_x - V_t) = 2V_t = 1V$ ,  $V_{GSM2} = V_x = V_{DD} - V_t = 2.0V$ , M2 is in linear mode.

When  $V_{in} = V_H$ ,  $V_{DS2} = V_{DD} - V_{DS1}$ , M2 is saturated if  $V_{DS1} < 2V_t$ .

c.  $V_{in} = 0$ ,  $V_{out} = V_{GSM2} - V_t = V_{DD} - 2V_t = 1.5V$ .

d. When  $V_{in}=V_{SP}$ ,  $V_{out}=V_{in}$ , considering that M1 and M2 are saturated.

$$0.5\beta_2(V_{DD}-2V_t)^2=0.5\beta_1(V_{SP}-V_t)^2,$$

$$V_{SP} = \frac{\sqrt{\frac{\beta_1}{\beta_2}} V_T + (V_{DD} - 2V_T)}{\sqrt{\frac{\beta_1}{\beta_2}}}$$

If  $W_1 \gg W_2$ ,  $\beta_1 \gg \beta_2$ ,  $V_{SP}=V_T=0.5V$ .

If  $W_1 \ll W_2$ ,  $\beta_1 \ll \beta_2$ ,  $V_{SP}=V_{DD}-2V_T=1.5V$ , M2 is not saturated  $V_{DSM2}=V_{DD}-V_{SP}=1.0V$ ,  $V_{GSM2}=V_{DD}-V_t=2.0V$ . But for M2 to conduct,  $V_{out}$  must be smaller or equal to 1.5V. Considering  $W_1 \ll W_2$ , the output voltage will have the maximum possible value 1.5V.

**1b50.**

a. When  $V_{in}=V_H$ , NMOS is in active mode, PMOS - saturated.

$$I_D=0.5\beta_p(V_{GSP}-|V_{tp}|)^2 = \beta_n((V_{GSn}-V_{tn})V_{DSn}-0.5V_{DSn}^2)$$

$$0.5\beta_p(2.5-0.4)^2 = \beta_n((2.5-0.5)V_{OL}-0.5V_{OL}^2);$$

$$\beta_n/\beta_p=(W_n/W_p)(k_n/k_p)=(4/0.5)(120/30)=64$$

$$V_{OL}=0.01V,$$

$$V_{OH}=V_{DD}=2.5V.$$

b.  $V_{SP}=V_{in}=V_{out}$ .

First consider the transistors are saturated.

$$0.5\beta_p(V_{DD}-|V_{tp}|)^2=0.5\beta_n(V_{SP}-V_{tn})^2,$$

$$V_{SP} = \frac{\sqrt{\frac{\beta_n}{\beta_p}} V_{Tn} + (V_{DD} - |V_{Tp}|)}{\sqrt{\frac{\beta_n}{\beta_p}}} = 0.7625V$$

But when  $V_{out}=0.7625V$ ,  $V_{DS}=2.5-0.7625 < V_{GS}-|V_{TP}|=2.1V$ . Therefore PMOS is not saturated. Then  $V_{SP}$  must be calculated from the following equation:

$$\beta_p((V_{DD}-|V_{tp}|)V_{SP}-V_{SP}^2/2) = 0.5\beta_n(V_{SP}-V_{tn})^2,$$

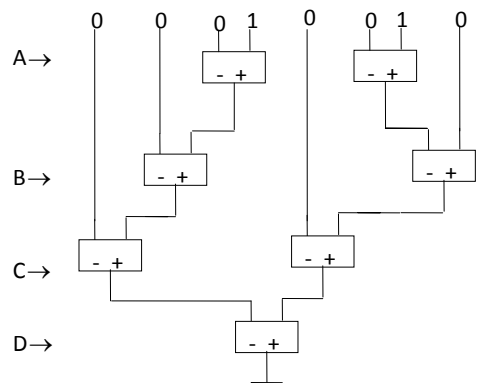
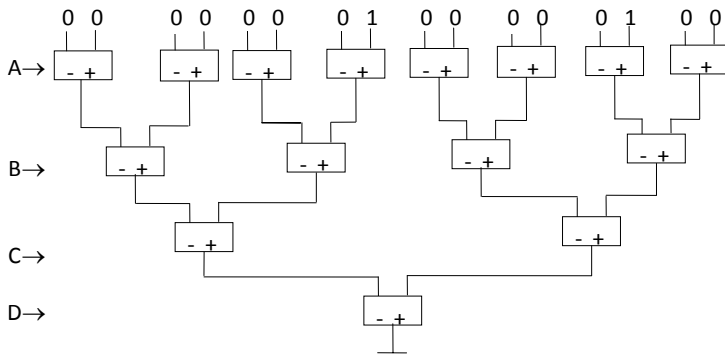
$$(2.5-0.4)V_{SP}-V_{SP}^2/2 = 32(V_{SP}-0.5)^2,$$

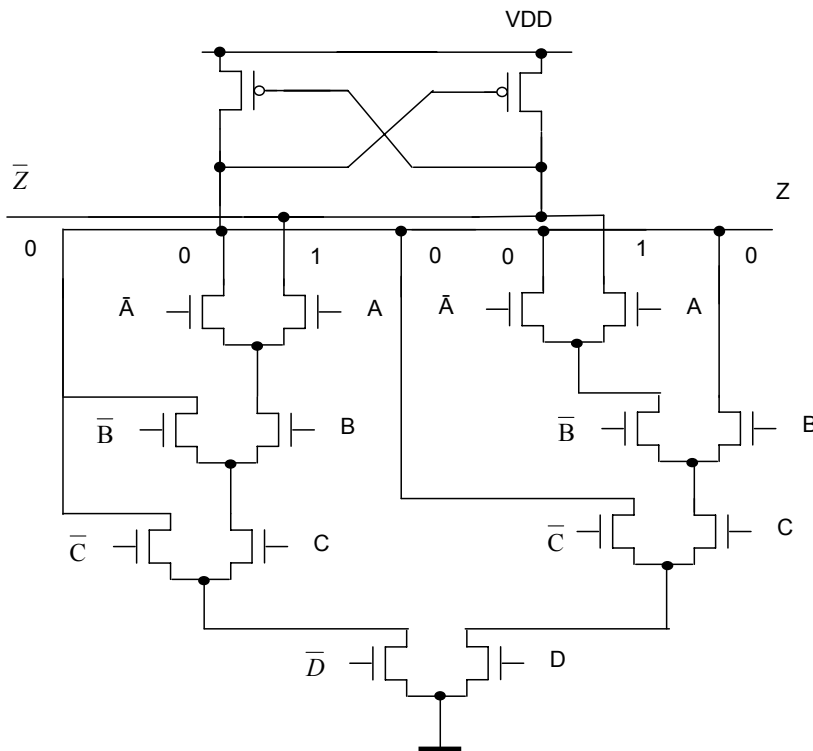
$$V_{SP}=0.695V.$$

**1b51.**

The truth table of  $F = ABC\bar{D} + \bar{A}\bar{B}CD$  function, the initial and simplified graphs of NMOS and the circuit are shown below.

|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| F | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| A | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 |
| B | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 |
| C | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| D | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |





**1b52.**

- |            |         |         |
|------------|---------|---------|
| 1. S1 = 0; | S2 = 1; | S3 = 0; |
| 2. S1 = 1; | S2 = 1; | S3 = 0; |
| 3. S1 = x; | S2 = x; | S3 = 1; |
| 4. S1 = x; | S2 = 0; | S3 = 0; |

**Verilog description of the circuit.**

```

module task_1(data, clk, S1,S2, S3, reset, out);
 input data, clk, S1,S2, S3, reset;
 output out, q1, q2, q3;

 always @(posedge clk)
 if (reset==0) q1=0; //synchronous reset
 else if (S1==1) q1=q1^data;//T
 flip-flop
 else q1=data;//D-flip-flop

 always @ (data or q1)
 if (S2) q3=q1;
 else q3=data;

 always @ (clk or data)
 if (clk) q2=data;
 else q2=q2;

 always @(q2 or q3)
 if (S3) out=q2;
 else out = q3;

endmodule

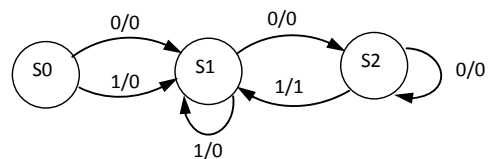
```

**1b53.**

The characteristic equation of D flip-flop is  $q^* = d$ ;  
 $d1 = \bar{x} \cdot q2$ ;  $d2=1$ ;  
 Hence  $q1^* = \bar{x} \cdot q2$ ;  $q2^* = 1$ ;  $y=x \cdot q1$ ;

| State | X=0    | X=1    |
|-------|--------|--------|
| 0 0   | 0 1, 0 | 0 1, 0 |
| 0 1   | 1 1, 0 | 0 1, 0 |
| 1 1   | 1 1, 0 | 0 1, 1 |

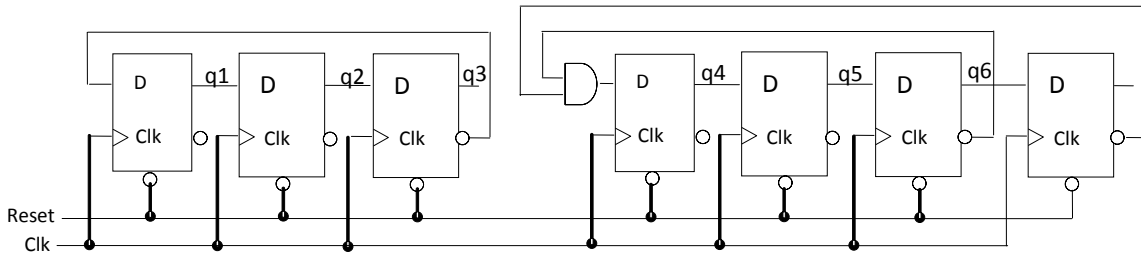
FSM has three states 00, 01, 11. The naming of states is 00 = S0, 01 = S1, S2=11.  
 The state diagram of FSM is shown.  
 The derived FSM is a detector of input sequencies 001 and 101.





**1b54.**

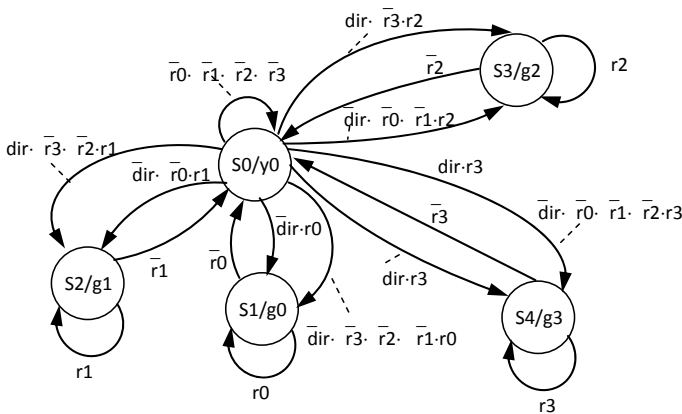
To obtain the module of count equal to 42, use two Johnson's counters with modules 6 and 7. Counter are moving in count independent. The circuit is represented below.



The state 1111 ( $d4 = \bar{q}_6 \cdot \bar{q}_7$ ) is skipped. The S5 state decoding function is  $f = \bar{q}_2 \cdot q_3 + \bar{q}_5 \cdot q_6$

**1b55.**

Construction arbiter state diagram. FSM input alphabet  $X = \{r_0, r_1, r_2, r_3, dir\}$ , where  $r_0, r_1, r_2, r_3$  are service request from devices. The input  $dir$  sets device priorities. If  $dir=0$ , the request  $r_0$  has the highest priority, then  $r_1, r_2, r_3$  are followed. If  $dir=1$ , priorities have inverse order. The state diagram of FSM is shown below.



- S0 - initial state of FSM, requests are absent
- S1 - execution of request r0
- S2 - execution of request r1
- S3 - execution of request r2
- S4 - execution of request r3

```

Verilog description of the arbiter
module arbiter(r,g,clk,reset, dir);
input [0:3] r;
input reset,clk,dir;
output [0:3] g;
parameter s0=3'b000,s1=3'b001,
s2=3'b010,s3=3'b011,s4=3'b100;
reg [2:0] next_state,state;
always @(posedge clk)
if(!reset) state=s0;
else state=next_state;
always @(r or state)
begin
case (state)
s0:casex({r,dir})
5'b0000x: next_state=s0;
5'b1xxx0,5'b10001: next_state=s1;
5'b01xx0,5'bx1001: next_state=s2;
5'b001x0,5'bxx101: next_state=s3;
4'b00010,5'bxxx11: next_state=s4;
default: next_state=s0;
endcase
s1: if (r[0]) next_state=s1;
else next_state=s0;
s2: if (r[1]) next_state=s2;
else next_state=s0;
s3: if (r[2]) next_state=s3;
else next_state=s0;
s4: if(r[3]) next_state=s4;
else next_state=s0;
default:next_state=s0;
endcase
end
assign g[0]=(state==s1);
assign g[1]=(state==s2);
assign g[2]=(state==s3);
assign g[3]=(state==s4);
endmodule

```

**1b56.**

a.  $data\_out[0] = (data\_in == 8'b0000\_0010) \vee (data\_in == 8'b0000\_1000) \vee (data\_in == 8'b0001\_0000) \vee (data\_in == 8'b1000\_0010)$

Therefore the answer will look like this.

$assign data\_out[0] = data\_in[1] | data\_in[3] | data\_in[5] | data\_in[7];$

b.  $assign data\_out[1] = data\_in[2] | data\_in[3] | data\_in[6] | data\_in[7];$

c.  $assign data\_out[2] = data\_in[4] | data\_in[5] | data\_in[6] | data\_in[7];$

## 2. ANALOG INTEGRATED CIRCUITS

### a) Test questions

|       |   |       |   |       |   |
|-------|---|-------|---|-------|---|
| 2a1.  | D | 2a26. | D | 2a50. | E |
| 2a2.  | D | 2a27. | E | 2a51. | D |
| 2a3.  | E | 2a28. | E | 2a52. | E |
| 2a4.  | E | 2a29. | D | 2a53. | C |
| 2a5.  | B | 2a30. | B | 2a54. | A |
| 2a6.  | E | 2a31. | A | 2a55. | D |
| 2a7.  | B | 2a32. | B | 2a56. | A |
| 2a8.  | E | 2a33. | E | 2a57. | D |
| 2a9.  | B | 2a34. | C | 2a58. | C |
| 2a10. | E | 2a35. | B | 2a59. | B |
| 2a11. | E | 2a36. | C | 2a60. | B |
| 2a12. | D | 2a37. | C | 2a61. | A |
| 2a13. | C | 2a38. | E | 2a62. | D |
| 2a14. | C | 2a39. | B | 2a63. | A |
| 2a15. | C | 2a40. | C | 2a64. | D |
| 2a16. | A | 2a41. | B | 2a65. | A |
| 2a17. | C | 2a42. | E | 2a66. | D |
| 2a18. | C | 2a43. | B | 2a67. | C |
| 2a19. | E | 2a44. | D | 2a68. | D |
| 2a20. | A | 2a45. | C | 2a69. | A |
| 2a21. | C | 2a46. | D | 2a70. | B |
| 2a22. | D | 2a47. | E | 2a71. | A |
| 2a23. | D | 2a48. | A | 2a72. | B |
| 2a24. | D | 2a49. | E | 2a73. | C |
| 2a25. | E |       |   |       |   |

## b) Problems

2b1.

$$I_{D2} = I_{ref} \left( \frac{W_2}{L_2} \right) / \left( \frac{W_1}{L_1} \right) = I_{ref}$$

$$|I_{D3}| = |I_{D2}|, \quad I_{D3} = I_{ref}$$

$$I_{D4} = I_{D3} \left( \frac{W_4}{L_4} / \frac{W_3}{L_3} \right)$$

$$I_{D4} = I_{ref}$$

2b2.

$$(V_{in} - V_{TH1})^2 \cdot W_1 \cdot \mu_n \cdot \frac{C_{ox}}{L_1} \cdot 2 = (V_{DD} - V_{TH1} - V_{out})^2 \cdot W_2 \cdot \mu_n \cdot \frac{C_{ox}}{L_2} \cdot 2$$

$$\sqrt{\frac{W_1}{L_1}} \cdot (V_{in} - V_{TH1}) = \sqrt{\frac{W_2}{L_2}} \cdot (V_{DD} - V_{TH1} - V_{out})$$

$$\sqrt{\frac{W_1}{L_1}} = \sqrt{\frac{W_2}{L_2}} \cdot \frac{dV_{out}}{dV_{in}}$$

$$K = \frac{dV_{out}}{dV_{in}} = -\sqrt{\frac{W_1 - L_2}{W_2 - L_1}}$$

2b3.

$$(V_{in} - V_{TH})^2 \cdot W_1 \cdot \mu_1 \cdot \frac{C_{ox}}{L_1} \cdot 2 = (V_{DD} - V_{TH} - V_{out})^2 \cdot W_2 \cdot \mu_n \cdot \frac{C_{ox}}{L_2} \cdot 2$$

Saturation condition  $V_{out} = V_{in} - V_{TH}$

$$(V_{in} - V_{TH})^2 \cdot W_1 \cdot \mu_n \cdot \frac{C_{ox}}{L_1} \cdot 2 = (V_{DD} - V_{TH} - (V_{in} - V_{TH}))^2 \cdot W_2 \cdot \mu_n \cdot \frac{C_{ox}}{L_2} \cdot 2$$

$$7 \cdot (V_{in} - V_{TH}) = 3 \cdot (V_{DD} - V_{TH})$$

$$10 \cdot V_{in} = 13.9$$

$$V_{in} = 1.39 \text{ V}$$

2b4.

$$I = \frac{V_{g2} + 2.5V}{R} = \frac{\beta}{2} \cdot (V_{g2} - V_{g1} - V_{TH2})^2 = \frac{\beta}{2} \cdot (V_{g1} - 2.5 - V_{TH1})^2$$

$$V_{g2} - V_{g1} - V_{TH2} = V_{g1} - 2.5 - V_{TH1}$$

$$V_{g1} = \frac{V_{g2} - 2.5}{2}$$

$$I = \frac{2 \cdot V_{g1}}{R} = \frac{\beta}{2} \cdot (V_{g1} - 2.5 - V_{TH1})^2$$

2b5.

$$I_{D2} = I_{ref} \left( \frac{W_2/L_2}{W_1/L_1} \right)$$

$$|I_{D3}| = |I_{D2}|$$

$$I_{D4} = I_{D3} \left( \frac{W_4/L_4}{W_3/L_3} \right)$$

$$I_{D4} = I_{ref} \left( \frac{W_2/L_2}{W_1/L_1} \right) \left( \frac{W_4/L_4}{W_3/L_3} \right)$$

$$I_{D4} = I_{ref} \left( \frac{W_2 L_1}{W_1 L_2} \right) \left( \frac{W_4 L_3}{W_3 L_4} \right)$$

$$I_{D4} = I_{ref} \frac{W_2 W_4 L_1 L_3}{W_1 W_3 L_2 L_4}$$

2b6.

$$R = \frac{V_{DD} - V_{ref}}{I}$$

$$I = \frac{\beta}{2} (V_{GS} - V_{THN})^2 = \frac{\beta}{2} (V_{ref} - V_{THN})^2$$

$$\beta = K_n \frac{W}{L} = \mu_n \frac{\epsilon_{Si} \cdot \epsilon_0}{t_{ox}} \cdot \frac{W}{L} = 120 \text{ uA/V}^2$$

$$I = 0.6 \cdot 10^{-4} (V_{ref} - 0.8)^2$$

$$V_{ref} = V_{DD} - IR$$

$$V_{ref} = 2 - 10^4 \cdot 10^{-4} \cdot 0.6 (V_{ref} - 0.8)^2$$

$$V_{ref} = 2 - 0.6 (V_{ref}^2 - 1.6V_{ref} + 0.64)$$

$$V_{ref} = 2 - 0.6V_{ref}^2 + 0.96V_{ref} - 0.384$$

$$-0.6V_{ref}^2 - 0.04V_{ref} + 1.616 = 0$$

$$0.6V_{ref}^2 + 0.04V_{ref} - 1.616 = 0$$

$$V_{ref1,2} = \frac{-0.04 \pm \sqrt{0.0016 + 3.8784}}{1.2}$$

$$V_{ref1,2} = \frac{-0.04 \pm 1.97}{1.2}$$

$$V_{ref} = \frac{-0.04 + 1.97}{1.2} = 1.675 \text{ V}$$

2b7.

a) When  $V_{DD}$  increases by 10%.

$$I_{out} = \frac{\beta}{2} (V_A - V_{TH})^2$$

$$V_A = \frac{R_2}{R_1 + R_2} V_{DD}$$

$$\Delta I_{out} = \frac{\beta}{2} \left[ \left( \frac{R_2}{R_1 + R_2} V_{DD} - V_{TH} \right)^2 - \left( \frac{R_2}{R_1 + R_2} 1.1V_{DD} - V_{TH} \right)^2 \right]$$

$$\beta = K_n \frac{W}{L} = 120 \cdot 10^{-6} \cdot \frac{50}{0.5} = 12 \mu\text{A/V}^2$$

$$\Delta I_{out} = 6 \cdot 10^{-3} [(0.78 - 0.5)^2 - (0.858 - 0.5)^2] = 6 \cdot 10^{-3} (0.0784 - 0.128) \approx -0.3$$

$$\Delta I_{out} = -0.3 \text{ mA (increases by 0.3 mA)}$$

b) When  $V_{DD}$  reduces by 10%.

$$\Delta I_{out} = \frac{\beta}{2} \left[ \left( \frac{R_2}{R_1 + R_2} V_{DD} - V_{TH} \right)^2 - \left( \frac{R_2}{R_1 + R_2} 0.9V_{DD} - V_{TH} \right)^2 \right]$$

$$\Delta I_{out} = 6 \cdot 10^{-3} (0.0784 - 0.0408) \approx 0.23$$

$$\Delta I_{out} = 0.23 \text{ mA (reduces by 0.23 mA)}$$

2b8.

$$K = -\frac{R_2 \parallel X_C}{R_1}$$

$$K = -\frac{R_2 \cdot \frac{1}{j\omega C}}{R_1 \left( R_2 + \frac{1}{j\omega C} \right)}$$

$$K = -\frac{R_2}{R_1(j\omega R_2 C + 1)} = -\frac{R_2}{R_1} \cdot \frac{1 - j\omega R_2 C}{1 + \omega^2 R_2^2 C^2}$$

$$|K| = \frac{R_2}{R_1} \cdot \frac{1}{\sqrt{1 + \omega^2 R_2^2 C^2}}$$

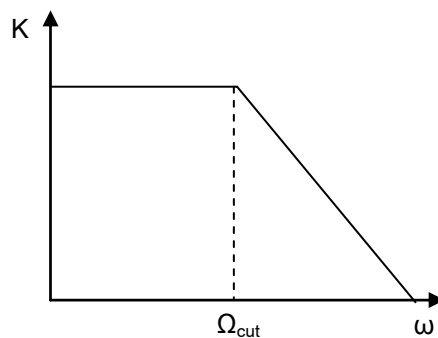
$$\omega=0 \Rightarrow K = -\frac{R_2}{R_1}, \frac{\frac{R_2}{R_1}}{\frac{R_2}{R_1}} = \sqrt{2}$$

$$\frac{1}{\sqrt{1 + \omega^2 R_2^2 C^2}} = \frac{1}{\sqrt{2}}$$

$$1 + \omega^2 R_2^2 C^2 = 2$$

$$\omega^2 R_2^2 C^2 = 1 \quad \omega_{\text{cut}} = \frac{1}{R_2 C}$$

$$\omega \gg \frac{1}{R_2 C} \Rightarrow K \rightarrow 0$$



2b9.

$$I_D = I_{OUT} = (V_{GS} - V_{TH}) \frac{g_m}{2}$$

$$V_{GS} = V_G = \frac{R_2}{R_1 + R_2} \cdot VDD$$

Putting together the following will be obtained:

$$I_{OUT} = \left( \frac{R_2}{R_1 + R_2} \cdot VDD - V_{TH} \right) \cdot \frac{g_m}{2}$$

2b10.

$$V_{OUT} = VDD - R_D \left( I_D + \frac{V_{OUT} - V_{in}}{R_F} \right)$$

$$V_{OUT} = \frac{VDD - R_D I_D - \frac{R_D}{R_F} V_{in}}{1 + \frac{R_D}{R_F}}$$

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})^2$$

$$V_{GS} = V_b - V_{in}$$

$$V_{OUT} = \frac{VDD - R_D \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_b - V_{in} - V_{TH})^2 - \frac{R_D}{R_F} V_{in}}{1 + \frac{R_D}{R_F}}$$

2b11.

$$V_- = -\frac{V_{out}}{K_A}$$

$$I_4 = I_1 + I_2 + I_3$$

$$\frac{V_- - V_{out}}{R_4} = \frac{V_{in1} - V_-}{R_1} + \frac{V_{in2} - V_-}{R_2} + \frac{V_{in3} - V_-}{R_3}$$

$$\frac{-\frac{V_{out}}{K_A} + V_{out}}{R_4} = \frac{V_{in1} + \frac{V_{out}}{K_A}}{R_1} + \frac{V_{in2} + \frac{V_{out}}{K_A}}{R_2} + \frac{V_{in3} + \frac{V_{out}}{K_A}}{R_3}$$

$$-V_{out} \left(1 + \frac{1}{K_A}\right) \cdot R_1 R_2 R_3 = V_{in1} \cdot R_4 R_2 R_3 + V_{out} \frac{1}{K_A} \cdot R_4 R_2 R_3 + V_{in2} R_4 R_1 R_3 +$$

$$+ V_{out} \cdot \frac{1}{K_A} \cdot R_4 R_1 R_3 + V_{in3} \cdot R_4 R_1 R_2 + V_{out} \frac{1}{K_A} R_4 R_1 R_2$$

$$V_{out} \left(\frac{1}{K_A} R_4 (R_2 R_3 + R_1 R_3 + R_1 R_2) + \left(1 + \frac{1}{K_A}\right) \cdot R_1 R_2 R_3\right) = -R_4 (V_{in1} R_2 R_3 + V_{in2} R_1 R_3 + V_{in3} R_1 R_2)$$

$$V_{out} = -\frac{R_4 (V_{in1} R_2 R_3 + V_{in2} R_1 R_3 + V_{in3} R_1 R_2)}{\frac{1}{K_A} R_4 (R_2 R_3 + R_1 R_3 + R_1 R_2) + \left(1 + \frac{1}{K_A}\right) \cdot R_1 R_2 R_3}$$

2b12.

$$I_{m1} = I_{ref}$$

$$I_{m2} = 5 \cdot I_{m1} = 5 \cdot I_{ref}$$

$$I_{m3} = I_{m2} = 5 I_{ref}$$

$$I_{m4} = 5 I_{m3} = 25 I_{ref}$$

$$V_{out} = R_1 \cdot I_{m4} = 25 I_{ref} R_1$$

2b13.

$$|I_{D1}| = |I_{ref}|, \quad I_{D2} = I_{D1} \frac{\left(\frac{W_2}{L_2}\right)}{\left(\frac{W_1}{L_1}\right)} = I_{ref}$$

$$|I_{D3}| = |I_{D2}|, I_{D3} = I_{ref}$$

$$I_{D4} = I_{D3} \frac{\left(\frac{W_4}{L_4}\right)}{\left(\frac{W_3}{L_3}\right)} = 0.5 I_{ref}$$

2b14.

$$I_{out} = \frac{1}{2} \mu_n c_{ox} \frac{W}{L} (V_A - V_{TH})^2, \quad V_A = \frac{V_{DD}}{R_1 + R_2} R_2$$

$$\Delta I_{out} = \frac{1}{2} \mu_n c_{ox} \frac{W}{L} \left[ \left( \frac{V_{DD}}{R_1 + R_2} R_2 - V_{TH} \right)^2 - \left( \frac{1.1 V_{DD}}{R_1 + R_2} R_2 - V_{TH} \right)^2 \right]$$

$$\Delta I_{out} = -\frac{1}{2} \mu_n c_{ox} \frac{W}{L} \left[ 0.21 \left( \frac{V_{DD}}{R_1 + R_2} R_2 \right)^2 - 0.2 \frac{V_{DD}}{R_1 + R_2} R_2 V_{TH} \right]$$

2b15.

$$R = \frac{V_{DD} - V_{ref}}{I}$$

$$I = I_1 + I_2 = \frac{V_{ref}}{R_1 + R_2} + \frac{\beta}{2} \left( \frac{V_{ref}}{R_1 + R_2} R_2 - V_{TH} \right)^2$$

$$R = \frac{V_{DD} - V_{ref}}{\frac{V_{ref}}{R_1 + R_2} + \frac{\beta}{2} \left( \frac{V_{ref}}{R_1 + R_2} R_2 - V_{TH} \right)^2}$$

2b16.

Relative to variable component

$$I_D = g_m V_{gs}$$

$$V_{out} = I_{D3} * R_1$$

$$\frac{I_{D3}}{g_{m3}} = \frac{I_{D2}}{g_{m2}}, \quad I_{D2} = I_{D1} = g_{m1} V_{in}$$

$$V_{out} = \frac{g_{m1} g_{m3} R_1}{g_{m2}} V_{in}$$

$$k = \frac{dV_{out}}{dV_{in}} = \frac{g_{m1} g_{m3} R_1}{g_{m2}}$$

2b17.

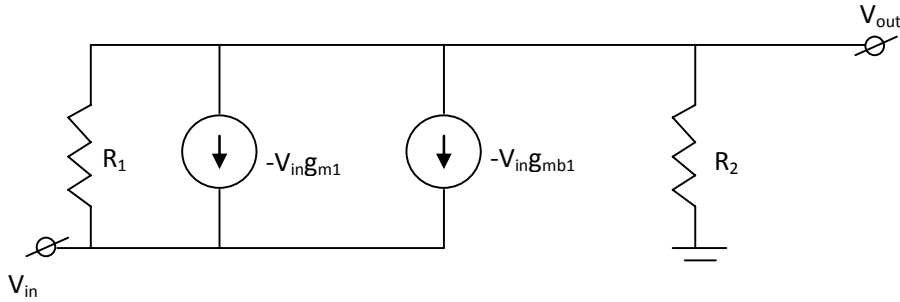
$$\left. \begin{aligned} V_{out} &= I_2 R_1 \\ \frac{I_2}{I_1} &= \frac{g_{m3}}{g_{m2}} \\ I_1 &= \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right)_1 (V_{in} - V_{THn})^2 \end{aligned} \right\} \Rightarrow V_{out} = R_1 \frac{g_{m3}}{g_{m2}} \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right)_1 (V_{in} - V_{THn})^2$$

$$A_V = \frac{\partial V_{out}}{\partial V_{in}} = R_1 \frac{g_{m3}}{g_{m2}} \mu_n C_{ox} \left( \frac{W}{L} \right)_1 (V_{in} - V_{TH}) = \frac{R_1 g_{m3} g_{m1}}{g_{m2}}$$

$$\text{Answer: } A_V = \frac{R_1 g_{m3} g_{m1}}{g_{m2}}$$

**2b18.**

The small signal model of the circuit is the following.



The following equation can be written for that:

$$-\frac{V_{out}}{R_2} + \frac{V_{in} - V_{out}}{R_1} + g_{m1} V_{in} + g_{mb1} V_{in} = 0$$

$$-V_{out} \frac{1}{R_2 \parallel R_1} = -V_{in} \left( \frac{1}{R_1} + g_{m1} + g_{mb1} \right)$$

$$A_V = \frac{V_{out}}{V_{in}} = R_2 \parallel R_1 \left( \frac{1}{R_1} + g_{m1} + g_{mb1} \right)$$

Answer:  $R_2 \parallel R_1 \left( \frac{1}{R_1} + g_{m1} + g_{mb1} \right)$

**2b19.**

The voltage of V1 can be obtained from

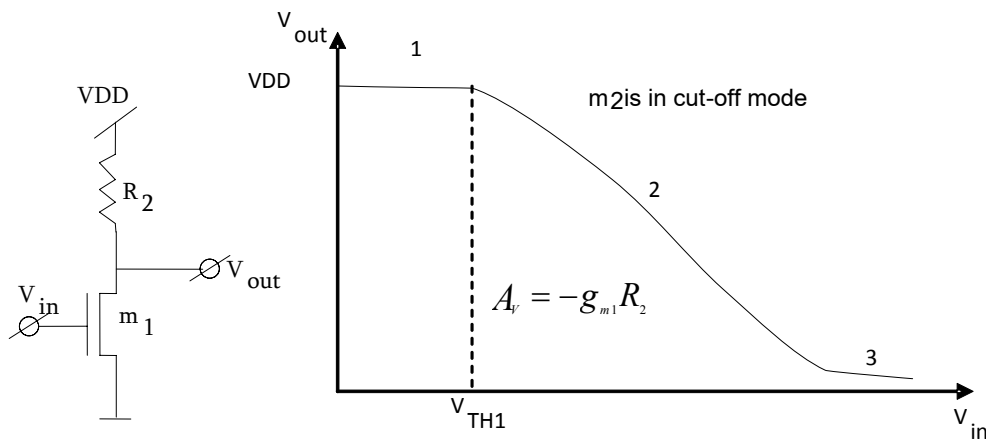
$$V_1 = V_{DD} - \frac{V_{DD} - V_{out}}{1 + \frac{R_2}{R_1}}$$

Depending on the ratio of R<sub>2</sub> and R<sub>1</sub> values, 5 cases can be obtained which are:

1)

$$R_1 \ll R_2 \Rightarrow \left( \frac{R_2}{R_1} + 1 \right) \rightarrow \infty \Rightarrow V_1 \approx V_{DD} - \frac{V_{DD} - V_{out}}{\infty} \approx V_{DD}$$

In this case m<sub>2</sub> will be in cut-off mode irrespective of V<sub>in</sub> value, and the circuit will have the following view:

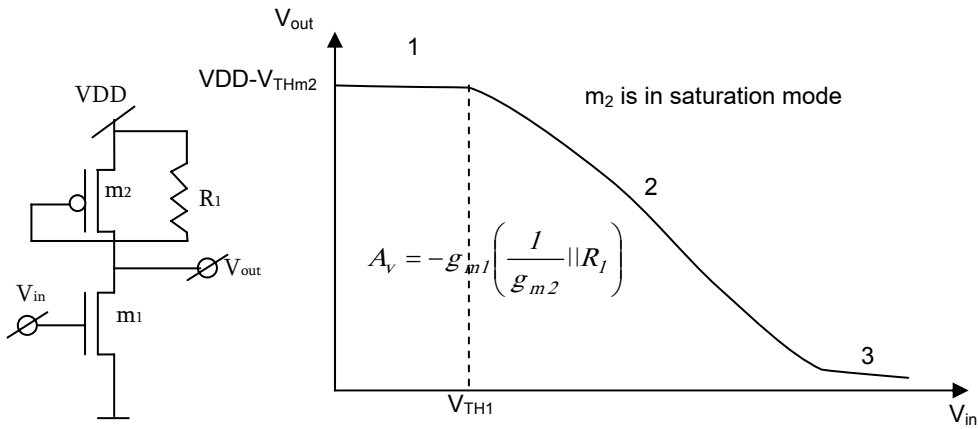


1. m<sub>1</sub> is in cut-off mode
2. m<sub>1</sub> is in saturation mode
3. m<sub>1</sub> is in triode mode

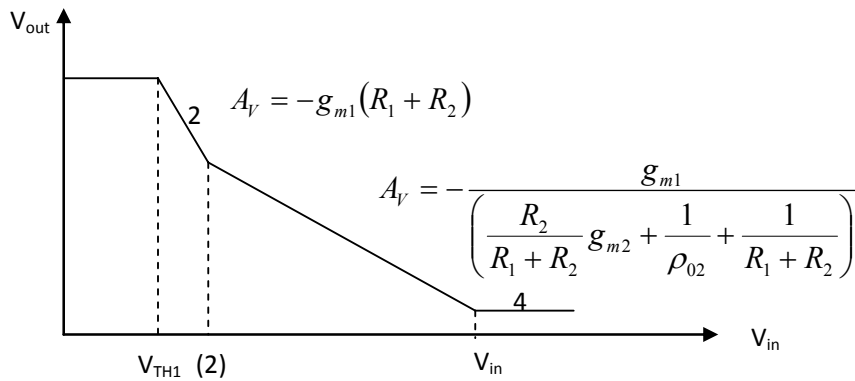


$$2) R_1 \gg R_2 \Rightarrow \frac{R_2}{R_1} \rightarrow 0 \Rightarrow V_1 = V_{DD} - V_{DD} + V_{out} = V_{out}$$

In this case  $m_2$  can be observed as a diode connection and it will be in saturation mode irrespective of  $V_{in}$  value, the circuit will have the following view:



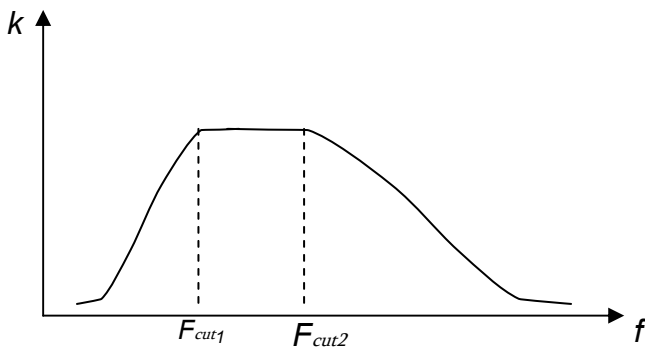
The values of  $R_1$  and  $R_2$  are proportional, in this case, depending on  $V_{out} - V_{in}$ , the following curve will be obtained:



1.  $m_1$  and  $m_2$  are in cut-off mode
2.  $m_1$  is in saturation mode,  $m_2$  - in cut-off mode
3.  $m_1$  and  $m_2$  are in saturation mode, (2) point of  $V_{in}$  axis is the value when  $|V_1 - V_{DD}| > |V_{TH2}|$
4.  $m_1$  is in triode mode,  $m_2$  is in saturation

**2b20.**

$$1) K_{cut} = \left(1 + \frac{R_3}{R_1}\right) = 11$$



$$f_{cut1} = \frac{1}{2\pi C_1 R_2} = \frac{1}{6,28 \cdot 10^{-5} \cdot 2 \cdot 10^3 \text{ ohm}} = \frac{1000}{12,56} \approx 8 \text{ Hz}$$

$$f_{cut2} = \frac{1}{2\pi C_2 R_3} = \frac{1}{6,28 \cdot 10^{-6} \cdot 10 \cdot 10^3 \text{ ohm}} = \frac{1000}{62,8} \approx 16 \text{ Hz}$$

2)  $C_1, R_2$  is a high pass filter.

$C_2, R_3$  is a low pass filter.

$$\Delta f = f_{kt2} - f_{kt1} = 16 \text{ Hz} - 8 \text{ Hz} = 8 \text{ Hz}$$

$$f_{kentr} = \frac{f_{kt1} + f_{kt2}}{2} = 12 \text{ Hz}$$

### 2b21.

After the sampling with the period of  $\Delta t = 1/f_s = 0.001$  seconds, a discrete signal can be obtained

$$u(n\Delta t) = \cos(2\pi f_0 n\Delta t) = \cos \frac{\pi f_0 n}{500} = x(n) = \cos \frac{\pi n}{4}, n \in \mathbb{Z}.$$

Equation  $\cos \frac{\pi f_0 n}{500} = \cos \frac{\pi n}{4}$  in view of the evenness and periodicity of the cosine function is performed for

all  $n$ , if  $\frac{\pi f_0}{500} n = \left( \pm \frac{\pi}{4} + 2\pi j \right) n$ , or  $f_0 = \pm 125 + 1000j$  (Hz), where number  $j$  is any integer.

Hence the two minimum values of the frequency:

$f_0 = 125$  Hz (no superposition of frequency as  $f_0 = 125 < f_s / 2 = 500$  Hz, conditions of Nyquist theorem are performed),  $f_0 = -125 + 1000 = 875$  Hz (conditions of Nyquist theorem are not performed).

Answer: 125 Hz, 875 Hz.

### 2b22.

See the solution of 2b21.

Answer 700 Hz, 900 Hz.

### 3. RF CIRCUITS

#### a) Test questions

- 3a1. A
- 3a2. A
- 3a3. B

#### 4. SEMICONDUCTOR PHYSICS AND ELECTRONIC DEVICES

##### a) Test questions

|       |   |       |   |       |   |
|-------|---|-------|---|-------|---|
| 4a1.  | D | 4a31. | D | 4a61. | D |
| 4a2.  | A | 4a32. | E | 4a62. | C |
| 4a3.  | B | 4a33. | C | 4a63. | C |
| 4a4.  | C | 4a34. | C | 4a64. | C |
| 4a5.  | B | 4a35. | B | 4a65. | C |
| 4a6.  | B | 4a36. | C | 4a66. | C |
| 4a7.  | C | 4a37. | C | 4a67. | C |
| 4a8.  | C | 4a38. | C | 4a68. | E |
| 4a9.  | A | 4a39. | B | 4a69. | C |
| 4a10. | B | 4a40. | D | 4a70. | C |
| 4a11. | A | 4a41. | D | 4a71. | D |
| 4a12. | C | 4a42. | D | 4a72. | B |
| 4a13. | B | 4a43. | B | 4a73. | E |
| 4a14. | D | 4a44. | B | 4a74. | E |
| 4a15. | E | 4a45. | B | 4a75. | D |
| 4a16. | A | 4a46. | B | 4a76. | E |
| 4a17. | C | 4a47. | C | 4a77. | C |
| 4a18. | C | 4a48. | A | 4a78. | A |
| 4a19. | C | 4a49. | D | 4a79. | E |
| 4a20. | B | 4a50. | B | 4a80. | D |
| 4a21. | E | 4a51. | A | 4a81. | C |
| 4a22. | C | 4a52. | B | 4a82. | C |
| 4a23. | C | 4a53. | C | 4a83. | B |
| 4a24. | C | 4a54. | C | 4a84. | A |
| 4a25. | E | 4a55. | B |       |   |
| 4a26. | C | 4a56. | B |       |   |
| 4a27. | B | 4a57. | B |       |   |
| 4a28. | C | 4a58. | C |       |   |
| 4a29. | C | 4a59. | B |       |   |
| 4a30. | A | 4a60. | D |       |   |

## b) Problems

### 4b1.

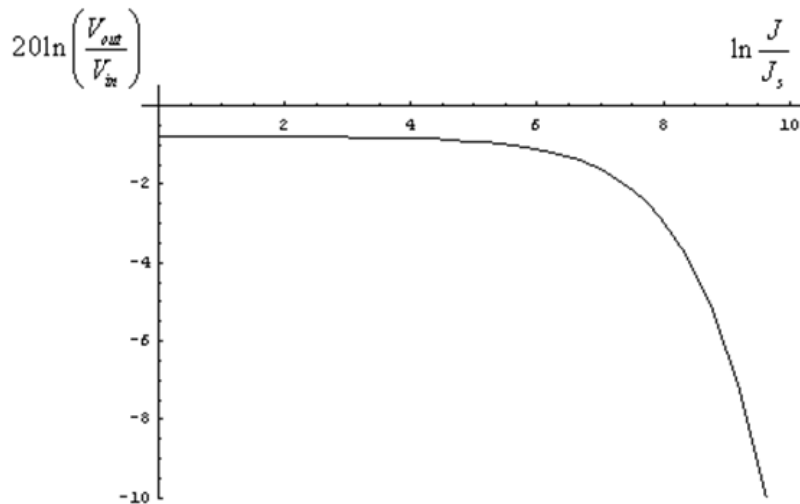
Diode's current-voltage characteristic:  $J = J_s(e^{\frac{eV}{kT}} - 1)$  can be introduced by the following way:

$\frac{dJ}{dV} = J_s \frac{e}{kT} \exp\left(\frac{eV}{kT}\right) = \frac{e}{kT}(J_s + J)$ , from which for diode's differential resistance this will be obtained:

$$r = \frac{dV}{dJ} = \frac{kT}{e(J_s + J)}.$$

In the shown circuit, for signal decay there is:  $\frac{V_{out}}{V_{in}} = \frac{r}{r + R} = \frac{\frac{kT}{e(J + J_s)}}{\frac{kT}{e(J + J_s)} + R}$

The final view of decay expressed by decibels:  $20 \ln \frac{V_{out}}{V_{in}} = 20 \ln \frac{\frac{kT}{e(J + J_s)}}{\frac{kT}{e(J + J_s)} + R}$ :



The obtained dependence for the given parameters is shown in the figure.

### 4b2.

It is enough to be limited by the approximation of full depletion layer, according to which formation of a  $p - n$  junction results in a depletion region at the p-n interface where the density of volume charge region is given by  $\rho = -ekx$  expression. To find potential distribution, it is necessary to solve Poisson equation in

volume charge's  $(-w, 0)$  and  $(0, w)$  regions:  $\frac{d^2\phi}{dx^2} = \frac{\rho}{\epsilon\epsilon_0} = -\frac{ekx}{\epsilon\epsilon_0}$ :

The general solution of this equation will be:  $\phi(x) = -\frac{ekx^3}{6\epsilon\epsilon_0} + C_1x + C_2$ ,

where the appeared unknown constants should be found from angle conditions requiring that  $\phi(0) = 0$  (start of potential calculation) and  $\frac{d\phi}{dx}(\pm w) = 0$ . In the result the following will be formed:

$$\phi(x) = -\frac{ekx^3}{6\epsilon\epsilon_0} + \frac{ekw^2}{2\epsilon\epsilon_0}x.$$

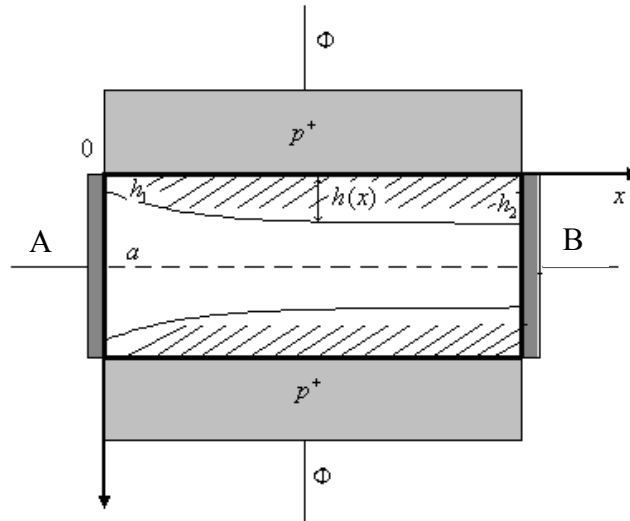
Considering that  $\varphi(w) - \varphi(-w) = \psi - V$  the following will be got:  $w = \sqrt[3]{\frac{\varepsilon\varepsilon_0(\psi - V)}{ek}}$  :

The value of volume charge:  $Q = A \int_0^w ekx dx = A \frac{ekw^2}{2} = A \frac{ek}{2} \left( \frac{\varepsilon\varepsilon_0(\psi - V)}{ek} \right)^{2/3}$  ,

from which for junction capacitance the following will be formed:  $C = \left| \frac{dQ}{dV} \right| = A \left( \frac{\varepsilon\varepsilon_0 ek}{12(\psi - V)} \right)^{1/3}$  :

Putting the data, the following will be written:  $C \approx 46.5$  pf.

**4b3.**



The structure of the transistor is shown in the figure. It is convenient to insert the following function, describing the integral density of the charge:

$$Q(y) = \int_0^y \rho(y) dy, \quad 0 \leq y \leq h(x),$$

where by the approximation of full depletion layer  $\rho(y) = eN_D(y)$ , and  $h(x)$  is the width of depletion layer in  $x = const$  plane. Reverse to  $V$  voltage, applied on  $p^+ - n$  junction, it is necessary to use Poisson equation in order to find the dependence of  $h(x)$ -and  $\rho(y)$ . Integrating it by  $y$  and considering that in  $y = h(x)$  point the component of field intensity should be zero, the following will come out:

$$-\varepsilon_y = \frac{\partial \varphi}{\partial y} = \frac{1}{\varepsilon\varepsilon_0} [Q(h) - Q(y)]$$

This equation can be once more integrated according to  $y - 0$  to  $h(x)$  considering that the change of potential should be equal to  $V$  .

$$V = \frac{1}{\varepsilon\varepsilon_0} \left[ Q(h) \int_0^h dy - \int_0^h Q(y) dy \right] = \frac{1}{\varepsilon\varepsilon_0} \left[ hQ(h) - \int_0^h Q(y) dy \right] = \frac{1}{\varepsilon\varepsilon_0} \int_0^h y \rho(y) dy$$

From here the following will be obtained:

$$\frac{dV}{dh} = \frac{h\rho(h)}{\varepsilon\varepsilon_0}$$

Now it is possible to go into transistor's calculation of current-voltage characteristic and transconductance. In every cross section of the channel  $x = const$  current value is constant and equals:

$$I_D = -eZ\mu_n \frac{dV}{dx} \int_{2a-h}^h N_D(y) dy = eZ\mu_n \frac{dV}{dx} 2 \int_h^a N_D(y) dy,$$

where  $\varepsilon_x = -\frac{dV}{dx}$  is the component of field intensity, and channel width equals  $2(a-h)$ . From the last equation, the following is formed:

$$I_D dx = 2eZ\mu_n dV \int_h^a N_D(y) dy = 2eZ\mu_n \frac{dV}{dh} dh \int_h^a N_D(y) dy,$$

In this equation it has been considered that according to x (along the channel) change of potential takes place and if there is integration in the left part according to x from x=0 (start of the channel) to x=L (end of the channel), then on the right part it equals the change of potential from 0 to drain's  $V_D$  voltage. Pass from potential integration, according to h integration is also convenient. In the result,

$$I_D = \frac{2Z\mu_n}{\varepsilon\varepsilon_0 L} \int_{h_1}^{h_2} [Q(a) - Q(h)] h \rho(h) dh,$$

Here  $h_1$  and  $h_2$  represent the edges of volume charge layer in x=0 and x=L cross sections (accordingly near the source and drain). According to definition, the transconductance will be:

$$g_m = \frac{\partial I_D}{\partial V_G} = \frac{\partial I_D}{\partial h_1} \frac{\partial h_1}{\partial V_G} + \frac{\partial I_D}{\partial h_2} \frac{\partial h_2}{\partial V_G}$$

where  $V_G$  is gate's ( $p^+$ -domain) voltage. The following is obtained:

$$g_m = \frac{2Z\mu_n}{L} [Q(h_2) - Q(h_1)].$$

#### 4b4.

$$R_h = 3.33 \cdot 10^{-4} \text{ m}^3/\text{Cl},$$

$$\rho = 8.93 \cdot 10^{-3} \text{ Ohm} \cdot \text{m},$$

$$\beta = 0.5 \text{ Ml},$$

$$\varphi = ?$$

$\varphi = \mu \cdot \beta$ , where  $\mu$  is particle's mobility

$$\mu = \frac{R_h}{\rho} = \frac{3.33 \cdot 10^{-4}}{8.93 \cdot 10^{-3}} \text{ m}^2/(\text{V} \cdot \text{s}) = 3.73 \cdot 10^{-2} \text{ m}^2/(\text{V} \cdot \text{s})$$

$$\varphi = 3.73 \cdot 10^{-2} \cdot 0.5 = 1.86 \cdot 10^{-2} \text{ rad}.$$

#### 4b5.

$$K_f = 100 \text{ uA/lm},$$

$$\Phi = 0.15 \text{ lm}$$

$$R = 400 \text{ kOhm},$$

$$I = 10 \text{ mA},$$

$$U = 220 \text{ V},$$

$$K_u = ?, K_p = ?$$

Photovoltaic cell current:

$$I_f = K_f \cdot \Phi = 15 \text{ uA}$$

Amplifier's input power:

$$\rho_m = I^2 R = (15 \cdot 10^{-6})^2 \cdot 4 \cdot 10^5 = 9 \cdot 10^{-5} \text{ Wt}$$

Power of relay:

$$\rho_n = U \cdot I = 220 \cdot 10 \cdot 10^{-3} = 2.2 \text{ Wt}$$

$$K_p = \frac{\rho_n}{\rho_m} = \frac{2.2}{9 \cdot 10^{-5}} = 2.44 \cdot 10^4$$

$$K_u = \frac{U_n}{U_R} = \frac{U_p}{I_f \cdot R} = \frac{220}{10 \cdot 10^{-6} \cdot 400 \cdot 10^3} = 36.7$$

#### 4b6.

$$I_0 = 8 \text{ uA},$$

$$E = 10 \text{ V}$$

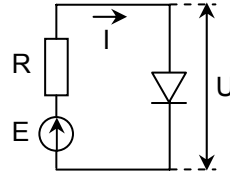
$$R = 1 \text{ kOhm},$$

$$T = 300 \text{ K},$$

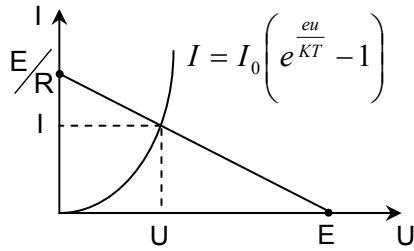
$$I = ?, U = ?$$

1.

$$I = I_0 \left( e^{\frac{u}{\phi_T}} - 1 \right) = I_0 \left( e^{\frac{E-IR}{(KT)/e}} - 1 \right)$$



2.



**4b7.**

$$I_{hmax} = 2 \text{ mA},$$

$$U_{gcut} = 5 \text{ V}$$

$$U_{g1} = -5 \text{ V}$$

$$U_{g2} = 0 \text{ V}$$

$$U_{g3} = -2.5 \text{ V}$$

$$I_h = ?, S = ?$$

$$1. \quad I_h = I_{hmax} \left( 1 - \frac{|U_g|}{U_{gcut}} \right)^2 \quad I_{h1} = 2 \left( 1 - \frac{|-5|}{5} \right)^2 = 0$$

$$I_{h2} = 2 \left( 1 - \frac{|0|}{5} \right)^2 = 2 \quad I_{h3} = 2 \left( 1 - \frac{|-2.5|}{5} \right)^2 = 0.5$$

$$2. \quad S = \frac{2 \cdot I_{hmax}}{U_{gcut}} \left( 1 - \frac{|U_g|}{U_{gcut}} \right)^2 \quad S_1 = \frac{2 \cdot 2}{5} \left( 1 - \frac{|-5|}{5} \right)^2 = 0$$

$$S_2 = \frac{2 \cdot 2}{5} \left( 1 - \frac{|0|}{5} \right)^2 = 0.8 \quad S_3 = \frac{2 \cdot 2}{5} \left( 1 - \frac{|-2.5|}{5} \right)^2 = 0.2$$

**4b8.**

a. By the above mentioned values calculate the densities of majority carriers in  $p$ - and  $n$ - domains:

$$n_n = N_d = \frac{\sigma_n}{q\mu_n} = \frac{10}{1.6 \cdot 10^{-19} \cdot 1300} = 4.8 \cdot 10^{16} \text{ cm}^{-3}, \quad P_p = N_a = \frac{\sigma_p}{q\mu_p} = \frac{5}{1.6 \cdot 10^{-19} \cdot 500} = 6.2 \cdot 10^{16} \text{ cm}^{-3}$$

When the external voltage is missing, the contact  $\phi_k$  difference of potentials can be calculated by the following:

$$\phi_k = \frac{kT}{q} \ln \frac{n_n p_p}{n_i^2} = \frac{1.38 \cdot 10^{-23} \cdot 300}{1.6 \cdot 10^{-19}} \ln \frac{4.8 \cdot 10^{16} \cdot 6.2 \cdot 10^{16}}{(1.4 \cdot 10^{10})^2} = 0.78 \text{ V},$$

b.  $d_p$  and  $d_n$  widths of both common  $d$  and  $p$  &  $n$  domains will be defined in the following way:

$$d = \sqrt{\frac{2\epsilon\epsilon_0\phi_k(N_d + N_a)}{qN_d N_a}} = \sqrt{\frac{2 \cdot 12 \cdot 8.86 \cdot 10^{-14} \cdot 0.78 \cdot (4.8 \cdot 10^{16} + 6.2 \cdot 10^{16})}{1.6 \cdot 10^{-19} \cdot 4.8 \cdot 10^{16} \cdot 6.2 \cdot 10^{16}}} = 0.196 \mu\text{m}$$

$$\frac{d_n}{d_p} = \frac{N_a}{N_d} = \frac{P_p}{n_n}, \quad d_p = d - d_n,$$



$$d_n = \frac{d * N_a}{N_d(1 + N_a/N_d)} = \frac{0.0000196 * 6.2 * 10^{16}}{4.8 * 10^{16} * \left(1 + \frac{6.2 * 10^{16}}{4.8 * 10^{16}}\right)} = 0.11 \text{ } \mu\text{m}$$

$$d_p = d - d_n = 0.196 - 0.11 = 0.086 \text{ } \mu\text{m}$$

c. The maximum  $E_m$  strength of an electrical field is defined by the following:

$$E_m = \frac{2 * \phi_k}{d} = \frac{2 * 0.78}{0.0000196} = 7.9 * 10^4 \text{ V/cm}$$

#### 4b9.

a.  $j_s$  density of photodiode's saturation current by ideal p-n junction can be defined as follows:

First, from Einstein equation define diffusion coefficients of electrons and holes

$$D_n = \frac{kT}{q} \mu_n = 0.026 * 1300 = 33.8 \text{ cm}^2/\text{v}, \quad D_p = \frac{kT}{q} \mu_p = 0.026 * 500 = 13 \text{ cm}^2/\text{v}, \text{ where } kT/q = 0.026\text{V},$$

then the densities of minority carriers.

$$p_n = \frac{(n_i)^2}{N_d} = \frac{(1.4 * 10^{10})^2}{10^{15}} = 1.96 * 10^5 \text{ cm}^{-3}, \quad n_p = \frac{(n_i)^2}{N_a} = \frac{(1.4 * 10^{10})^2}{5 * 10^{15}} = 3.9 * 10^4 \text{ cm}^{-3}$$

And finally the current density:

$$j_s = q \left( \frac{D_p p_n}{L_p} + \frac{D_n n_p}{L_n} \right) = 1.6 * 10^{-19} \left( \frac{13 * 1.96 * 10^5}{0.006} + \frac{33.8 * 3.9 * 10^4}{0.01} \right) = 8.8 * 10^{-11} \text{ A/cm}^2$$

b. In order to define open circuit voltage, first define the photo current by the above mentioned values:

$$I_f = q \alpha W \beta S \Phi = 1.6 * 10^{-19} * 10^3 * 0.01 * 0.7 * 10^{-4} * 10^{18} = 1.1 * 10^{-4} \text{ A}$$

$$\text{then open circuit voltage } V_{xx} = \frac{kT}{q} \ln \left( \frac{I_f}{I_s} + 1 \right) \approx \frac{kT}{q} \ln \left( \frac{I_f}{I_s} \right) = 0.026 * \ln \left( \frac{1.1 * 10^{-4}}{8.8 * 10^{-11}} \right) = 0.3 \text{ V}$$

#### 4b10.

a. Define  $h_1/h_2$  ratio of  $h_1$  and  $h_2$  widths of the channel near the source and drain. Near source reverse biasing voltage applied on p-n junction  $V_{p-n} = V_G + V_1 = 1 + 0.5 = 1.5\text{V}$ , and near the drain  $V_{p-n} = V_G + V_2 = 1 + 1 = 2\text{V}$ . Near source width of charge layer  $d = (2\epsilon\epsilon_0\mu_n\rho V_{p-n})^{1/2} = (2 * 12 * 8.85 * 10^{-14} * 1300 * 0.5 * 1.5)^{1/2} = 0.45 * 10^{-4} \text{ cm}$ , and near the drain

$$d_2 = (2\epsilon\epsilon_0\mu_n\rho V_{p-n})^{1/2} = (2 * 12 * 8.85 * 10^{-14} * 1300 * 0.5 * 2)^{1/2} = 0.52 * 10^{-4} \text{ cm}.$$

$h_1$  and  $h_2$  widths of source and drain of the channel will be defined:

$$h_1 = \alpha - 2d_1 = 2 * 10^{-4} - 2 * 0.45 * 10^{-4} = 1.1 * 10^{-4} \text{ cm},$$

$$h_2 = \alpha - 2d_2 = 2 * 10^{-4} - 2 * 0.52 * 10^{-4} = 0.96 * 10^{-4} \text{ cm},$$

Thus  $h_1/h_2$  ratio of  $h_1$  and  $h_2$  widths will be

$$h_1/h_2 = 1.1/0.96 = 1.15$$

b. cutoff voltage

$$V_{G0} = \frac{\alpha^2}{8\epsilon\epsilon_0\mu_n\rho} = \frac{4 * 10^{-8}}{8 * 12 * 8.85 * 10^{-14} * 1300 * 0.5} = 7.24 \text{ V}$$

#### 4b11.

a. Define C capacitance of the gate in depletion mode

$$S = \ell \cdot b = 10^{-2} * 10^{-2} = 10^{-4} \text{ cm}^2 \quad C = \frac{\epsilon\epsilon_0 S}{d} = \frac{12 * 8.85 * 10^{-14} * 10^{-4}}{0.5 * 10^{-4}} = 2.12 * 10^{-12} \text{ F}$$

b. In order to define the cutoff voltage, first find the density of majority carriers in the channel

$$n = N_C \exp\left[-\frac{E_C - E_F}{kT}\right] = 10^{19} * \exp\left[-\frac{0.2}{0.025}\right] = 3.2 * 10^{15} \text{ cm}^{-3}, \text{ then the cutoff voltage}$$

$$V_{G0} = \frac{q\alpha bl}{C} = \frac{1.6 * 10^{-19} * 3.2 * 10^{15} * 2 * 10^{-4} * 10^{-2} * 10^{-2}}{2.12 * 10^{-12}} = 4.8 \text{ V}$$

**4b12.**

Capacitance of the abrupt p-n junction is given by

$$C = \sqrt{\frac{\epsilon\epsilon_0 N_A N_D e}{2(N_a + N_0)(V_r + k)}} S,$$

where  $N_A$  and  $N_D$  are the donor and acceptor densities and  $S$  is cross section area of diode.

When  $V_r=2 \text{ V}$ ,  $C_1=200 \text{ pF}$

$$C_1 = S \sqrt{\frac{\epsilon\epsilon_0 N_A N_D e}{2(N_a + N_0)(V_{r1} + k)}},$$

When reverse bias is  $V_r$ , then  $C_2 = 50 \text{ pF}$ .

$$C_2 = S \sqrt{\frac{\epsilon\epsilon_0 N_A N_D e}{2(N_a + N_0)(V_{r2} + k)}},$$

Therefore

$$\frac{C_1}{C_2} = \sqrt{\frac{V_{r2} + k}{V_{r1} + k}},$$

Hence

$$V_{r2} = \left(\frac{C_1}{C_2}\right)^2 (V_{r1} + k) - k = 44.1 \text{ V}.$$

**4b13.**

$$\sigma = q\mu_n N_D$$

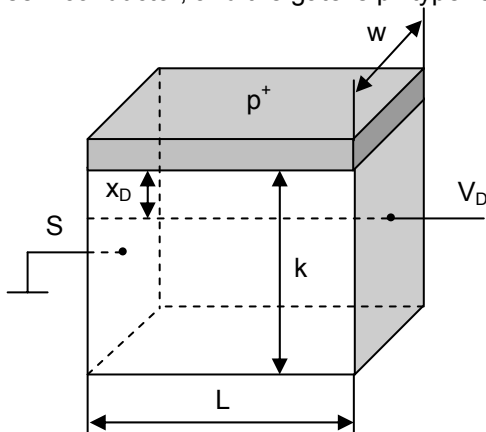
$$N_D = \frac{\sigma}{q\mu_n} = 10^{21} \mu^{-3}$$

The "pinch-off" voltage of the channel for the abrupt p-n junction is  $U_{po} = \frac{\epsilon N_D a^2}{2\epsilon\epsilon_0}$ ,

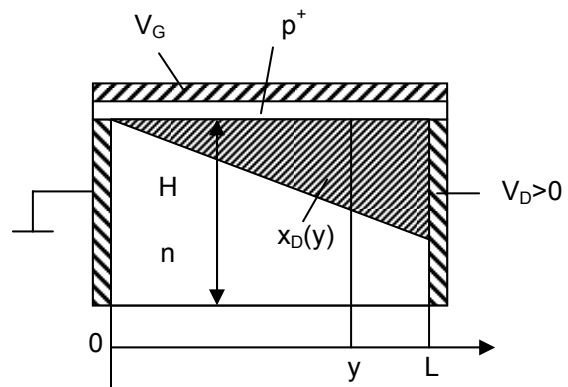
where  $a = \frac{w}{2} = 4 \mu \text{ m}$ , hence  $U_{po} = 12$ .

**4b14.**

For simplification, an asymmetric field effect transistor will be considered, the channel of which is an n-type semiconductor, and the gate is  $p^+$ -type region which forms the  $p^+$ -n junction with the channel.



$k \ll L \ll w$   
a)



b)

The n-channel of the transistor has length  $L$  (a distance from source to drain), width “ $w$ ” and depth “ $H$ ” (a distance between the insulating substrate and metallurgical boundary of the gate). When the source is grounded and  $V_D=0$ , the channel effective width (the width of its quasi-neutral part) is uniform in any cross section  $y=\text{const}$  and equals  $(H-x_D)$ , where

$$x_D = \sqrt{\frac{2\epsilon\epsilon_0(V_{bi} - V_G)}{eN_D}}$$

Here  $V_{bi}$  is the contact potential difference,  $V_G$ -gate voltage,  $N_D$  is the channel doping level,  $\epsilon$ - dielectric,  $\epsilon_0$ - electrical constant and  $e$  is an elementary charge.

Assume that  $p^+$ -region of the gate is strongly doped and therefore almost all space charge region can be considered as fully expanded in the n- channel region.

$L \gg H$  and the Schottky graduate channel approximation is valid. Also for the uniform doped channel the approximation of complete depletion layer is applicable. When  $V_D > 0$ , the drain current in the  $y$  section is

$$I_D = eN_D w [H - x_D(y)] v(y),$$

where  $v(y)$ - is the electron drift velocity, which is defined by  $\epsilon_y$ -component of electrical field in the section  $y$ :

$$v(y) = \frac{\mu_n \epsilon_y}{1 + \frac{\epsilon_y}{\epsilon_c}},$$

and

$$x_D = \sqrt{\frac{2\epsilon\epsilon_0(V(y) + V_{bi} - V_G)}{eN_D}}$$

is depletion layer width in the same section.

As  $L \gg H$ , then  $\epsilon_y = \frac{dV(y)}{dy}$ , and

$$I_D = eN_D w \left[ H - \sqrt{\frac{2\epsilon\epsilon_0(V(y) + V_{bi} - V_G)}{eV_D}} \right] \frac{\mu_n}{1 + \frac{1}{\epsilon_c} \frac{dv}{dy}} \frac{dv}{dy}$$

or

$$I_D = \left\{ eN_D w \left[ H - \sqrt{\frac{2\epsilon\epsilon_0(V(y) + V_{bi} - V_G)}{eV_D}} \right] \mu_n - \frac{I_D}{\epsilon_c} \right\} \frac{dv}{dy}$$

Integrating this equation  $y=0$  ( $V(0)=0$ ) up to  $y=L$  ( $V(L)=V_D$ ), considering that  $I_D = \text{Const}$ , the following can be obtained:

$$\begin{aligned} I_D \cdot L &= \int_0^{V_D} \left\{ eN_D w \left[ H - \sqrt{\frac{2\epsilon\epsilon_0(V(y) + V_{bi} - V_G)}{eV_D}} \right] \mu_n - \frac{I_D}{\epsilon_c} \right\} dv = \\ &= eN_D w H \mu_n V_D - eN_D w \mu_n \int_0^{V_D} \sqrt{\frac{2\epsilon\epsilon_0(V(y) + V_{bi} - V_G)}{eV_D}} dv - \frac{I_D}{\epsilon_c} V_D \end{aligned}$$

Or

$$I_D \left( 1 + \frac{V_D}{\epsilon_c L} \right) = \frac{eN_D w H \mu_n V_D}{L} - \frac{eN_D w \mu_n}{L} \sqrt{\frac{2\epsilon\epsilon_0}{eN_D}} \int_0^{V_D} \sqrt{V + V_{bi} - V_G} dv$$

After some transformation the final expression for the drain current follows:

$$\frac{I_D}{I_p} = \frac{3 \frac{V_D}{V_{bi} - V_T} - 2 \left[ \left( \frac{V_D + V_{bi} - V_G}{V_{bi} - V_T} \right)^{\frac{3}{2}} - \left( \frac{V_{bi} - V_G}{V_{bi} - V_T} \right)^{\frac{3}{2}} \right]}{1 + \frac{V_D}{\epsilon_c L}},$$

where

$$I_p = \frac{3N_D w H \mu_n}{3L} (V_{bi} - V_T)$$

$$V_T = V_{bi} - \frac{eN_D H^2}{2\epsilon\epsilon_0}$$

**4b15.**

For a "limited source" condition with the total amount of impurities  $Q$ , the solution of the Fick equation is given by the Gaussian function.

$$N(x, t) = \frac{Q}{\sqrt{\pi Dt}} \exp\left[-\left(\frac{x}{2\sqrt{Dt}}\right)^2\right]$$

$p-n$  junction is formed at that place, where boron concentration equals to the concentration of impurities in

the bulk silicon substrate, that is:  $\frac{Q}{\sqrt{\pi Dt}} \exp\left[-\left(\frac{x_j}{2\sqrt{Dt}}\right)^2\right] = N_D$

Solving this equation with respect to  $x_j$ , the following is obtained  $x_j = 2\sqrt{Dt} \sqrt{\ln\left[\frac{Q}{N_D \sqrt{\pi Dt}}\right]}$

Substituting the numerical values of  $Q$ ,  $D$ ,  $N_D$  and  $t$ , into this expression yields  $x_j = 2,7 \mu\text{m}$ .

**4b16.**

First find equilibrium heights of emitter and collector junctions:

$$\varphi_{0E} = \frac{kT}{e} \ln \frac{N_{AE} \cdot N_{DB}}{n_i^2} = 0.856 \text{ eV},$$

$$\varphi_{0K} = \frac{kT}{e} \ln \frac{N_{DB} \cdot N_{AK}}{n_i^2} = 0.635 \text{ eV}.$$

The width of emitter junction depletion region in the base of the transistor is

$$W_{nE} = \sqrt{\frac{2\epsilon\epsilon_0(e\varphi_{0E} - U_{EB})}{e^2 N_{DB}^2 \left(\frac{1}{N_{AE}} + \frac{1}{N_{DB}}\right)}} = 0.215 \mu\text{m},$$

and correspondingly the width of collector junction depletion region in the base is

$$W_{nK} = \sqrt{\frac{2\epsilon\epsilon_0(e\varphi_{0K} - U_{BK})}{e^2 N_{DB}^2 \left(\frac{1}{N_{AK}} + \frac{1}{N_{DB}}\right)}} = 0.258 \mu\text{m}.$$

Therefore the width of the neutral base will be the following:

$$W_B = W - W_{nE} - W_{nK} = 0.527 \mu\text{m}.$$

The concentration of minority carries (holes) at the emitter-base junction will equal the following:

$$p_n(0) = \frac{n_i^2}{N_{DB}} \exp\left(\frac{eU_{EB}}{kT}\right) = 5.18 \cdot 10^{12} \text{ cm}^{-3}.$$

Finally, the total charge of minority carries accumulated in the base is

$$Q = \frac{eS p_n(0) W_B}{2} = 6.4 \cdot 10^{-13} \text{ K}.$$

**4b17.**

Bipolar diffusion coefficient is determined as

$$D = \frac{n + p}{\frac{n}{D_p} + \frac{p}{D_n}}$$

For intrinsic semiconductor

$$D = \frac{2D_n D_p}{D_n + D_p}.$$

Using Einstein relations one obtains

$$D = \frac{k_B T}{e} \frac{2\mu_n \mu_p}{\mu_n + \mu_p} \approx 20,7 \text{ cm}^2/\text{s}.$$

#### 4b18.

First a density of donors in the semiconductor can be defined:

$$N_D = \frac{1}{e\mu_n \rho} = 1.6 \cdot 10^{15} \text{ cm}^{-3}.$$

The built-in potential barrier for the electrons is equal to

$$\varphi_m = \Phi_{Au} - \Phi_{Ge},$$

Here the semiconductor work function can be represented as

$$\Phi_{Ge} = \chi_{Ge} + E_c - E_F = \chi_{Ge} + kT \ln \frac{N_c}{N_D} = \chi_{Ge} + kT \ln \frac{N_c}{n_i} + kT \ln \frac{n_i}{N_D} = \chi_{Ge} + \frac{E_g}{2} - \varphi_0$$

where it is meant, that

$$\frac{E_g}{2} = kT \ln \frac{N_c}{n_i},$$

and

$$\varphi_0 = kT \ln \frac{N_D}{n_i} = 0.11 \text{ eV}$$

Therefore,  $\Phi_{Ge} = 4.22 \text{ eV}$ , and  $\varphi_m = 0.78 \text{ eV}$ .

#### 4b19.

$$a) \gamma = \frac{I_{cp}}{I_{cp} + I_{cn}} = \frac{3}{3 + 0.01} \approx 0.9967,$$

$$b) \alpha_T = \frac{I_{cp}}{I_{cp}} = \frac{2.99}{3} = 0.9967,$$

$$c) \alpha_0 = \gamma \alpha_T = 0.9934$$

$$I_c = I_{cp} + I_{cn} = 3.01 \text{ mA}$$

$$I_c = I_{cp} + I_{cn} = 2.99 + 0.001 = 2.991 \text{ mA}$$

$$I_{cp0} = I_c - \alpha_0 I_c = 2.991 - 0.9934 \cdot 3.01 = 0.000866 = 0.87 \text{ uA}.$$

#### 4b20.

$$V_T = V_{FB} + 2\psi_B + \frac{\sqrt{2\varepsilon_S q N_a (2\psi_B)}}{C_{ox}}$$

$$C_{ox} = \frac{\varepsilon_{ox}}{d} = \frac{3.9 \cdot 8.85 \cdot 10^{-14} \text{ F}}{5 \cdot 10^{-7} \text{ cm} \cdot \text{cm}} = 6.9 \cdot 10^{-7} \text{ F/cm}^2$$

$$V_{FB} = \psi_S - \frac{Q_{ox}}{C_{ox}} \approx -0.98 - \frac{1.6 \cdot 10^{-19} \cdot 5 \cdot 10^{11}}{6.9 \cdot 10^{-7}} \approx -1.1 \text{ V}$$

$$V_T = -1.1 + 0.84 - \frac{\sqrt{2 \cdot 11.9 \cdot 8.85 \cdot 10^{-14} \cdot 1.6 \cdot 10^{-19} \cdot 10^{17} \cdot 0.84}}{6.9 \cdot 10^{-7}} \approx 0.02 \text{ V}$$

The substrate accumulation charge (Boron) will lead to flat-band bias by  $\frac{qN_{bor}}{C_{ox}}$ , and therefore

$$0.6 = -0.02 + \frac{qN_{bor}}{C_{ox}}$$

$$N_{bor} = \frac{0.62 \cdot 6.9 \cdot 10^{-7}}{1.6 \cdot 10^{-19}} \approx 2.67 \cdot 10^{12} \text{ cm}^{-2}$$

4b21.

$$\left. \begin{aligned} \sigma_1 &= \sigma_o e^{-\frac{\Delta E}{2kT_1}} \\ \sigma_2 &= \sigma_o e^{-\frac{\Delta E}{2kT_2}} \end{aligned} \right| \begin{aligned} \Delta\sigma &= \sigma_1 - \sigma_2 = \sigma_o \left( e^{-\frac{\Delta E}{2kT_2}} - e^{-\frac{\Delta E}{2kT_1}} \right) \\ \frac{\Delta\sigma}{\sigma_1} &= \frac{e^{-\frac{\Delta E}{2kT_2}} - e^{-\frac{\Delta E}{2kT_1}}}{e^{-\frac{\Delta E}{2kT_1}}} = e^{\frac{\Delta E}{2k} \left( \frac{1}{T_1} - \frac{1}{T_2} \right)} - 1 \approx 1.18 - 1 \approx 0.18 \end{aligned}$$

Answer: will increase by 18%.

4b22.

$$\left. \begin{aligned} \sigma &= cn\mu_n + cp\mu_p \\ \sigma &= cn\mu_n + e \frac{n_i^2}{n} \mu_p \end{aligned} \right| p = \frac{n_i^2}{n}$$

Find the minimum value from  $\frac{d\sigma}{dn} = 0$  condition:

$$\frac{d\sigma}{dn} = c\mu_n - c \frac{n_i^2}{n^2} \mu_p = 0$$

$$\mu_n = \frac{n_i^2}{n^2} \mu_p \Rightarrow n = n_i \sqrt{\frac{\mu_p}{\mu_n}}$$

$$p = \frac{n_i^2}{n} = \frac{n_i^2}{n_i \sqrt{\frac{\mu_p}{\mu_n}}} = n_i \sqrt{\frac{\mu_n}{\mu_p}}$$

4b23.

$$\mu = \frac{e\tau}{m}; \quad L_d = \sqrt{D\tau}; \quad D = \frac{kT}{e} \mu$$

$$L_d = \sqrt{\frac{kT}{e} \mu \tau} = \sqrt{\frac{kT\tau^2}{m}} = \tau \sqrt{\frac{kT}{m}}$$

$$L_D - \frac{L_D}{10} = \sqrt{\frac{kT}{e} \mu \left(1 + \frac{1}{20}\right) \cdot \tau \left(1 + \frac{x}{100}\right)} = \sqrt{\frac{kT\tau^2}{m} \left(1 + \frac{1}{20}\right) \cdot \left(1 + \frac{x}{100}\right)}$$

$$L_D \left(1 + \frac{1}{10}\right) = \tau \sqrt{\frac{kT}{m} \left(1 + \frac{1}{20}\right) \cdot \left(1 + \frac{x}{100}\right)} = \sqrt{\frac{21}{20} \cdot \frac{100 + x}{100}}$$

Answer: will increase by 15%.

4b24.

a. Using Bohr model, the ionization energy of donors in any semiconductor is expressed by:

$$E_d = \left( \frac{\epsilon_o}{\epsilon_s} \right)^2 \cdot \left( \frac{m^*}{m_o} \right) \cdot E_H$$

$$E_d = \frac{1}{(17)^2} \cdot 0.014 \cdot 13.6 \approx 6.58 \cdot 10^{-4} \text{ eV}$$

b. The radius of the first orbit (n=1) is connected to the corresponding radius of hydrogen atom

$$r = \left( \frac{\varepsilon \cdot m_0}{m^*} \right) r_H$$

$$r_{nsb} = \left( \frac{17}{0.014} \right) \cdot 0.53 = 643.6 \text{ } \overset{\circ}{A}$$

c. Density of conducting electrons

$$n \approx \frac{N_d}{1 + g \exp\left(-\frac{E_d}{kT}\right)} \approx \frac{N_d}{1 + 2 \exp\left(-\frac{E_d}{kT}\right)}$$

$$kT = 0.00034$$

$$n = \frac{1 \cdot 10^{14}}{1 + 2 \exp(-2)} = 0.78 \times 10^{14}$$

4b25.

Field tension in space for metal  $E = \frac{V_k}{d} = 10^7 \text{ V/cm}$ .

Surface density of charge

$$Q_s = E_k \cdot \varepsilon_0, \quad Q_s = 10^7 \cdot 8.85 \cdot 10^{-14} = 8.85 \cdot 10^7 \text{ K/cm}^2$$

$$\text{Number of electrons } n = \frac{Q_s}{q} = \frac{8.85 \cdot 10^{-7}}{1.6 \cdot 10^{-19} \text{ K}} \cdot \frac{\text{K}}{\text{cm}^2} \approx 5.5 \cdot 10^{12} \text{ cm}^{-2}$$

in order to provide the difference of the same potentials (for metal-n type semiconductor contact) it will be

necessary to deplete  $W = \frac{n}{n_{ss}} = \frac{5 \cdot 10^{12}}{10^{15}} = 5 \cdot 10^{-3} \text{ cm}$  width layer from semiconductor.

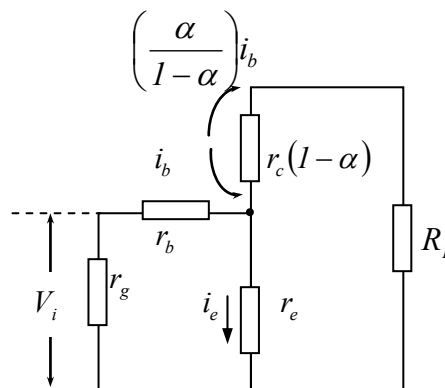
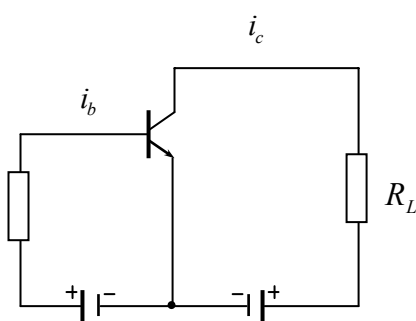
4b26.

According to AC, the input voltage

$$V_i = i_e r_e + i_b r_b$$

$$\alpha = \frac{i_c}{i_e},$$

$$i_b = (1 - \alpha) i_e$$



Input power

$$P_i = V_i \cdot i_b = [i_e r_e + (1 - \alpha) \cdot i_e r_b] (1 - \alpha) i_e$$

Output power

$$P_{out} = i_c^2 R_L = i_e^2 \alpha R_L$$

Amplification coefficient according to power

$$G_p = \frac{\alpha^2 R_L}{[r_e + (1 - \alpha) r_b] (1 - \alpha)}$$

$$\alpha = 0.98, \quad r_e = 20 \text{ Ohm}, \quad r_b = 500 \text{ Ohm}, \quad R_L \approx 30 \text{ kOhm}$$

$$G_p = \frac{0.98^2 \cdot 3 \cdot 10^4}{(20 + 0.02 \cdot 500)(0.02)} = 5 \cdot 10^4$$

**4b27.**

As it is known

$$n \sim T^{3/2} \exp(-E_g / 2k_B T)$$

Therefore

$$\frac{n_1}{n_2} = \left( \frac{T_1}{T_2} \right)^{3/2} \exp \left[ -\frac{E_g(0)}{2k_B} \left( \frac{1}{T_1} - \frac{1}{T_2} \right) \right], \text{ where } E_g = E_g(0) - \alpha T.$$

Therefore

$$E_g(0) = 2k_B \frac{T_1 T_2}{T_1 - T_2} \ln(n_1 T_2^{3/2} / n_2 T_1^{3/2}) = 0.26 \text{ eV.}$$

**4b28.**

In the given case continuity equation has the following form:  $D_p \frac{d^2 \Delta p}{dx^2} - E \mu_p \frac{d \Delta p}{dx} - \frac{\Delta p}{\tau_p} = 0$ ,  $x \neq 0$ , or

$$\frac{d^2 \Delta p}{dx^2} - \frac{eE}{k_B T} \frac{d \Delta p}{dx} - \frac{\Delta p}{L_p^2} = 0:$$

It is obvious that  $\Delta p \rightarrow 0$ , when  $x \rightarrow \pm\infty$ . Therefore

$$\Delta p = \begin{cases} \Delta p(x=0) \exp(k_1 x), & x < 0 \\ \Delta p(x=0) \exp(k_2 x), & x > 0 \end{cases}$$

where  $k_{1,2} = eE / 2k_B T \pm (e^2 E^2 / 4k_B^2 T^2 + L_p^{-2})^{1/2}$ . It is obvious that  $k_1 > 0$  and  $k_2 < 0$ .

**4b29.**

From the determination of the injection coefficient it follows that  $j_p(0) = \xi [j_p(0) + j_n(0)]$ . Therefore

$$j_n(0) = j_p(0)(1 - \xi) / \xi. \text{ Hole current density at } x = 0 \text{ equals } j_p(0) = -eD_p \left. \frac{d \Delta p}{dx} \right|_{x=0}.$$

Taking into account that in the considered case  $\Delta p = \Delta p(x=0) \exp(-x / L_p)$ , one obtains  $j_p(0) = eD_p \Delta p(0) / L_p$ ,

therefore  $j_n(0) = eD_p \Delta p(0)(1 - \xi) / \xi L_p = 1.68 \text{ mA.cm}^{-2}$ .

**4b30.**

Sample resistivity  $\rho = RS / l = 0.03 \text{ Ohm}$ . On the other hand,  $\rho = 1 / (e n \mu_n + e p \mu_p) = 1 / (e \frac{n_i^2}{p} \mu_n + e p \mu_p)$ . Hence

$$\rho = \frac{1 + \sqrt{1 - 4e^2 \rho^2 \mu_n \mu_p n_i^2}}{2e p \mu_n} = 4 \times 10^{21} \text{ m}^{-3}.$$

**4b31.**

a. The density of radiation, coming to surface is  $F$ , and in  $d$  depth, after absorption, is  $F e^{-\alpha d}$ . Therefore, the number of quantum, absorbed in  $d$  depth, is  $F(1 - e^{-\alpha d})$ , and the photocurrent, occurred in the result of absorption, is  $I_{ph} = qSF(1 - e^{-\alpha d})$ .

b. Find the photocurrent, occurred in the result of absorption of  $\lambda_1$  wave radiation.

$$I_{ph1} = qSF (1 - e^{-\alpha_1 d}) \approx 10^{-9} \text{ A}$$

c. Find the photocurrent, occurred in the result of absorption of  $\lambda_2$  wave radiation.

$$I_{ph1} = qSF (1 - e^{-\alpha_1 d}) \approx 0.15 \cdot 10^{-9} \text{ A}$$

d. Find the ratio of photocurrents.

$$\frac{I_{ph1}}{I_{ph2}} \approx 6.7$$



**4b32.**

In general, the performance of photodiode is characterized by three processes 1.  $\tau_{RC}$  time constant, 2.  $\tau_d$  diffusion time of non-equilibrium charges through the base and 3.  $\tau$  time of non-equilibrium charges through bulk charge domain.

Time constant of photodiode is defined by base resistance and charge capacitance of p-n junction.

$$\tau_{RC} = RC :$$

Base resistance and charge capacitance of p-n junction are defined by the following expressions:

$$R = \frac{w}{Sq_n\mu_n} \approx 0,3 \text{ Ohm}, \quad C = \frac{S\varepsilon\varepsilon_0}{d} \approx 10^{-10} \text{ F}.$$

Therefore,  $\tau_{RC} = RC \approx 3 \cdot 10^{-11} \text{ s}$ .

Diffusion time of non-equilibrium charges through the base can be defined by the following expression:

$$\tau_d = w^2/2D = 2,9 \cdot 10^{-7} \text{ s}.$$

The drift time of non-equilibrium charges through bulk charge domain can be defined by the following expression:

$$\tau = \frac{d}{V_{\max}} = 2 \cdot 10^{-11} \text{ s}.$$

Therefore, photodiode performance, which is defined by the largest time constant, in the given case diffusion time of non-equilibrium charges through the base, will equal  $2,9 \cdot 10^{-7} \text{ s}$ .

**4b33.**

In order to define the power density equivalent to the noise of photodiode, its basic noise must be taken into account, which has fluctuation nature and is defined by average square value of current fluctuation by the following expression:

$$\bar{I}_{dr}^2 = 2qI\Delta f,$$

as well as photosensitivity, according to power, by the following expression:

$$S_i(\lambda) = \frac{I_{L\phi}}{P} :$$

Consider that minimum signal, detected against the background of noise is defined by the value of signal noise ratio, equal to one.

$$\frac{I}{\sqrt{\bar{I}_{dr}^2}} = \frac{S_i(\lambda)P}{\sqrt{\bar{I}_{dr}^2}} = 1 :$$

Threshold sensitivity can be defined, i.e. the minimum power of radiation which will be detected against the background of the noise.

$$P = \frac{\sqrt{\bar{I}_{dr}^2}}{S_i(\lambda)}$$

$\bar{I}^2$  is proportional to  $\Delta f$  range of bandwidth, hence, the power density, equivalent to noise in the unit layer of frequency will be.

$$P = \frac{1}{S_i(\lambda)} \sqrt{\frac{\bar{I}_{dr}^2}{S\Delta f}} = 8 \cdot 10^{-12} \text{ VtHz}^{-1/2} \text{ cm}^{-1}$$

**4b34.**

Near the source,  $d$  width of p-n junction will be defined by the following expression:

$$d_1 = \left( \frac{2\varepsilon\varepsilon_0}{q} \cdot \frac{1}{N_d} \right)^{1/2} = 0,36 \cdot 10^{-4} \text{ cm}.$$

and near the drain:

$$d_2 = \left( \frac{2\varepsilon\varepsilon_0}{q} \cdot \frac{q+V}{N_d} \right)^{1/2} = 1,79 \cdot 10^{-4} \text{ cm}.$$

therefore narrowing size of channel near the source will equal  $d_1 - d_2 = 1,43 \cdot 10^{-5} \text{ cm}$ .

**4b35.**

The saturation voltage of the transistor  $V_{DS\text{ sat}} = V_{GS} - V_t = 3 - 1 = 2\text{V}$ .

$V_{DS} > V_{DS\text{ sat}}$  therefore the transistor operates in saturation mode. In this mode the drain current of the transistor will equal  $I_D = 0,5 \mu_n C_{ox} (W/L) (V_{GS} - V_t)^2$ . Calculate  $C_{ox} = (\epsilon_0 \epsilon_{si} \epsilon_{o2}) / t_{ox} = (3,9 \times 8,85 \times 10^{-14}) / 10 \times 10^{-7} = 3,45 \times 10^{-7} \text{ F/cm}^2$ .

$$I_D = 0,5 \times 300 \times 3,45 \times 10^{-7} (3-1)^2 \times (10/1) = 2,07 \text{ uA.}$$

Transconductance  $g_m = d I_D / d V_{GS}$

$$g_m = \mu_n C_{ox} (W/L) (V_{GS} - V_t) = 300 \times 3,45 \times 10^{-7} \times 10 \times 2 = 2 \text{ uA}$$

Answer: 2,07 uA and 2 uA.

**4b36.**

$$\varphi_s^{(1)} = 0$$

$$\varphi_s^{(2)} = \varphi_o$$

$$\varphi_s^{(3)} = 2\varphi_o$$

$$Q_{sc}, C_{sc} = ?$$

1. If  $\varphi_s = 0$ ,  $Q_{sc}$  charge is missing, and  $C_{sc}$  equals  $C_{FB}$  of flat regions.

$$Q_{sc} = 0;$$

$$C_{si} = C_{FB} = \sqrt{\frac{\epsilon_s \epsilon_o \cdot q \cdot N_w}{kT/q}}$$

Calculate the necessary characteristics.

$$N_a = \frac{1}{q \cdot \mu_p \cdot \rho} = \frac{1}{1,6 \cdot 10^{-19} \cdot 600 \cdot 10} = 1 \cdot 10^{15} \text{ cm}^{-3}$$

$$\varphi_o = \frac{kT}{q} \ln \frac{N_d}{n_i} = 0,0259 \cdot \ln \frac{1 \cdot 10^{15}}{1,6 \cdot 10^{10}} = 0,29 \text{ V.}$$

$$C_{FB} = \sqrt{\frac{11,8 \cdot 8,85 \cdot 10^{-14} \cdot 1,6 \cdot 10^{-19} \cdot 1 \cdot 10^{15}}{0,0259}} = 8 \cdot 10^{-8} \text{ F/cm}^2.$$

2. If  $\varphi_s = \varphi_o$  the semiconductor will be intrinsic which is the boundary value from depletion region to transition into low inversion:

$$Q_{sc} = Q_B = \sqrt{2 \cdot \phi_s \epsilon_o \cdot q \cdot N_w \left( \varphi_o - \frac{kT}{q} \right)} =$$

$$= \sqrt{2 \cdot 11,8 \cdot 8,85 \cdot 10^{-14} \cdot 1,6 \cdot 10^{-19} \cdot 1 \cdot 10^{15} \cdot 0,26} = 9,3 \cdot 10^{-9} \text{ S/cm}^2$$

$$C_{sc} = C_B = \sqrt{\frac{\epsilon_s \epsilon_o \cdot q \cdot N_w}{2 \cdot \left( \varphi_o - \frac{kT}{q} \right)}} =$$

$$= \sqrt{\frac{11,8 \cdot 8,85 \cdot 10^{-14} \cdot 1,6 \cdot 10^{-19} \cdot 1 \cdot 10^{15}}{2 \cdot (0,29 - 0,03)}} = 5,7 \cdot 10^{-8} \text{ F/cm}^2$$

3. If  $\varphi_s = 2\varphi_o$ , which corresponds to transition from low inversion into high inversion, then

$$Q_{sc} = 2 \cdot \epsilon_s \epsilon_o \cdot q \cdot N_w \cdot 2\varphi_o =$$

$$= \sqrt{2 \cdot 11,8 \cdot 8,85 \cdot 10^{-14} \cdot 1,6 \cdot 10^{-19} \cdot 1 \cdot 10^{15} \cdot 0,58} = 1,4 \cdot 10^{-8} \text{ S/cm}^2$$

$$C_{sc} = \sqrt{\frac{\epsilon_s \epsilon_o \cdot q \cdot N_w}{2\varphi_o}} =$$

$$= \sqrt{\frac{11,8 \cdot 8,85 \cdot 10^{-14} \cdot 1,6 \cdot 10^{-19} \cdot 1 \cdot 10^{15}}{0,58}} = 1,7 \cdot 10^{-8} \text{ F/cm}^2$$

**4b37.**

pSi

$$N_w = 10^{18} \text{ cm}^{-3},$$

$$T = 300 \text{ S}$$

$$N_{ss} = 2 \cdot 10^{12} \text{ cm}^{-2} \cdot \text{eV}^{-1}$$

$$\varphi_s^{(1)} = 0$$

$$\varphi_s^{(2)} = \varphi_o$$

$$\varphi_s^{(3)} = 2\varphi_o$$

$$Q_{ss} = ? \quad Q_{sc} = ?$$

The charge of surface states:

 $Q_{ss} = -qN_{ss}(\varphi_s - \varphi_o)$ , and  $Q_{sc}$  charge is conditioned by ionized acceptors:

$$Q_{sc} = -\sqrt{2q\xi_s\xi_o N_a \varphi_s}$$

$$\varphi_o = \frac{kT}{q} \ln \frac{N_a}{n_i} = 0.0259 \cdot \ln \frac{10^{18}}{1.6 \cdot 10^{10}} = 0.46 \text{ V}$$

$$Q_{ss}^{(1)} = qN_{ss}\varphi_o = 1.6 \cdot 10^{-19} \cdot 10^{12} \cdot 0.46 = 1.5 \cdot 10^{-7} \text{ S/cm}^2$$

$$Q_{ss}^{(2)} = 0$$

$$Q_{ss}^{(3)} = -qN_{ss}\varphi_o = -1.6 \cdot 10^{-19} \cdot 2 \cdot 10^{12} \cdot 0.46 = -1.5 \cdot 10^{-7} \text{ S/cm}^2$$

$$Q_{sc}^{(2)} = 0$$

$$Q_{sc}^{(2)} = -\sqrt{2q\xi_s\xi_o N_w \varphi_s} = -2 \cdot 1.6 \cdot 10^{-19} \cdot 8.85 \cdot 10^{-14} \cdot 11.8 \cdot 10^{18} \cdot 0.46 = -3.9 \cdot 10^{-7} \text{ S/cm}^2$$

$$Q_{sc}^{(3)} = -\sqrt{2q\xi_s\xi_o N_w 2\varphi_o} = \sqrt{-2 \cdot 1.6 \cdot 10^{-19} \cdot 8.85 \cdot 10^{-14} \cdot 11.8 \cdot 10^{18} \cdot 0.92} = -5.5 \cdot 10^{-7} \text{ S/cm}^2$$

**4b38.**

$$n = \frac{N_i}{2e^{\frac{E_i - F}{kT}} + 1}, \text{ when } F = E_i \Rightarrow n = \frac{N_d}{3}.$$

**4b39.**

$$\Delta n(t) = \Delta n(0) \cdot e^{-t/\tau_n}; \quad \Delta n(0) = 2.5 \cdot 10^{-20} \text{ m}^{-3}$$

$$\left. \frac{d(\Delta n(t))}{dt} \right|_{t=0} = -\frac{\Delta n(0)}{\tau_n} = 2.8 \cdot 10^{24}$$

$$a) \tau_n = \left| \frac{\Delta n(0)}{2.8 \cdot 10^{24}} \right| = 89 \text{ um / sec}$$

$$b) \Delta n(2 \text{ um}) = 2.5 \cdot 10^{-20} \cdot e^{-\frac{2 \cdot 10^{-3}}{89 \cdot 10^{-6}}} \approx 4.4 \cdot 10^{10} \text{ m}^{-3}$$

4b40.

$$R = 0$$

$$R = \frac{1}{e} \cdot \frac{p\mu_p^2 - n\mu_n^2}{(p\mu_p + n\mu_n)^2} = 0 \Rightarrow n\mu_n^2 = p\mu_p^2 \Rightarrow \frac{n}{p} = \frac{\mu_p^2}{\mu_n^2}$$

$$\frac{\sigma_p}{\sigma} = \frac{ep\mu_p}{en\mu_n + ep\mu_p} = \frac{1}{\frac{n\mu_n}{p\mu_p} + 1} = \frac{1}{\frac{\mu_p^2\mu_n}{\mu_n^2\mu_p} + 1} = \frac{1}{\frac{\mu_p}{\mu_n} + 1} \approx 0.7; \quad \frac{\sigma_p}{\sigma} \approx 70\%$$

4b41.

$$\mu = \frac{e\tau}{m}; \quad L = \sqrt{D\tau}; \quad D = \frac{kT}{e} \mu; \quad L = \sqrt{\frac{kT}{e} \mu \tau} \Rightarrow L \sqrt{\frac{kT}{m} \tau^2} = \tau \sqrt{\frac{kT}{m}}$$

$$L + \frac{L}{10} = \sqrt{\frac{kT}{e} \left( \mu + \frac{\mu}{20} \right) \cdot \left( \tau + \frac{\tau \cdot x}{100} \right)}$$

$$L \left( 1 + \frac{1}{10} \right) = \sqrt{\frac{kT}{e} \mu \tau \left( 1 + \frac{1}{20} \right) \cdot \left( \tau + \frac{x}{100} \right)}$$

$$1 + \frac{1}{10} = \sqrt{\frac{21}{20} \cdot \frac{100 + x}{100}} \Rightarrow x = 15\%$$

4b42. If there are scattering centers of  $N_d$  concentration with the S cross section, the average path will be

$$l = \frac{l}{N_d S}, \quad l = \frac{l}{N_d \pi r^2} \approx 0.64 \text{ } \mu\text{m}$$

$$\tau = \frac{l}{v}; \quad \frac{m^* \bar{v}^2}{z} = \frac{3}{2} kT; \quad \bar{v} = \left( \frac{3kT}{m^*} \right)^{1/2}$$

$$\tau = l \left( \frac{m^*}{3kT} \right)^{1/2} \approx 0.69 \cdot 10^{-11} \text{ sec}$$

$$\mu = \frac{e\tau}{m^*} \approx 0.77 \text{ m}^2 / \sqrt{\text{V}} \cdot \text{sec}$$

4b43.

Using the orbital moving equation of the electron bounded on the donor state  $\frac{mv^2}{r} = \frac{e}{4\pi\epsilon\epsilon_0}$ , the relation

between the electron wavelength and momentum  $\lambda = \frac{2\pi\hbar}{mv}$ , as well as quantization condition  $n\lambda = 2\pi r$  for

the radius of electron orbit  $r = \frac{4\pi\hbar^2 \epsilon\epsilon_0}{me^2} n^2$  is obtained, where  $n$  is the main quantum number. From the

obvious condition of impurity band originates  $N_d^{-1/3} = 2r(n=1)$ , for the minimal concentration of donor

impurity  $N_d = \left( me^2 / 8\pi\hbar^2 \epsilon\epsilon_0 \right)^3 = 7.5 \times 10^{14} \text{ cm}^{-3}$  is obtained.

4b44.

Condition of band-to-band impact ionization is  $mv^2 / 2 \geq E_g$ . Electron velocity determined as  $v = \mu E + v_T$ ,

where  $v_T$  is the thermal electron velocity. Therefore  $E = \frac{1}{\mu} \left( \sqrt{2E_g / m} - \sqrt{3k_B T / m} \right)$ : For silicon

$$E_g \gg k_B T 3/2, \text{ therefore } E = \frac{\sqrt{2E_g / m}}{\mu} \cong 7.4 \times 10^4 \text{ V/cm.}$$

**4b45.**

As well known  $n_i = (N_c N_v)^{1/2} e^{-E_g/2k_B T}$ , where  $N_{c,v} = 2(m_{n,p} k_B T / 2\pi\hbar^2)^{3/2}$ ,  $E_g(T) = E_g(0) - \alpha T$ ,  $\alpha$  is the coefficient of temperature expansion of semiconductor forbidden gap. Therefore from the equations follows that

$$n_i^2(T_1) = N_c(T_1)N_v(T_1)e^{-\frac{E_g(0)-\alpha T_1}{k_B T_1}} \quad \text{and} \quad n_i^2(T_2) = N_c(T_2)N_v(T_2)e^{-\frac{E_g(0)-\alpha T_2}{k_B T_2}}$$

$$\alpha = \frac{k_B T_1}{T_1 - T_2} \ln \frac{n_i^2(T_1)}{N_c(T_1)N_v(T_1)} - \frac{k_B T_2}{T_1 - T_2} \ln \frac{n_i^2(T_2)}{N_c(T_2)N_v(T_2)} = 5 \times 10^{-3} \text{ eV/K.}$$

**4b46.**

The condition of the electron reflection from the atomic planes is determined by Wolf-Breg formula  $2d \sin \theta = n\lambda$ . The distance between atomic planes is defined as  $d = a / (h^2 + k^2 + l^2)^{1/2} = a / (1^2 + 0^2 + 0^2)^{1/2} = a$ . Using the relation between electron energy and wave-length  $\varepsilon = p^2 / 2m = \hbar^2 / 2m\lambda^2$  one obtains  $\varepsilon = \hbar^2 n^2 / 2m(2d \sin \theta)^2$ . Therefore, in the case  $n = 1$  for the electron energy one obtains  $\varepsilon \cong 4.8 \text{ eV}$ .

**4b47.**

1. First find the densities of minority carriers:

$$p_n = \frac{(n_i)^2}{N_d} = \frac{(1.4 \cdot 10^{10})^2}{10^{15}} = 1.96 \cdot 10^5 \text{ cm}^{-3}, \quad n_p = \frac{(n_i)^2}{N_a} = \frac{(1.4 \cdot 10^{10})^2}{5 \cdot 10^{15}} = 3.9 \cdot 10^4 \text{ cm}^{-3},$$

2. Then, from Einstein relation, the coefficients of electron and hole diffusion:

$$D_n = \frac{kT}{q} \mu_n = 0.026 \cdot 1300 = 33.8 \text{ cm}^2/\text{s}, \quad D_p = \frac{kT}{q} \mu_p = 0.026 \cdot 500 = 13 \text{ cm}^2/\text{s},$$

3. The density of saturation current will be:

$$j_s = q \left( \frac{D_p p_n}{L_p} + \frac{D_n n_p}{L_n} \right) = 1.6 \cdot 10^{-19} \left( \frac{13 \cdot 1.96 \cdot 10^5}{0.006} + \frac{33.8 \cdot 3.9 \cdot 10^4}{0.01} \right) = 8.8 \cdot 10^{-11} \text{ A/cm}^2$$

4. And the photocurrent

$$I_F = S q F_0,$$

therefore

$$\frac{I_F}{I_s} = \frac{S q F_0}{S j_s} = \frac{q F_0}{j_s} = \frac{1.6 \cdot 10^{-19} \cdot 10^{18}}{8.8 \cdot 10^{-11}} \approx 2 \cdot 10^{10} \text{ A.}$$

5. Thus, for idle state voltage, there is:

$$V_{xx} = \left( \frac{kT}{q} \right) \ln \left( \frac{I_F}{I_s} + 1 \right) = 0.026 \cdot \ln(2 \cdot 10^{10}) = 0.62 \text{ V.}$$

**4b48.**

1. First find the densities of majority carriers in n and p domains:

$$n_n = N_C \exp \left[ -\frac{E_C - E_F}{kT} \right] = 2.8 \cdot 10^{19} \cdot \exp \left[ -\frac{0.2}{0.026} \right] = 1.28 \cdot 10^{16} \text{ cm}^{-3},$$

$$p_p = N_V \exp \left[ -\frac{E_F - E_V}{kT} \right] = 1.02 \cdot 10^{19} \cdot \exp \left[ -\frac{0.1}{0.026} \right] = 2.17 \cdot 10^{17} \text{ cm}^{-3};$$

2. Then n – p junction's contact potential difference:

$$\varphi_k = \frac{kT}{q} \ln \frac{p_p \cdot n_n}{n_i^2} = 0.026 \cdot \ln \frac{1.28 \cdot 10^{16} \cdot 2.17 \cdot 10^{17}}{2.56 \cdot 10^{20}} \approx 0.78 \text{ eV.}$$

**4b49.**

As in the proposed structure the junctions are connected to each other sequentially, are symmetric, therefore they have the same width, the general charge capacitance of the structure will be:

$$C = \frac{C_1 C_2}{C_1 + C_2},$$

where  $C_1 = C_2 = \frac{\epsilon\epsilon_0 S}{d/2}$ . Using  $C_{1,2}$  expression,  $C = \frac{\epsilon\epsilon_0 S}{d} \approx 10^{-12}$  F is obtained.

#### 4b50.

$p$ - $n$  junction barrier capacitance is given by  $C = \left[ \frac{\epsilon\epsilon_0 q N_a N_d}{2(N_a + N_d)} \right]^{1/2} * (V + \phi)^{-1/2} = K * (V + \phi)^{-1/2}$ ,

Hence the change of capacitance in the result of lowering  $V$  voltage twice will be given by

$$\frac{C_1}{C_2} = \frac{(2 + 0.6)^{-1/2}}{(1 + 0.6)^{-1/2}} = 0.784, \text{ or } C_2 = C_1 * 0.784 = 78.4 \text{ pF}$$

Answer: 78.4 pF.

#### 4b51.

For calculation of Miller indices it is necessary:

- To find intercepts of the given plane with the three basis axes  $x$ ,  $y$ ,  $z$  of cubic crystal;
- To take the reciprocals of these numbers;
- To reduce them to the smallest three integers having the same ratio.

For the given plane, there is:

- Intercepts are equal to:  $S_x = 2$ ;  $S_y = 1/2$ ;  $S_z = 1$ .
- Reciprocals values are equal to:  $1/2$ ;  $2$ ;  $1$
- Smallest three integers are equal to:  $1$ ;  $4$ ;  $2$ .

So, Miller indices of the given plane are as follows (142).

#### 4b52.

Determine the resistivity.

$$\rho = RS/l = 150 \cdot 1 \cdot 10^{-3} \cdot 2 \cdot 10^{-3} / (10 \cdot 10^{-3}) = 0.03 \text{ } \Omega \cdot \text{m}.$$

Resistivity of the  $n$ -type silicon sample is given by

$$\rho = [q(n\mu_n + p\mu_p)]^{-1} :$$

Substituting the corresponding values of the quantities from the problem condition the following is obtained

$$0.03 = [1.6 \cdot 10^{-19} (0.12 \cdot n + p \cdot 0.05)]^{-1} ,$$

or

$$0.12/n + 0.05p = 2.08 \cdot 10^{20} :$$

As  $np = n_i^2$ , then

$$p = \frac{n_i^2}{n} = \frac{(1.5 \cdot 10^{16})^2}{n} :$$

The following is obtained:

$$0.12n + 0.05(1.5 \cdot 10^{16})^2 / n = 2.08 \cdot 10^{20} ,$$

or

$$0.12n^2 + 0.05(1.5 \cdot 10^{16})^2 - 2.08 \cdot 10^{20} n = 0 :$$

Hence for majority carriers:

$$n = \frac{2.08 \cdot 10^{20} + \sqrt{(2.08 \cdot 10^{20})^2 - 4 \cdot 0.12 \cdot 0.05 \cdot (1.5 \cdot 10^{16})^2}}{2 \cdot 0.12} = 1.73 \cdot 10^{21} \text{ m}^{-3}.$$

As all the mixed atoms are ionized,  $N_d = n = 1.73 \cdot 10^{21} \text{ m}^{-3}$ .

#### 4b53.

For the resistivity of semiconductor:

$$\sigma = q(n\mu_n + p\mu_p) \text{ and } np = n_i^2$$

Thus

$$\frac{\sigma}{q} = n\mu_n + \frac{n_i^2 \mu_p}{n}$$

This expression has the minimum, when

$$\frac{d(\sigma/q)}{dn} = 0,$$

or when  $\mu_n - n_i^2 \mu_p / n^2 = 0$  or  $n = n_i \sqrt{\mu_p / \mu_n}$ .

Since  $\frac{d^2(\sigma/q)}{dn^2}$  the value of this expression is positive, then in this point  $n = n_i \sqrt{\mu_p / \mu_n}$  resistivity has the minimum value.

#### 4b54.

For the p-type silicon  $\sigma_p = q\rho_p\mu_p$ . From this expression for the hole densities in the p-region, the following is obtained:

$$p_p = \sigma_p / (q\mu_p) = 10^4 / (0,19 \cdot 1,6 \cdot 10^{-19}) = 3,29 \cdot 10^{23} \text{ m}^{-3}.$$

Similarly for the n-type silicon:

$$n_n = \sigma_n / (q\mu_n) = 100 / (0,39 \cdot 1,6 \cdot 10^{-19}) = 1,6 \cdot 10^{21} \text{ m}^{-3}$$

For the hole densities in the n-region:

$$p_n = n_i^2 / n_n = (2,5 \cdot 10^{19})^2 / (1,60 \cdot 10^{21}) = 3,91 \cdot 10^{17} \text{ m}^{-3}$$

Therefore the built-in potential is equal to:

$$\varphi_k = \frac{kT}{q} \ln\left(\frac{p_p}{p_n}\right) = \frac{1,38 \cdot 10^{-20} \cdot 300}{1,6 \cdot 10^{-19}} \ln\left(\frac{3,29 \cdot 10^{23}}{3,91 \cdot 10^{17}}\right) = 0,35 \text{ V}.$$

#### 4b55.

Let us find the diode DC current for the DC voltage ( $V=0,1\text{V}$ ) by the following formula

$$I = I_0 \exp\left(\frac{qV}{kT} - 1\right) = 25 \cdot 10^{-6} (\exp(1,6 \cdot 10^{-19} \cdot 0,1 / 1,38 \cdot 10^{-23} \cdot 300) - 1) = 1,17 \text{ mA}$$

In this case DC resistance of diode is equal to

$$R_0 = V / I = 0,1 / (1,17 \cdot 10^{-3}) = 85 \ \Omega.$$

#### 4b56.

For wet oxidation at 950°C

$$d_0^2 + Ad_0 = Bt$$

Rearranging the equation

$$d_0 = \frac{Bt}{d_0} - A$$

Thus, a plot of  $d_0$  versus  $t/d_0$  will give B as the slope and A as the intercept.

|                             |       |       |       |       |       |
|-----------------------------|-------|-------|-------|-------|-------|
| $d_0$ ( $\mu\text{m}$ )     | 0.041 | 0.100 | 0.128 | 0.153 | 0.177 |
| $t/d_0$ (h/ $\mu\text{m}$ ) | 2.683 | 3.000 | 3.125 | 3.268 | 3.390 |

From the plot, the intercept of the line yields  $A = 0.50 \ \mu\text{m}$ .

The slope of the line yields  $B = 0.2 \ \mu\text{m}^2\text{h}$ .

#### 4b57.

For a "limited source" condition with the total amount of impurities  $Q$ , the solution of Fick equation is given by the Gaussian function.

$$N(x,t) = \frac{Q}{\sqrt{\pi Dt}} \exp\left[-\left(\frac{x}{2\sqrt{Dt}}\right)^2\right]$$

$p-n$  junction is formed at the place, where boron concentration equals the concentration of impurities in the bulk silicon substrate, that is:

$$\frac{Q}{\sqrt{\pi Dt}} \exp\left[-\left(\frac{x_j}{2\sqrt{Dt}}\right)^2\right] = N_D$$

Solving this equation with respect to  $x_j$ ,

$$x_j = 2\sqrt{Dt} \sqrt{\ln\left[\frac{Q}{N_D \sqrt{\pi Dt}}\right]}$$

Substituting the numerical values of  $Q$ ,  $D$ ,  $N_D$  and  $t$ , into this expression yields

$$x_j = 2,7 \mu\text{m}.$$

#### 4b58.

The capacitance of quantum dot with a radius  $r$ :

$$C = 4\pi\epsilon\epsilon_0 r,$$

where  $\epsilon$  is the dielectric constant of the material, surrounding the quantum dot (for GaAs,  $\epsilon=13.2$ )  $\epsilon_0=8.85 \cdot 10^{-12}\text{F/m}$ . If there is one electron on the dot, due to the presence of the second electron, the electrostatic energy of the dot will increase by  $\Delta E=e^2/c$ . This means that the change in point's potential

$$\Delta V = \frac{\Delta E}{e} = \frac{e}{C} = \frac{e}{4\pi\epsilon\epsilon_0 r} = 11\text{mV}$$

Hence it follows that the change of gate voltage about 10mV will give rise to the step on the current-voltage characteristics, i.e. coulomb step of coulomb staircase is about of 10 mVr.

#### 4b59.

In the mode of photocurrent, the diode is under bias voltage and when lighting the current  $I=-(I_0+I_\Phi)$  flows through it where  $I_\Phi$  is photocurrent, which in the case of internal quantum yield equal to one is

$$I_\Phi = e \frac{p}{hv} = e \frac{p\lambda}{hc} = 6 \cdot 10^{-3}\text{A}.$$

Here  $e$  is the value of electron charge,  $c$  is the light speed,  $v$  is the light frequency.

$$\text{Thus, } I = -(I_0 + I_\Phi) = -(I_0 + \frac{ep}{hv}) \approx -6\text{mA} \quad (I_0 \ll I_\Phi).$$

In photoemf mode  $I = I_0(e^{\frac{eV_{xx}}{kT}}) - I_\Phi = 0$ , therefore

$$V_{xx} = \frac{kT}{e} \ln\left(1 + \frac{I_\Phi}{I_0}\right) \approx \frac{kT}{e} \ln \frac{I_\Phi}{I_0},$$

where  $k$  is the Boltzmann constant. Putting the numerical values, the following is obtained:

$$V_{xx} = 0.35\text{V}.$$

#### 4b60.

Estimate the value of field strength by considering the equilibrium conditions, when in each cross section of the base, current density of majority carriers is equal to zero.

$$j_n = j_{nD} + j_{nE} = eD_n \frac{dn}{dx} + e\mu_n E \cdot n(x) = 0.$$

Here  $D_n = \frac{kT}{e} \mu_n$  is the electron diffusion coefficient,  $\mu_n$  is mobility,  $kT$  is the quantum of thermal energy,  $e$  is the elementary charge,  $E$  is the electric field strength.

Using the local electro neutrality condition  $n_0(x) = N_D(x)$  from (1) it can be written as:

$$E(x) = -\frac{D_n}{\mu_n n(x)} \frac{dn(x)}{dx} = -\frac{kT}{e} \frac{1}{N_D(x)} \frac{dN_D(x)}{dx} = \frac{kT}{e} \frac{1}{L_0} = \text{const}(y).$$

Such an electrical field also affects the movement minority carriers / holes/, injected to the base. The drift time of their movement through the base is

$$t_{dr} = \frac{W}{\mu_p E} = \frac{WL_0}{D_p}.$$

And the diffusion transition time of holes is:  $t_{dif} = \frac{W^2}{2D_p}$ .

Dividing these values to each other the following is obtained:

$$\frac{t_{dr}}{t_{dif}} = \frac{WL_0}{D_p} \bigg/ \frac{W^2}{2D_p} = \frac{2L_0}{W}.$$



## 5. SEMICONDUCTOR TECHNOLOGY

### a) Test questions

|       |   |       |   |       |   |
|-------|---|-------|---|-------|---|
| 5a1.  | E | 5a21. | B | 5a42. | C |
| 5a2.  | D | 5a22. | C | 5a43. | C |
| 5a3.  | D | 5a23. | B | 5a44. | A |
| 5a4.  | D | 5a24. | C | 5a45. | A |
| 5a5.  | B | 5a25. | B | 5a46. | A |
| 5a6.  | C | 5a26. | A | 5a47. | C |
| 5a7.  | B | 5a27. | E |       |   |
| 5a8.  | B | 5a28. | D |       |   |
| 5a9.  | B | 5a30. | C |       |   |
| 5a10. | A | 5a31. | A |       |   |
| 5a11. | B | 5a32. | A |       |   |
| 5a12. | B | 5a33. | C |       |   |
| 5a13. | C | 5a34. | A |       |   |
| 5a14. | A | 5a35. | B |       |   |
| 5a15. | B | 5a36. | D |       |   |
| 5a16. | D | 5a37. | B |       |   |
| 5a17. | C | 5a38. | C |       |   |
| 5a18. | D | 5a39. | A |       |   |
| 5a19. | C | 5a40. | A |       |   |
| 5a20. | C | 5a41. | E |       |   |

## b) Problems

### 5b1.

$$\rho_p = 10^{-4} \text{ Ohm}\cdot\text{m}$$

$$\rho_n = 10^{-2} \text{ Ohm}\cdot\text{m}$$

$$\mu_p = 0,05 \text{ m}^2 \text{V}^{-1} \text{ s}^{-1}$$

$$\mu_n = 0,13 \text{ m}^2 \text{V}^{-1} \text{ s}^{-1}$$

$$n_i = 1,38 \times 10^{16} \text{ m}^{-3}$$

$$T = 300 \text{ K}$$

$$\varphi_c = ?$$

The contact potential is obtained from  $\varphi_c = (kT/q) \ln p_p/p_n = 0,0258 \ln p_p/p_n$

The densities of holes and electrons are obtained using the specific resistance expressions:

$$p_p = \frac{1}{\rho_p q \mu_p} = \frac{1}{10^{-4} \cdot 1,6 \cdot 10^{-19} \cdot 0,05} = 1,25 \cdot 10^{24} \text{ m}^{-3}$$

$$n_n = \frac{1}{\rho_n q \mu_n} = \frac{1}{10^{-2} \cdot 1,6 \cdot 10^{-19} \cdot 0,13} = 4,2 \cdot 10^{21} \text{ m}^{-3}$$

The holes' density in n region is obtained using the mass action law:

$$p_n = \frac{n_i^2}{n_n} = \frac{2 \cdot 10^{32}}{4,2 \cdot 10^{21}} = 4,8 \cdot 10^{10} \text{ m}^{-3}$$

The contact potential of p-n junction

$$\varphi_c = 0,0258 \ln p_p/p_n = \ln 1,25 \times 10^{24} / 4,8 \times 10^{10} = 0,8 \text{ V.}$$

### 5b2.

$$\sigma_p = 100 \text{ S/cm}$$

$$\mu_p = 1900 \text{ cm}^2 \text{V}^{-1} \text{ s}^{-1}$$

$$T = 300 \text{ K}$$

$$n_i = 2,5 \times 10^{13} \text{ atom/cm}^3$$

$$p_p, n_p = ?$$

Using the expression of the specific conductance

$\sigma_p = 1/q\mu_p p_p$ , the holes' density

$$p_p = \frac{\sigma_p}{q\mu_p} = \frac{100}{1,6 \cdot 10^{-19} \cdot 1900} = 3,29 \cdot 10^{17} \text{ cm}^{-3}$$

The concentration of electrons is obtained using the mass action law:

$$n_p p_p = n_i^2$$
$$n_p = \frac{n_i^2}{p_p} = \frac{6,25 \cdot 10^{26}}{3,29 \cdot 10^{17}} = 1,9 \cdot 10^9 \text{ cm}^{-3}$$

### 5b3.

$$N_d = 10^{17} \text{ atom/cm}^3$$

$$L = 100 \text{ }\mu\text{m}$$

$$W = 10 \text{ }\mu\text{m}$$

$$d = 1 \text{ }\mu\text{m}$$

$$\mu_n = 1500 \text{ cm}^2 \text{V}^{-1} \text{ s}^{-1}$$

$$T = 300 \text{ K}$$

$$R \text{ and } \rho_s = ?$$

Assuming that the impurities are fully ionized, the electrons concentration  $n = N_d$

The specific resistance

$$\rho_n = 1/\sigma = 1/qn\mu_n, \rho_n = 1/1,6 \times 10^{-19} \times 10^{17} \times 1500 = 0,042 \text{ Ohm}\cdot\text{cm}$$

The resistance

$$R = \rho_n L / W \cdot d = 0,042 \times 100 \cdot 10^{-4} / 10 \times 10^{-4} \times 1 \times 10^{-4} = 4,2 \text{ kOhm.}$$

The sheet resistance

$$\rho_s = \rho_n / d = 0,042 / 1 \times 10^{-4} = 420 \text{ Ohm/mm.}$$

### 5b4.

$$C_{ox} = 100 \text{ nF/cm}^2 = 100 \times 10^{-9} \text{ F/cm}^2$$

$$\epsilon_{SiO_2} = 3,9 \times 8,85 \times 10^{-14} \text{ F/cm}$$

$$t_{ox} = ?$$

Calculate the thickness of SiO<sub>2</sub> layer by the intrinsic formula of MOS-capacitor's permittivity.

$$C_{ox} = \epsilon_{SiO_2} / t_{ox} \text{ and } t_{ox} = \epsilon_{SiO_2} / C_{ox}$$

$$t_{ox} = 3,9 \times 8,85 \times 10^{-14} / 100 \times 10^{-9}$$

$$t_{ox} = 3,45 \times 10^{-6} \text{ cm} = 0,0345 \text{ } \mu\text{m}.$$

To grow the high quality gate oxide, the dry oxidation process will be used.

**5b5.**

The gate capacitance is defined as

$$C = C_{ox} S = (\epsilon_0 \epsilon_{ox} / t_{ox}) WL$$

After the scaling by the  $\alpha$  factor

$$C_{sc} = (\epsilon_0 \epsilon_{ox} / t_{ox} / \alpha) (W / \alpha) L / \alpha = (\epsilon_0 \epsilon_{ox} / t_{ox}) WL / \alpha = C / \alpha.$$

$$\text{For } \alpha = 2, C_{sc} = C / 2.$$

The capacitance will decrease by factor two.

**5b6.**

For direct voltage, the current density:

$$\text{when } T=T_1 \quad j_1 \rightarrow \exp\left(-\frac{E_g - qV}{kT_1}\right) = \exp\left(-\frac{0,7 - 0,4 \cdot 1,6 \cdot 10^{-19}}{0,022}\right) = 1,2 \cdot 10^{-6} \text{ A/cm}^2.$$

$$\text{when } T=T_2 \quad j_2 \rightarrow \exp\left(-\frac{E_g - qV}{kT_2}\right) = \exp\left(-\frac{0,7 - 0,4 \cdot 1,6 \cdot 10^{-19}}{0,026}\right) = 9,7 \cdot 10^{-6} \text{ A/cm}^2.$$

$$\text{therefore } \frac{j_2}{j_1} = 8,08.$$

**5b7.**

Near the drain, the width of p-n junction when the voltage applied to drain is  $V = 0,1V$

$$d_1 = \left(\frac{2\epsilon\epsilon_0 \cdot \phi_f + V}{q \cdot N_i}\right)^{1/2} = \left(\frac{2 \cdot 12 \cdot 8,86 \cdot 10^{-14} \cdot 0,6 + 0,1}{1,6 \cdot 10^{-19} \cdot 10^{15}}\right)^{1/2} = 9,64 \cdot 10^{-5} \text{ cm}$$

$$\text{and in case of voltage absence: } d_2 = \left(\frac{2\epsilon\epsilon_0 \cdot \phi_f}{q \cdot N_i}\right)^{1/2} = 8,93 \cdot 10^{-5} \text{ cm}$$

therefore, near the drain, the channel will narrow by  $d_1 - d_2 = 0,71 \cdot 10^{-5} \text{ cm}$ .

**5b8.**

Saturation voltage of the transistor

$$V_{DSsat} = V_{GS} - V_t = 3 - 1 = 2V.$$

$$V_{DS} > V_{DSsat}$$

therefore the transistor operates in saturation mode. In this mode the drain current of the transistor equals:

$$I_D = 0,5 \mu_n C_{ox} (W/L) (V_{GS} - V_t)^2.$$

Calculate

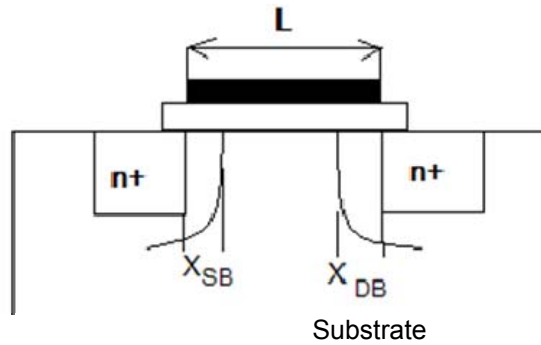
$$C_{ox} = (\epsilon_0 \epsilon_{SiO_2}) / t_{ox} = (3,9 \times 8,85 \times 10^{-14}) / 10 \times 10^{-7} = 3,45 \times 10^{-7} \text{ F/cm}^2.$$

$$I_D = 0,5 \times 300 \times 3,45 \times 10^{-7} (3-1)^2 \times (10/1) = 2,07 \text{ mA}.$$

Transconductance  $g_m = d I_D / d V_{GS}$

$$g_m = \mu_n C_{ox} (W/L) (V_{GS} - V_t) = 300 \times 3,45 \times 10^{-7} \times 10 \times 2 = 2 \text{ mA}$$

5b9.



$$L_{sum} = L - X_{SB}(V_S) - X_{DB}(V_D)$$

By data, \$L\_{eff} = 0\$, hence

$$X_{DB}(V_D) = L - X_{SB}(V_S)$$

Use the width formula of depletion region:

$$2\epsilon_0 \epsilon_{Si} (\phi_c - V_D) / q N_{sub} = (L - X_{SB}(0))^2$$

$$V_D = \phi_c - q N_{sub} (L - X_{SB}(0))^2 / 2\epsilon_0 \epsilon_{Si}$$

Calculate

$$\phi_c = \phi_T \ln N_{sub} N_{don} / n_i^2$$

In a room temperature

$$\phi_T \approx 0,026 \text{ V and } \phi_c \approx 0,935 \text{ V.}$$

$$X_{SB}(0) = \sqrt{2\epsilon_0 \epsilon_{Si} \phi_c / q N_{sub}}$$

$$X_{SB}(0) = \sqrt{2 \times 8,85 \times 10^{-14} \times 11,8 \times 0,935 / 1,6 \times 10^{-19} \times 10^{16}}$$

$$X_{SB}(0) \approx 0,349 \times 10^{-4} \text{ cm.}$$

Put and calculate

$$|V_D| \approx 2,3 \text{ V.}$$

5b10.

Density of electronic current

$$J_n = e D_n dN/dx$$

According to Einstein formula:

$$D_n = \phi_T \mu_n \text{ and } dN/dx = k$$

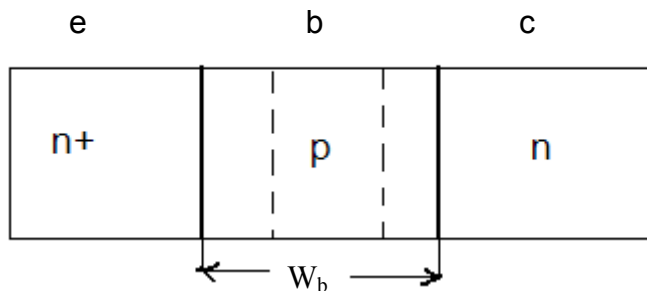
$$J_n = e \phi_T \mu_n k$$

In a room temperature

$$\phi_T = 0,026 \text{ V}$$

$$J_n = 1,6 \cdot 10^{-19} \times 0,026 \times 1200 \times 8 \times 10^{18} = 39,936 \text{ A/cm}^2.$$

5b11.



a. Calculate the difference of contact potentials in E-B and C-B p-n junctions. At a room temperature

$$\phi_T \approx 0,026 \text{ V}$$

$$\phi_{ce} = \phi_T \ln N_e N_b / n_i^2 \quad \phi_{ce} = 0,026 \ln 10^{19} \times 10^{15} / 2,25 \times 10^{20} \approx 0,8759 \text{ V}$$

$$\phi_{cc} = \phi_T \ln N_b N_c / n_i^2 \quad \phi_{cc} = 0,026 \ln 10^{16} \times 5 \times 10^{15} / 2,25 \times 10^{20} \approx 0,7 \text{ V}$$

b. Calculate E-B depletion layer width when the external voltage = 0.

$$X_{eb}(0) = \sqrt{2\epsilon_0 \epsilon_{Si} \phi_{ce} / q N_b} \approx 3,38 \times 10^{-5} \text{ cm} = 0,338 \text{ um.}$$

c. For base punch-through, \$X\_{cb}\$ depletion layer width must be

$$X_{cb} = 0,6 - 0,338 = 0,262 \text{ um.}$$

d. The voltage of the collector can be obtained from the following equation:

$$X_{cb} = \sqrt{2\epsilon_0 \epsilon_{Si} N_c (\varphi_{cc} - V) / q N_b (N_b + N_c)}.$$

Hence

$$V = \varphi_{cc} - X_{cb}^2 q N_b (N_b + N_c) / 2\epsilon_0 \epsilon_{Si} N_c \quad \text{and} \quad |V| \approx 0,8765 \text{ V}$$

**5b12.**

As  $N_d > N_{acc}$ , so silicon has an n – type conductivity. The active concentration of the impurities is  $N_{act} = N_{don} - N_{acc} = 9 \times 10^{16} \text{ cm}^{-3}$ . At room temperature  $T = 300 \text{ K}$  all impurity atoms are ionized,  $N_{act} \approx N_{don} = n$ . The specific conductivity of the sample at room temperature

$$\begin{aligned} \sigma_n &= e n \mu_n \\ \sigma_n &= 1,6 \times 10^{-19} \text{ C} \times 9 \times 10^{16} \text{ cm}^{-3} \times 1400 \text{ cm}^2/\text{V}\cdot\text{s} \\ \sigma_n &= 20,16 \text{ (Ohm}\cdot\text{cm)}^{-1}. \end{aligned}$$

**5b13.**

The time delay in the  $n^+$  polysilicon interconnect line  $t = RC$ . The interconnect line resistance  $R = \rho_{\square} l/b$ :

$$R = 200 \times 1 \times 10^3 / 1 = 200 \text{ kOhm.}$$

The interconnect line capacitance to the substrate  $C = C_0 S$ ,  $C = 60 \times 10^3 \times 1 = 60000 \text{ aF}$ . The time delay

$$t = 200 \times 10^3 \times 60000 \times 10^{-18} = 12 \text{ ns.}$$

**5b14.**

The time delay in the metal interconnect line  $t = RC$ . The resistance of the metal 1 interconnect line  $R = \rho_{\square} l/b$

$$R = 0,1 \times 1 \times 10^3 / 0,2 = 500 \text{ Ohm}$$

The capacitance of the line to the substrate  $C = C_0 S$ .

$$C = 23 \times 10^3 \times 0,2 = 4600 \text{ aF.}$$

$$\text{The time delay } t = RC = 500 \times 4600 \times 10^{-18} = 2,3 \text{ ps.}$$

**5b15.**

The p-n junction barrier layer capacitance  $C = (\epsilon_0 \epsilon_{Si} / x_{p-n}) S_{p-n}$ , where  $x_{p-n}$  is the depletion width of the p-n junction. For the one-sided p-n junction  $x_{p-n}$  is defined as

$$x_{p-n} \approx x_n = \sqrt{2\epsilon_0 \epsilon_{Si} (\varphi_{li} - U) / e N_d}$$

$$x_{p-n} = \sqrt{2 \times 8,85 \times 10^{-14} \times 11,8 \times (0,7 - 0) / 1,6 \times 10^{-19} \times 10^{16}} = 3 \times 10^{-5} \text{ cm, when } U = 0 \text{ V.}$$

$$x_{p-n} = \sqrt{2 \times 8,85 \times 10^{-14} \times 11,8 \times (0,7 + 5) / 1,6 \times 10^{-19} \times 10^{16}} = 8,6 \times 10^{-5} \text{ cm, when } U = -5 \text{ V.}$$

The barrier layer capacitance

$$C_0 = (\epsilon_0 \epsilon_{Si} / x_{p-n}) S_{p-n} = (8,85 \times 10^{-14} \times 11,8 / 3 \times 10^{-5}) \times 10^{-5} = 34,8 \times 10^{-14} \text{ F} = 0,348 \text{ pF.}$$

$$C_5 = (\epsilon_0 \epsilon_{Si} / x_{p-n}) S_{p-n} = (8,85 \times 10^{-14} \times 11,8 / 8,6 \times 10^{-5}) \times 10^{-5} = 12,14 \times 10^{-14} \text{ F} = 0,12 \text{ pF.}$$

## 6. NUMERICAL METHODS AND OPTIMIZATION

### a) Test questions

|       |   |       |   |       |   |
|-------|---|-------|---|-------|---|
| 6a1.  | E | 6a34. | A | 6a67. | C |
| 6a2.  | D | 6a35. | A | 6a68. | C |
| 6a3.  | D | 6a36. | D | 6a69. | B |
| 6a4.  | B | 6a37. | B | 6a70. | C |
| 6a5.  | B | 6a38. | D | 6a71. | C |
| 6a6.  | A | 6a39. | C | 6a72. | C |
| 6a7.  | A | 6a40. | D | 6a73. | B |
| 6a8.  | B | 6a41. | B | 6a74. | D |
| 6a9.  | A | 6a42. | D | 6a75. | D |
| 6a10. | A | 6a43. | B | 6a76. | D |
| 6a11. | A | 6a44. | D | 6a77. | B |
| 6a12. | C | 6a45. | A | 6a78. | A |
| 6a13. | C | 6a46. | A | 6a79. | B |
| 6a14. | D | 6a47. | A | 6a80. | B |
| 6a15. | C | 6a48. | C |       |   |
| 6a16. | A | 6a49. | D |       |   |
| 6a17. | D | 6a50. | A |       |   |
| 6a18. | D | 6a51. | B |       |   |
| 6a19. | B | 6a52. | C |       |   |
| 6a20. | B | 6a53. | D |       |   |
| 6a21. | B | 6a54. | B |       |   |
| 6a22. | A | 6a55. | B |       |   |
| 6a23. | A | 6a56. | A |       |   |
| 6a24. | D | 6a57. | B |       |   |
| 6a25. | C | 6a58. | C |       |   |
| 6a26. | E | 6a59. | D |       |   |
| 6a27. | D | 6a60. | A |       |   |
| 6a28. | C | 6a61. | B |       |   |
| 6a29. | C | 6a62. | C |       |   |
| 6a30. | B | 6a63. | B |       |   |
| 6a31. | D | 6a64. | D |       |   |
| 6a32. | D | 6a65. | A |       |   |
| 6a33. | E | 6a66. | E |       |   |

## b) Problems

### 6b1.

Define low and upper values of the game and their corresponding optimal strategies. There is:

$$\alpha = \max_i \min_j h_{ij}, \alpha=2, i=3, j=1;$$

$$\beta = \max_j \min_i h_{ij}, \beta=3, i=1, j=1:$$

As  $\alpha \neq \beta$ , i.e. there is mixed strategy matrix game in case of which vector elements of  $P=(p_1, p_2, p_3)^T$  probabilities are the probabilities of selecting A side corresponding strategy.  $Q=(q_1, q_2, q_3)^T$  vector elements are the ones of selecting B side corresponding strategy.

$$\alpha \leq V \leq \beta,$$

$$\begin{cases} 3p_1 - p_2 + 2p_3 \geq V, & 3q_1 - 2q_2 + 4q_3 \leq V, \\ -2p_1 + 4p_2 + 2p_3 \geq V, & -q_1 + 4q_2 + 2q_3 \leq V, \\ 4p_1 + 2p_2 + 6p_3 \geq V, & 2q_1 + 2q_2 + 6q_3 \leq V, \\ p_1 + p_2 + p_3 = 1, & q_1 + q_2 + q_3 = 1, \end{cases}$$

In the result of solving the system the following is obtained:

$$\begin{array}{lll} p_1=0, & p_2=0, & p_3=1; \\ q_1=2/5, & q_2=3/5, & q_3=0; \\ & V=2. & \end{array}$$

### 6b2.

Define  $\Phi(t, t_0)$  by matrix method.

There is:

$$M(A(t))_{3 \times 3} = E_{3 \times 3} + Q^1(A(t))_{3 \times 3} =$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} \int_2^1 t dt & \int_2^1 t^2 dt & \int_2^1 t^3 dt \\ \int_2^1 1 dt & \int_2^1 t dt & \int_2^1 (2-t) dt \\ \int_2^1 t^2 dt & \int_2^1 t dt & \int_2^1 t dt \end{bmatrix} = \begin{bmatrix} \left(\frac{t^2}{2} - 1\right) & \left(\frac{t^3}{3} - \frac{8}{3}\right) & \left(\frac{t^4}{4} - 4\right) \\ (t-1) & \left(\frac{t^2}{2} - 1\right) & \left(2t - \frac{t^2}{2} - 2\right) \\ \left(\frac{t^3}{3} - \frac{8}{3}\right) & \left(\frac{t^2}{2} - 2\right) & (t-1) \end{bmatrix} = \Phi(t, t_0).$$

### 6b3.

There is

$$H = \begin{bmatrix} 0 & 0.4472 & 0.4851 \\ 0 & 0 & 0.4851 \\ 1 & 0.8944 & 0.7276 \end{bmatrix},$$

The opposite of which

$$H^{-1} = \begin{bmatrix} -2 & 0.5 & 1 \\ 2.2361 & -2.2361 & 0 \\ 0 & 2.0616 & 0 \end{bmatrix}.$$

Therefore

$$\Phi(t) = H \cdot \text{diag}(e^{-\lambda_i}) \cdot H^{-1} = \begin{bmatrix} e^{-2t} & (-e^{-2t} + e^{-3t}) & 0 \\ 0 & e^{-3t} & 0 \\ 2(e^{-2t} - e^{-t}) & \left(\frac{1}{2}e^{-t} 2e^{-2t} + \frac{3}{2}e^{-3t}\right) & e^t \end{bmatrix}.$$

### 6b4.

There is

$$\lambda_1=4.4142, \lambda_2=1.5858, \lambda_3=0;$$

$$H = \begin{bmatrix} -0.3366 & -0.5615 & -0.2182 \\ -0.4760 & 0.7941 & -0.8729 \\ -0.8125 & 0.2326 & 0.4364 \end{bmatrix}, H^{-1} = \begin{bmatrix} -0.6731 & -0.2380 & -0.8125 \\ -1.1230 & 0.3971 & 0.2326 \\ -0.6547 & -0.6547 & 0.6547 \end{bmatrix}.$$

Therefore

$$\text{diag}(A) = H \cdot A \cdot H^{-1} = \begin{bmatrix} 4.4142 & 0 & 0 \\ 0 & 1.5858 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

**6b5.**

As the constant term of characteristic polynomial (P3) is equal to the determinant of matrix, therefore it will be

$$P_3 = \det \begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & 2 \\ -1 & 0 & 1 \end{bmatrix} = 1 \cdot 1 \cdot 1 + 2 \cdot 2 = 5.$$

**6b6.**

As

$$\|A\|_2 = \left( \sum_{i=1}^3 \sum_{j=1}^3 a_{ij}^2 \right)^{1/2}$$

then

$$\|A\|_2 = \sqrt{1^2 + (-2)^2 + 1^2 + 2^2 + (-1)^2 + 1^2} = \sqrt{12}$$

**6b7.**

As coefficient of characteristic polynomial term containing  $\lambda^2$  is  $P_1 = -\sum_{i=1}^3 \lambda_i$ ,

$$P_1 = -(1 + 2 + 3) = -6:$$

**6b8.**

As characteristic polynomial coefficient of the term which contains  $\lambda^2$

$$P_1 = \sum_{i,j=1}^4 \lambda_i \cdot \lambda_j,$$

then

$$P_2 = \lambda_1 \lambda_2 + \lambda_1 \lambda_3 + \lambda_1 \lambda_4 + \lambda_2 \lambda_3 + \lambda_2 \lambda_4 + \lambda_3 \lambda_4 = 0 \cdot 1 + 0 \cdot 2 + 0 \cdot 3 + 1 \cdot 2 + 1 \cdot 3 + 2 \cdot 3 = 11.$$

**6b9.**

It is known that linear interpolation error is defined by the following formula:

$$R_2 = \frac{1}{8} M_2 h^2,$$

where  $h$  is the length of the step,  $M_2 = \max_{x \in [0,1000]} |f''(x)|$ . As  $f(x) = \log x$ , then  $M_2 = 1$ . Therefore, in

order to provide the necessary accuracy,  $h$  should be taken  $\sqrt{0.008} \approx 0.089$ .

Taking into consideration that  $f''(x) = -x^{-2}$ , i.e. for large  $x$ -es the derivative is small, then dividing the range into parts, it is possible to enlarge the division step. For example, divide  $[0,1000]$  range into  $[0,10]$ ,  $[10,100]$ ,  $[100,1000]$  ranges. In that case,  $h_1$  division step in the 1<sup>st</sup> range equals 0.089 (as  $M_{2,1} = \max_{x \in [0,10]} |f''(x)| = M_2 = 1$ ). In the 2<sup>nd</sup> range  $M_{2,2} = 0.01$ , therefore,  $h_2$  step is defined from

$\frac{1}{8} \cdot 0.01 \cdot h_2^2 < 0.001$  non-equation, i.e.  $h_2 = 0.894$ . Similarly, in the 3<sup>rd</sup> range  $M_{2,3} = 0.0001$ , and the step

equals  $h_3 = 8.94$ . Thus, in order to provide the anticipated accuracy it is necessary to take approximately 100 points in each part. Other types of divisions of initial range can also be observed. The minimum number of points can be obtained by changing length of the next range in each step, i.e. in  $k^{\text{th}}$  step  $h_k$  is selected by

the following formula:  $8^{-1} |x_k|^{-2} h_k^2 < 0.001$ ,  $x_{k+1} = x_k + h_k$ .



**6b10.**

The formula of generalized trapeziums, applied for the given integral, is  $\frac{h}{2} \left( \frac{1}{b^2} + \frac{e}{1+b^2} + 2 \sum_{i=1}^{n-1} \frac{e^{x_i}}{x_i^2 + b^2} \right)$ . In

the given case the error is defined by inaccuracy formula of generalized trapeziums' formula  $\frac{M_2}{12} (1-0)h^2$ ,

where  $h = x_{i+1} - x_i$  -  $[0,1]$  is interval division step. Noticing  $M_2 = O(b^{-4})$ , it is seen that the given formula does not operate in case of small  $b$ -s. Therefore, for  $O(h^2)$  class provision it is necessary to divide  $[0,1]$  part into subparts and to use smaller division step in 0 range. For example, by dividing  $[0,1]$  into two subparts -  $[0,l]$  and  $[l,1]$ ,  $h_1$  division step in the first range should be selected in a way that  $h_1 \approx hb^2$ , and in the second range  $h_2 \approx h(l^2 + b^2)$ . Other forms of division can also be discussed.

**6b11.**

As  $\frac{f(x)}{\sqrt{x}}$  function has uniqueness in 0 point, it is not possible to apply squarization formula at once. For that

reason divide  $[0,1]$  part into  $[0,10^{-8}]$  and  $[10^{-8},1]$  ranges. The integral with the first range allows

$\left| \int_0^{10^{-8}} \frac{f(x)}{\sqrt{x}} dx \right| = |f(c)| \cdot 2\sqrt{10^{-8}} \approx C \cdot 10^{-4}$  mark ( $0 < c < 10^{-8}$ ). This mark allows ignoring the first integral and

calculating only the integral with the second range, where the integrating function has no uniqueness and therefore, squarization formula is possible to apply. But as  $\sqrt{x}$  accepts small values in that range, it is desirable to improve the integrating function in advance by using integration in parts of

$\int_{10^{-8}}^1 \frac{f(x)}{\sqrt{x}} dx = \int_{10^{-8}}^1 f(x) d2\sqrt{x} = 2f(1) - 2f(10^{-8})10^{-4} - \int_{10^{-8}}^1 2\sqrt{x} f'(x) dx$ . For the final integral, the formula of

generalized rectangular can be applied, the error of which is estimated by  $M_1 \frac{h}{4} (1-10^{-8})$  formula where

$M_1 \leq m_1 \cdot 10^2$  ( $m_1 = \max_{x \in [0,1]} |f'(x)|$ ). In the last inequality  $|f''(x)| \leq 1$  condition has been used. Eventually,  $h \approx 10^{-6}$ .

**6b12.**

Notice that  $x^3 - 20x + 1 = 0$  equation is solved which has three  $z_1 < z_2 < z_3$  real roots. Write the formula of

iteration process in the following view  $x_{n+1} - x_n = \frac{x_n^3 - 20x_n + 1}{20}$ . From this it follows that in  $x_0 < z_1$  or

$x_0 > z_3$  case, the method diverges (the difference in the left part is monotonous);  $x_0 = z_1$  and  $x_0 = z_3$

points are static; in  $z_1 < x_0 < z_3$  case the method converges  $z_2$ .

**6b13.**

The constants  $\alpha$  and  $\beta$  cannot be found uniquely, because the term 'smallest error' is not defined in the formulation of problem. First, it must be defined exactly what 'the smallest error' means. For example, the distance between functions  $f$  and  $g$  can be defined as  $\|f - g\| = \max_{x \in [-10,10]} |f(x) - g(x)|$  (uniform norm). In

this case the best approximating linear function will be  $y = 2.5$  (it follows from Chebyshev theorem). On the

other hand, the function  $y = \frac{\arctg 100}{20} \approx \frac{\pi}{40}$  gives the best approximation, when the distance between

functions  $f$  and  $g$  is defined as  $\|f - g\|_2 = \sqrt{\int_{-10}^{10} |f(x) - g(x)|^2 dx}$  (root-mean-square norm). Also the weighting

norms  $\|f - g\|_{2,p} = \sqrt{\int_{-10}^{10} |f(x) - g(x)|^2 p(x) dx}$  may be considered where the weighting function  $p$  is a fixed positive function. In this case the coefficients  $\alpha, \beta$  will depend on  $p$ .

**6b14.**

The integration interval is not bounded; therefore, the multiple-application rectangle rule cannot be applied immediately. Assume the function  $f$  on interval  $[1, \infty)$  is bounded by the constant  $C : |f(x)| \leq C$ . The integral can be represented as:

$$\int_1^{\infty} \frac{f(x) dx}{1+x^2} = \int_1^{1.5C\varepsilon^{-1}} \frac{f(x) dx}{1+x^2} + \int_{1.5C\varepsilon^{-1}}^{3C\varepsilon^{-1}} \frac{f(x) dx}{1+x^2} + \int_{3C\varepsilon^{-1}}^{\infty} \frac{f(x) dx}{1+x^2} \equiv I_1 + I_2 + I_3$$

The estimate of the function  $f$  implies inequalities  $|I_2| < \frac{\varepsilon}{3}$  and  $|I_3| < \frac{\varepsilon}{3}$ . Applying multiple-application rectangle rule for the first integral, this is obtained:

$$I_1 \approx \tilde{I}_1 = h \sum_{k=1}^N \frac{f(x_k)}{1+x_k^2}, \text{ where } N = \frac{1.5C}{h\varepsilon}.$$

The reminder term of this formula is the following:

$$R_1 = \frac{M_1(1.5C - \varepsilon)h}{2\varepsilon}, \text{ where } M_1 = \max_{x \geq 1} \left| \left( \frac{f(x)}{1+x^2} \right)' \right|$$

Hence for  $h = \frac{4\varepsilon^2}{9CM_1}$  the necessary accuracy is obtained:

$$\left| \int_1^{\infty} \frac{f(x) dx}{1+x^2} - \tilde{I}_1 \right| < \varepsilon.$$

Notice that the obtained step  $h$  is too small, so in this case non-equidistant intervals can be used. The rectangle rule for non-equidistant intervals takes this form:

$$\tilde{I}_2 = \sum_{k=1}^N \frac{f(x_k)}{1+x_k^2} (x_{k+1} - x_k),$$

with the reminder term

$$R_1 = \sum_{k=1}^N \frac{M_k (x_{k+1} - x_k)^2}{2}$$

Taking into account that

$$M_k = \max_{x \in [x_k, x_{k+1}]} \left| \left( \frac{f(x)}{1+x^2} \right)' \right| \leq \frac{M}{x_k^2} + \frac{C}{x_k^4}, \text{ where } M = \max_{x \geq 1} |f'(x)|,$$

this is obtained

$$R_1 \leq \frac{M_0}{2} \sum_{k=1}^N \frac{1}{x_k^2} (x_{k+1} - x_k)^2, \text{ where } M_0 = \max(M, C).$$

Thus, taking the partition points  $x_k$  such that  $R_1 < \frac{\varepsilon}{3}$ , the final formula is obtained.

**6b15.**

The polynomial  $P_3$  is represented as

$P_3(x) = (a_0 + a_1(x - x_1))(x - x_2)^2 + (a_2 + a_3(x - x_2))(x - x_1)^2 \equiv P_{3,1}(x) + P_{3,2}(x)$ . The advantage of this representation is the following. For the function  $P_{3,k}$  there is  $P_{3,k}(x_j) = P'_{3,k}(x_j) = 0$  where  $j \neq k, j, k = 1, 2$ .

Using the conditions  $P_3^{(k)}(x_1) = P_{3,1}^{(k)}(x_1) = f^{(k)}(x_1)$ , where  $k = 0, 1, a_0 = \frac{f(x_1)}{(x_2 - x_1)^2}$ ,

$a_1 = \frac{f'(x_1)}{(x_1 - x_2)^2} - \frac{2f(x_1)}{(x_1 - x_2)^3}$  is obtained. The same formulas exist for  $a_2$  and  $a_3$ . Finally, the polynomial  $P_3$  is presented as follows:

$$P_3(x) = \left( f(x_1) + \left( f_1'(x_1) - \frac{2f(x_1)}{x_1 - x_2} \right) (x - x_1) \right) \frac{(x - x_2)^2}{(x_1 - x_2)^2} + \\ + \left( f(x_2) + \left( f_2'(x_2) - \frac{2f(x_2)}{x_2 - x_1} \right) (x - x_2) \right) \frac{(x - x_1)^2}{(x_2 - x_1)^2}$$

Using the same considerations the needed formula can be obtained for arbitrary number of the points  $x_k$ .

#### 6b16.

There are many ways to solve this problem. For example, using successive approximations  $x_{k+1} = \frac{x_k^4 + 1}{10}$ ,  $x_0 = 0$ , the root of the equation on the interval  $[0,1]$  with prescribed accuracy can be found after four steps. If the bisection method on the same interval is used, seven steps will be necessary.

#### 6b17.

Substituting  $t = x - 1$ , the problem is reduced to the equivalent problem of definition of quadratic function  $G(t) = a_1 t^2 + b_1 t + c_1$ , which gives the best approximation for the function

$$F(t) = \frac{1}{2+t} + \frac{1}{2-t}$$

in  $[-1,1]$  interval. Taking into account, that  $F$  function is even in symmetric interval, and defining distance between two functions as  $D_1 = \max_{[-1,1]} |F(t) - G(t)|$ , or  $D_2 = \int_{-1}^1 (F(t) - G(t))^2 dt$ ,  $b_1 = 0$  is obtained. Consider,

for example,  $D_1$  distance case. It is represented as:

$$D_1 = \max_{[-1,1]} |F(t) - G(t)| = \max_{t \in [-1,1]} \left| \frac{4}{4-t^2} - a_1 t^2 - c_1 \right| = \max_{z \in [0,1]} \left| \frac{4}{4-z} - a_1 z - c_1 \right|.$$

As the function  $\frac{4}{4-z}$  is convex in  $[0,1]$  interval and denoting  $L = \min_{a_1, c_1} D_1$ , the system for the definition of the best approximation polynomial of first order is obtained.

$$\begin{cases} \frac{4}{4-0} - c_1 = L \\ \frac{4}{4-1} - a_1 - c_1 = L \\ \frac{4}{(4-z_0)^2} - a_1 = 0 \\ \frac{4}{4-z_0} - a_1 z_0 - c_1 = -L \end{cases}$$

Here  $0, z_0, 1$  points are Chebyshev's alternance points. Solving that system this is obtained:

$$a_1 = \frac{1}{3}, \quad c_1 = \frac{4\sqrt{3}-1}{6}, \quad L = \frac{7-4\sqrt{3}}{6}.$$

So, the best approximation polynomial will be  $y = \frac{1}{3}(x-1)^2 + \frac{4\sqrt{3}-1}{6}$ , with minimal distance

$$L = \min D_1 = \frac{7-4\sqrt{3}}{6} \approx 0.012.$$

The case of distance  $D_2$  is easier. Also, one can consider the case of weighted approximation, when

$$D_\rho = \int_{-1}^1 (F(t) - G(t))^2 \rho(t) dt, \quad \text{where } \rho \text{ is the given positive function.}$$

**6b18.**

Calculating  $\Delta_n$ ,  $\Delta_{n+1} = 2\Delta_n - \Delta_{n-1}$  recurrent formula for  $n > 1$  is obtained. Solving it,  $\Delta_n = C_1 + C_2 n$  is obtained, where  $C_1$  and  $C_2$  are unknown constants. As  $\Delta_1 = 2$  and  $\Delta_2 = 3$ ,  $\Delta_n = n + 1$ . So,

$$\lim_{n \rightarrow \infty} \frac{\Delta_n}{n} = 1 \text{ is obtained.}$$

**6b19.**

One of possible ways. Assume the equation of  $AB$  line is  $l(x, y) \equiv ax + by + c = 0$ . This line splits the plane into two half-planes:  $\{(x, y) : l(x, y) > 0\}$  and  $\{(x, y) : l(x, y) < 0\}$ . If the point  $D$  lies in triangle's interior, then substituting the coordinates of that and  $C$  point into  $l(x, y)$ , values of the same sign will be obtained (i.e.  $D$  and  $C$  belong to the same half-plane). The same considerations can be done for  $BC$  and  $AC$  lines. An arbitrary polygon case can be reduced to triangle case by partitioning and checking if  $D$  point lies in one of partition triangles.

**6b20.**

The polynomial  $P_4$  is represented as:

$$P_4 = (x - x_2)(a_0 + a_1(x - x_1) + a_2(x - x_1)^2 + a_3(x - x_1)^3) + (x - x_1)^4 b_0.$$

Substituting that polynomial into the given equalities, the system for determination of unknown  $a_k$  and  $b_0$  is obtained:

$$(x_2 - x_1)^4 b_0 = y_4, \begin{cases} (x_1 - x_2)a_0 = y_0 \\ a_0 + (x_1 - x_2)a_1 = y_1 \\ 2a_1 + 2(x_1 - x_2)a_2 = y_2 \\ 6a_2 + 6(x_1 - x_2)a_3 = y_3 \end{cases}.$$

The general case may be considered analogously.

**6b21.**

As characteristic polynomial's absolute term ( $P_3$ ) equals to determinant of matrix, then it will be

$$P_3 = \det \begin{bmatrix} 1 & -3 & 0 \\ 0 & 1 & 2 \\ -1 & 0 & 1 \end{bmatrix} = 1 \cdot 1 \cdot 1 + 3 \cdot 2 = 7.$$

**6b22.**

As

$$\|A\|_2 = \left( \sum_{i=1}^3 \sum_{j=1}^3 a_{ij}^2 \right)^{1/2} = \sqrt{1^2 + (-3)^2 + 1^2 + 2^2 + (-1)^2 + 1^2} = \sqrt{17}.$$

**6b23.**

As the coefficient of first degree of  $\lambda$  in the characteristic polynomial  $P_1 = \sum_{i=1}^3 \lambda_i$ , then  $P_1 = 1 + 4 + 2 = 7$ :

**6b24.**

A As the coefficient of first degree of  $\lambda^2$  in the characteristic polynomial  $P_1 = \sum_{i,j=1}^4 \lambda_i \cdot \lambda_j$ , then

$$P_2 = \lambda_1 \lambda_2 + \lambda_1 \lambda_3 + \lambda_1 \lambda_4 + \lambda_2 \lambda_3 + \lambda_2 \lambda_4 + \lambda_3 \lambda_4 = 4 \cdot 1 + 4 \cdot 2 + 4 \cdot 3 + 1 \cdot 2 + 1 \cdot 3 + 2 \cdot 3 = 35$$

**6b25.**

There are five linearly independent conditions for determination of the  $P_n$  polynomial coefficients. Therefore, the minimal value of  $n$  for which the problem has a solution for arbitrary values  $y_j$  and  $z_k$  cannot be less

than four. Forth order polynomial which satisfies given conditions, can be found explicitly, similar to Lagrange interpolation polynomial. First, find basic polynomials  $\Phi_j$  and  $\Psi_k$ , satisfying conditions:

$$\begin{aligned} \Phi_j^{(i)}(1) &= \delta_{ij}, \quad i = 0,1,2, & \Phi_j^{(q)}(0) &= 0, \quad q = 0,1, & \text{for } j &= 0,1,2 \\ \Psi_k^{(i)}(1) &= 0, \quad i = 0,1,2, & \Psi_k^{(q)}(0) &= \delta_{kq}, \quad q = 0,1 & \text{for } k &= 0,1. \end{aligned}$$

Here  $\delta_{ij}$  is a Kronecker symbol ( $\delta_{ij} = 0$  for  $i \neq j$  and  $\delta_{ii} = 1$ ). After that, the required polynomial is found by the formula

$$P_4(x) = \sum_{j=0}^2 y_j \Phi_j(x) + z_0 \Psi_0(x) + z_1 \Psi_1(x).$$

Show how to construct basic polynomials. Consider, for example,  $\Phi_1$ .

Search this polynomial in the form

$$\Phi_1(x) = x^2(x-1)(a(x-1)+b).$$

Then the constants  $a, b$  will be determined from the conditions  $\Phi_1'(1) = 1$ ,  $\Phi_1''(1) = 0$ , or

$b = 1$ ,  $2a + 4b = 0$ , from which  $a = -2$ ,  $b = 1$ . The remaining basic polynomials can be found analogously.

The polynomials  $P_4$  can be found also directly, by writing it down as:  $P_4(x) = x^2(a(x-1)^2 + b(x-1) + c) + (x-1)^3(dx + e)$ , and determining coefficients  $a, b, c, d, e$  from problem conditions by constants  $y_j$  and  $z_k$ .

#### 6b26.

As the function  $f(x) = x^3$  is odd, and an interval  $[-1, 1]$  is symmetric relative to origin, then the required polynomial has to be an odd function, too. Hence,  $P_1(x) = ax$ , where the constant  $a$  must be determined. For the determination of this constant, consider the function  $g(x) = f(x) - P_1(x) = x^3 - ax$  on the interval  $[0, 1]$ . There is  $g(0) = 0$  and  $g(1) = 1 - a$ . The function  $g$  is convex, therefore  $a$  can be found from the condition, that the minimal value of function  $g$  at some point  $x_0$  of open interval  $(0, 1)$  is equal to  $g(1) = 1 - a$  with opposite sign. Then in the interval  $[-1, 1]$  four points ( $z_1 = -1, z_2 = -x_0, z_3 = x_0, z_4 = 1$ ) are obtained, where  $f(z_k) - P_1(z_k) = (-1)^k \|f - P_1\|$ , so, by the Chebyshev theorem, the polynomial  $P_1 = ax$  will be the required polynomial of best approximation. So:

$g'(x) = 3x^2 - a$ , from which  $x_0 = \sqrt{\frac{a}{3}}$ . Further,  $g\left(\sqrt{\frac{a}{3}}\right) = -\frac{2}{3}a\sqrt{\frac{a}{3}}$ , and therefore the constant  $a$  can be found

from equation  $1 - a = \frac{2}{3}a\sqrt{\frac{a}{3}}$ . Solving this equation,  $a = \frac{3}{4}$  is obtained. Finally, the polynomial of best approximation is  $P_1(x) = \frac{3}{4}x$ . This problem can be solved easier, using geometric considerations.

#### 6b27.

Assume  $\lambda$  is eigenvalue of  $A$  matrix. In that case  $\det(A - \lambda E) = 0$  ( $E$  is unit matrix). If  $A$  matrix satisfies

$A^n = 0$  condition, where  $n$  is a natural number, then there is

$$-\lambda^n E = A^n - \lambda^n E = (A - \lambda E)(A^{n-1} + \lambda A^{n-2} + \lambda^2 A^{n-3} + \dots + \lambda^{n-1} E),$$

therefore, using the properties of determinants

$$-\lambda^n = \det(-\lambda^{n-1} E) = \det(A - \lambda E) \det(A^{n-1} + \lambda A^{n-2} + \lambda^2 A^{n-3} + \dots + \lambda^{n-1} E) = 0$$

i.e.  $\lambda = 0$ . Thus if  $A^n = 0$ , where  $n$  is a natural number, the eigenvalue of  $A$  matrix equals zero.

**6b28.**

Substituting 1 in the expansion of the logarithm function alternating series  $\ln 2 = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{1}{k}$  is obtained

which converges very slowly as, according to Leibniz theorem,  $|\ln 2 - S_N| \leq \frac{1}{N+1}$ , where  $S_N$  is a partial

sum of the given series. Therefore, to provide the necessary accuracy it is necessary to take not less than 1000 terms of the series which leads to significant increase of round-off error. Therefore it is better to apply

the following method. Define  $t$  number from equation  $\frac{1+t}{1-t} = 2$ .  $t = \frac{1}{3}$  is obtained hence, the seeking number is presented as:

$$\ln 2 = \ln\left(1 + \frac{1}{3}\right) - \ln\left(1 - \frac{1}{3}\right) = \sum_{k=0}^{\infty} \left((-1)^{k+1} + 1\right) \frac{1}{k3^k}$$

This series in the right part may be estimated by geometrical progression with  $\frac{1}{3}$  denominator. Therefore in

order to provide the necessary accuracy it is enough to take only 8 summands of the given series. The same method may be applied again. Other methods are also possible.

**6b29.**

As the absolute term ( $P_3$ ) of characteristic polynomial equals to determinant of matrix, then it will be

$$P_3 = \det \begin{bmatrix} 2 & -6 & 0 \\ 0 & 2 & 4 \\ -2 & 0 & 2 \end{bmatrix} = 2 \cdot 2 \cdot 2 + (-6) \cdot (-2) \cdot 4 = 56 :$$

**6b30.**

As the coefficient of  $\lambda^2$  in the characteristic polynomial is equal to  $P_2 = \sum_{\substack{i,j=1 \\ i \neq j}}^4 \lambda_i \cdot \lambda_j$ , then

$$\begin{aligned} P_2 &= \lambda_1 \lambda_2 + \lambda_1 \lambda_3 + \lambda_1 \lambda_4 + \lambda_2 \lambda_3 + \lambda_2 \lambda_4 + \lambda_3 \lambda_4 = \\ &= 2 \cdot 0.5 + 2 \cdot 1 + 2 \cdot 1.5 + 0.5 \cdot 1 + 0.5 \cdot 1.5 + 1 \cdot 1.5 = \\ &= 1 + 2 + 3 + 0.5 + 0.75 + 1.5 = 8.75 : \end{aligned}$$

**6b31.**

As the characteristic equation looks as follows,

$$|\lambda E - A| = \begin{vmatrix} \lambda + 2 & 0 \\ -1 & \lambda + 1 \end{vmatrix} = (\lambda + 1) \cdot (\lambda + 2) = \lambda^2 + 3\lambda + 2 = 0,$$

the eigenvalues  $\lambda_1 = -2, \lambda_2 = -1$ . The eigenvector, which corresponds to  $\lambda_1$ , has to be found from the homogeneous linear system  $\mathbf{h}_1 = (h_{11}, h_{21})^T$ :

$$\begin{bmatrix} -2 & 0 \\ 1 & -1 \end{bmatrix} \cdot \begin{pmatrix} h_{11} \\ h_{21} \end{pmatrix} = -2 \cdot \begin{pmatrix} h_{11} \\ h_{21} \end{pmatrix},$$

and in the same way, the eigenvector  $\mathbf{h}_2 = (h_{12}, h_{22})^T$ , which corresponds to  $\lambda_2$ , may be found from

$$\begin{bmatrix} -2 & 0 \\ 1 & -1 \end{bmatrix} \cdot \begin{pmatrix} h_{12} \\ h_{22} \end{pmatrix} = -1 \cdot \begin{pmatrix} h_{12} \\ h_{22} \end{pmatrix},$$

hence  $h_{12} = 0$ , and for  $h_{22}$ ,  $h_{22} = 1$  can be chosen.

Solving these systems, the following is obtained:

$$\begin{pmatrix} h_{11} \\ h_{21} \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \begin{pmatrix} h_{12} \\ h_{22} \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

So transform matrix is

$$H = \begin{bmatrix} -1 & 0 \\ 1 & 1 \end{bmatrix}$$

for which  $H^{-1} \equiv H$ .

Finally

$$\Phi(t) = H \cdot \begin{bmatrix} e^{\lambda_1 t} & 0 \\ 0 & e^{\lambda_2 t} \end{bmatrix} \cdot H^{-1} = \begin{bmatrix} e^{-2t} & 0 \\ e^{-t} - e^{-2t} & e^{-t} \end{bmatrix}.$$

$$\Phi(t) = \begin{bmatrix} e^{-2t} & 0 \\ e^{-t} - e^{-2t} & e^{-t} \end{bmatrix}$$

**6b32.**

It is obvious that

$$B = \begin{bmatrix} 0 & 0.5 \\ 0.5 & 0 \\ 0.5 & 1 \end{bmatrix}, \quad A \cdot B = \begin{bmatrix} 0.5 & 1.25 \\ 0.25 & 0.25 \\ 0.5 & 0.5 \end{bmatrix}, \quad A^2 \cdot B = \begin{bmatrix} 0.75 & 1.125 \\ 0.375 & 0.75 \\ 0.375 & 0.375 \end{bmatrix} :$$

Therefore

$$\text{rang} L_x = \text{rang} [B : AB : A^2 B] = \left[ \begin{array}{cc|cc|cc} 0 & 0.5 & 0.5 & 1.25 & 0.75 & 1.125 \\ 0.5 & 0 & 0.25 & 0.25 & 0.375 & 0.75 \\ 0.5 & 1 & 0.5 & 0.5 & 0.375 & 0.375 \end{array} \right] = 3,$$

then the system is fully controllable.

**6b33.**

Representing the function  $f$  by the first order Taylor formula in the point  $\frac{a+b}{2}$ , the following is obtained:

$$f(x) = f\left(\frac{a+b}{2}\right) + \frac{f'\left(\frac{a+b}{2}\right)}{1!} \left(x - \frac{a+b}{2}\right) + \frac{f''(c)}{2!} \left(x - \frac{a+b}{2}\right)^2, \quad \text{where } c \in (a, b).$$

Substituting this expansion in the integral  $I$ , the following inequality is obtained:

$$|I - \tilde{I}| = \left| \int_a^b \left( f(x) - f\left(\frac{a+b}{2}\right) \right) dx \right| = \left| \int_a^b \frac{f''(c)}{2!} \left(x - \frac{a+b}{2}\right)^2 dx \right| \leq \frac{M_2}{2!} \int_a^b \left(x - \frac{a+b}{2}\right)^2 dx = \frac{M_2}{24} (b-a)^3, \quad \text{where}$$

$M_2 = \max_{x \in [a, b]} |f''(x)|$ . The final expression is the seeking estimation.

**6b34.**

The main part of the proof is the following: taking convexity of the function  $e^x$  into account, inequalities  $\tilde{I}_1 < I < \tilde{I}_2$  are obtained. After that, substituting values  $I, \tilde{I}_1, \tilde{I}_2$ , the required inequality is proved.

**6b35.**

$n$  times apply Rolle's theorem first to the function  $f$ , then to the derivatives of this function.

**6b36.**

The function  $g(x) = \frac{1}{1+x}$  is not a contraction on the half-line  $[0, \infty)$ , hence modify the successive approximation process, for example, in the following way. Applying the successive approximations formula, this is obtained:

$$x_{n+2} = \frac{1+x_n}{2+x_n}, \quad n \geq 0.$$

The function  $g_1(x) = \frac{1+x}{2+x}$  is a contraction (because of  $|g_1'(x)| = \frac{1}{(2+x)^2} \leq \frac{1}{4} < 1$  for  $x \geq 0$ ), therefore the

sequences  $\{x_{2k}\}_{k=1}^{\infty}$  and  $\{x_{2k+1}\}_{k=1}^{\infty}$  converge to the same limit, which is a limit of the sequence  $\{x_k\}_{k=1}^{\infty}$ . This

limit is a positive root of the equation  $x^2 + x - 1 = 0$ , that is equal to  $\frac{\sqrt{5}-1}{2}$ .

**6b37.**

Apply standard approximate formulas (rectangles formulas, trapezoidal formulas and so on), but in this case determine the maximal value of the modules of the derivative of the function  $\frac{\sin x}{x}$  for the estimation of the remainder term. Therefore it is better to expand the function  $\sin x$  in a Taylor series, divide by  $x$  and then integrate it term by term. Finally,

$$I = \sum_{k=0}^{\infty} (-1)^k \frac{1}{2^k (2k+1)!(2k+1)}.$$

It is alternate series; therefore, its remainder is less than the first omitted term. The integral will be calculated with prescribe accuracy if only two first summands are obtained.

**6b38.**

As the absolute term ( $P_3$ ) of characteristic polynomial equals to determinant of matrix, then it will be

$$P_3 = \det \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 4 \\ -2 & 0 & 2 \end{bmatrix} = 2 \cdot 2 \cdot 2 + 1 \cdot (-2) \cdot 4 = 0.$$

**6b39.**

As the coefficient of  $\lambda^2$  in the characteristic polynomial is equal to  $P_2 = \sum_{\substack{i,j=1 \\ i \neq j}}^4 \lambda_i \cdot \lambda_j$ , then

$$\begin{aligned} P_2 &= \lambda_1 \lambda_2 + \lambda_1 \lambda_3 + \lambda_1 \lambda_4 + \lambda_2 \lambda_3 + \lambda_2 \lambda_4 + \lambda_3 \lambda_4 = \\ &= 2 \cdot 0.5 + 2 \cdot 1 + 2 \cdot 3 + 0.5 \cdot 1 + 0.5 \cdot 3 + 1 \cdot 3 = \\ &= 1 + 2 + 6 + 0.5 + 1.5 + 3 = 14 : \end{aligned}$$

**6b40.**

As the characteristic equation looks as follows:

$$|\lambda E - A| = \begin{vmatrix} \lambda + 1 & 0 \\ -2 & \lambda + 2 \end{vmatrix} = (\lambda + 1) \cdot (\lambda + 2) = \lambda^2 + 3\lambda + 2 = 0,$$

the eugenvalues  $\lambda_1 = -1, \lambda_2 = -2$ . The eugenvector, which corresponds to  $\lambda_1$ , has to be found from the homogeneous linear system  $\mathbf{h}_1 = (h_{11}, h_{21})^T$ .

$$\begin{bmatrix} -1 & 0 \\ 2 & -2 \end{bmatrix} \cdot \begin{pmatrix} h_{11} \\ h_{21} \end{pmatrix} = -1 \cdot \begin{pmatrix} h_{11} \\ h_{21} \end{pmatrix},$$

hence  $h_{11} = 1, h_{21} = 2$ .

and in the same way, the eugenvector  $\mathbf{h}_2 = (h_{12}, h_{22})^T$  which corresponds to  $\lambda_2$ , may be found from

$$\begin{bmatrix} -1 & 0 \\ 2 & -2 \end{bmatrix} \cdot \begin{pmatrix} h_{12} \\ h_{22} \end{pmatrix} = -2 \cdot \begin{pmatrix} h_{12} \\ h_{22} \end{pmatrix},$$

hence  $h_{12} = 0, h_{22} = 1$ .

So, transform matrix is

$$H = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

for which  $H^{-1} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$ .

Finally



$$\begin{aligned}\Phi(t) &= H \cdot \begin{bmatrix} e^{\lambda_1 t} & 0 \\ 0 & e^{\lambda_2 t} \end{bmatrix} \cdot H^{-1} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} e^{-t} & 0 \\ 0 & e^{-2t} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} = \\ &= \begin{bmatrix} e^{-t} & 0 \\ 2e^{-t} & e^{-2t} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} e^{-t} & 0 \\ (2e^{-t} - 2e^{-2t}) & e^{-2t} \end{bmatrix} . \\ \Phi(t) &= \begin{bmatrix} e^{-t} & 0 \\ (2e^{-t} - 2e^{-2t}) & e^{-2t} \end{bmatrix} .\end{aligned}$$

**6b41.**

It is obvious that

$$B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 1 & 2 \end{bmatrix}, \quad A \cdot B = \begin{bmatrix} 2 & 5 \\ 1 & 1 \\ 2 & 2 \end{bmatrix}, \quad A^2 \cdot B = \begin{bmatrix} 6 & 9 \\ 3 & 6 \\ 3 & 3 \end{bmatrix}.$$

Therefore

$$\text{rang} L_x = \text{rang}[B : AB : A^2 B] = \left[ \begin{array}{cc|cc|cc} 0 & 1 & 2 & 5 & 6 & 9 \\ 1 & 0 & 1 & 1 & 3 & 6 \\ 1 & 2 & 2 & 2 & 3 & 3 \end{array} \right] = 3,$$

The system is fully controllable.

## 7. DISCRETE MATHEMATICS AND THEORY OF COMBINATIONS

### a) Test questions

|       |   |       |   |       |   |
|-------|---|-------|---|-------|---|
| 7a1.  | E | 7a26. | D | 7a50. | C |
| 7a2.  | A | 7a27. | B | 7a51. | B |
| 7a3.  | B | 7a28. | C | 7a52. | C |
| 7a4.  | A | 7a29. | E | 7a53. | C |
| 7a6.  | A | 7a30. | A | 7a54. | B |
| 7a7.  | C | 7a31. | E | 7a55. | C |
| 7a8.  | E | 7a32. | D | 7a56. | C |
| 7a9.  | E | 7a33. | E | 7a57. | B |
| 7a10. | D | 7a34. | B | 7a58. | B |
| 7a11. | C | 7a35. | A | 7a59. | B |
| 7a12. | D | 7a36. | C | 7a60. | B |
| 7a13. | A | 7a37. | D | 7a61. | C |
| 7a14. | B | 7a38. | B | 7a62. | D |
| 7a15. | A | 7a39. | A | 7a63. | C |
| 7a16. | C | 7a40. | B | 7a64. | B |
| 7a17. | D | 7a41. | C | 7a65. | C |
| 7a18. | D | 7a42. | B | 7a66. | B |
| 7a19. | E | 7a43. | B |       |   |
| 7a20. | A | 7a44. | C |       |   |
| 7a21. | E | 7a45. | B |       |   |
| 7a22. | A | 7a46. | E |       |   |
| 7a23. | B | 7a47. | E |       |   |
| 7a24. | B | 7a48. | D |       |   |
| 7a25. | E | 7a49. | C |       |   |

**b) Problems**

**7b1.**

As  $\xi_1=w, \xi_2=w, \xi_3=w$ , but  $\xi_1+\xi_2=w, \xi_1+\xi_3=w$  and  $\xi_2+\xi_3=w$ , therefore the corresponding threshold function will be:

$$x_1x_2 \vee x_1x_3 \vee x_2x_3,$$

to get Zhegalkin polynomial of which it is enough to use the following equation:

$$a \vee b = a \oplus b \oplus ab,$$

$$x_1x_2 \vee x_1x_3 \vee x_2x_3 = (x_1x_2 \oplus x_1x_3 \oplus x_1x_2x_3) \vee x_2x_3 = x_1x_2 \oplus x_1x_3 \oplus x_1x_2x_3 \oplus x_2x_3 \oplus (x_1x_2 \oplus x_1x_3 \oplus x_1x_2x_3) * x_2x_3 = x_1x_2 \oplus x_1x_3 \oplus x_1x_2x_3 \oplus x_2x_3 \oplus x_1x_2x_3 \oplus x_2x_3 \oplus x_1x_2x_3 \oplus x_1x_2x_3 = x_1x_2 \oplus x_1x_3 \oplus x_2x_3.$$

**7b2.**

To verify the wholeness of the system it is enough to use Post theorem, i.e. find out if the system is fully included in any of classes of  $T_0, T_1, S, M, L$ ?

- It is not included in  $T_0$  as  $x_1 \rightarrow x_2 \notin T_0$ ;
- It is not included in  $T_1$  as  $x_1x_2x_3 \notin T_1$ ;
- It is not included in  $S$  as  $x_1 \vee x_2 \notin S$  (the functions depending on two variables are not self-dual at all);
- It is not included in  $M$  as  $x_1 \rightarrow x_2 \notin M$  (00 set proceeds 10 set, whereas  $0 \rightarrow 0 = 1$  and  $1 \rightarrow 0 = 0$ , i.e. monotony condition is violated);
- It is not included in  $L$  as  $x_1 \oplus x_2 \oplus x_1x_2$  function is  $x_1 \vee x_2$  Zhegalkin polynomial and contains sum of variables, i.e. it is not linear -  $x_1 \vee x_2 \notin L$

According to Post theorem, the system is complete.

**7b3.**

It is clear  $\bar{x} \notin T_0, \bar{x} \notin T, 1 \notin S$  (as its table consists only of 1s and self-duality condition is violated – anti-symmetry towards middle line),  $\bar{x} \notin M$  (as  $0 \uparrow 1$ , but  $\bar{0} > \bar{1}$ ),  $(x_1 \rightarrow x_2) \rightarrow x_3 \notin L$ , as  $a \rightarrow b = \bar{a} \vee b$ , therefore  $(x_1 \rightarrow x_2) \rightarrow x_3 = (x_1 \vee x_2) \rightarrow x_3 = \overline{x_1 \vee x_2} \vee x_3 = x_1 \bar{x}_2 \vee x_3$ , and from  $a \vee b = a \oplus b \oplus ab$  equation it follows that  $x_1 \bar{x}_2 \vee x_3 = x_1x_2 \oplus x_3 \oplus x_1x_2x_3 = x_1(1 \oplus x_2) \oplus x_3 \oplus x_1x_3(1 \oplus x_2) = x_1 \oplus x_1x_2 \oplus x_3 \oplus x_1x_3 \oplus x_2x_3 \notin L$ .

According to Post theorem, the system is complete.

**7b4.**

Search the solution in the form of indefinite coefficients.

| $x_1$ | $x_2$ | $x_3$ | $(x_1 \rightarrow x_2)^{\bar{x}_3}$ |
|-------|-------|-------|-------------------------------------|
| 0     | 0     | 0     | 1                                   |
| 0     | 0     | 1     | 0                                   |
| 0     | 1     | 0     | 1                                   |
| 0     | 1     | 1     | 0                                   |
| 1     | 0     | 0     | 0                                   |
| 1     | 0     | 1     | 1                                   |
| 1     | 1     | 0     | 1                                   |
| 1     | 1     | 1     | 0                                   |

$$(x_1 \rightarrow x_2)^{\bar{x}_3} = a_0 \oplus a_1x_1 \oplus a_2x_2 \oplus a_3x_3 \oplus a_{12}x_1x_2 \oplus a_{13}x_1x_3 \oplus a_{23}x_2x_3 \oplus a_{123}x_1x_2x_3$$

$$1 = a_0$$

$$0 = a_3 \oplus 1, a_3 = 1$$

$$1 = 1 \oplus a_2, a_2 = 0$$

$$0 = 1 \oplus 1 \oplus a_{23}, a_{23} = 0$$

$$0 = 1 \oplus a_1, a_1 = 1$$

$$1 = 1 \oplus 1 \oplus 1 \oplus a_{13}, a_{13} = 0$$

$$1 = 1 \oplus 1 \oplus a_{12}, a_{12} = 1$$

$$0 = 1 \oplus 1 \oplus 1 \oplus 1 \oplus a_{123}, a_{123} = 0$$

Answer  $1 \oplus x_1 \oplus x_3 \oplus x_1x_2$ .

**7b5.**

Applying “divide to own” standard tactics, it is possible to get the solution of the problem by the following method:

- a. Counting the number of units in couples
- b. Summing couples
- c. Counting the number of units in tetrads
- d. Summing tetrads
- e. The same continues for objects, 16-bit, 32-bit and other sequences
- f. At the end of the process the number contains the number of units.

An example of solution for 16-bit number:

```

Count (n)
 n = (n & 0x5555) + ((n >> 1) & 0x5555)
 n = (n & 0x3333) + ((n >> 2) & 0x3333)
 n = (n & 0x0F0F) + ((n >> 4) & 0x0F0F)
 n = (n & 0x00FF) + ((n >> 8) & 0x0F0F)
return n;

```

**7b6.**

Taking binary arithmetic property into consideration, that  $N \& (N-1)$  reduces the amount of units in the number by 1, which follows from the fact that  $N-1$  makes all zeros at the end of the number into 1, and junior class unit - 0, leaving all high classes without change. Now, applying this action in the cycle, the amount of units in the number can be counted.

```

Count (n)
C = 0;
while (n <> 0)
 n = (n & (n-1))
 C = C + 1
Return C

```

**7b7.**

Applying partition procedure, which is applied in QuickSort family algorithms, it is possible to get the following linear algorithm:

```

select(a, k, left, right)
pivotNewIndex = partition(a, left, right, pivotIndex)
while (k <> pivotNewIndex)
 if k < pivotNewIndex
 right = pivotNewIndex-1
 else
 left = pivotNewIndex + 1
 pivotNewIndex = partition(a, left, right, pivotIndex)
return k;

```

**7b8.**

Heap building is implemented in  $O(N \log N)$  period of time. Accordingly, applying the following algorithm, the elements can be classified in  $O(N \log N)$  period of time. The first (minimum or maximum) element is selected, substituted by the latter, after which the last element is shifted down its position in the heap, using ShiftDown standard procedure which requires  $O(\log N)$  time. Applying N-1 procedure, sorted array in  $O(N \log N)$  period of time is obtained.

**7b9.**

Disjunctive normal form of this function having the mentioned table will be:

$$f = \bar{x}_1 \bar{x}_2 \bar{x}_3 \vee \bar{x}_1 \bar{x}_2 x_3 \vee \bar{x}_1 x_2 x_3 \vee x_1 \bar{x}_2 \bar{x}_3 \vee x_1 x_2 \bar{x}_3 \vee x_1 x_2 x_3$$

| x <sub>1</sub> | x <sub>2</sub> | x <sub>3</sub> | f |
|----------------|----------------|----------------|---|
| 0              | 0              | 0              | 1 |
| 0              | 0              | 1              | 1 |
| 0              | 1              | 0              | 0 |
| 0              | 1              | 1              | 1 |
| 1              | 0              | 0              | 1 |
| 1              | 0              | 1              | 0 |
| 1              | 1              | 0              | 1 |
| 1              | 1              | 1              | 1 |

Applying the first part of Qwine algorithm ("assembling"), the following will be obtained:

$$\bar{x}_1\bar{x}_2\bar{x}_3 \vee \bar{x}_1\bar{x}_2x_3 \vee \bar{x}_1x_2x_3 \vee x_1\bar{x}_2\bar{x}_3 \vee x_1x_2\bar{x}_3 \vee x_1x_2x_3 \vee \bar{x}_1\bar{x}_2 \vee \bar{x}_2\bar{x}_3 \vee \bar{x}_1x_3 \vee x_2x_3 \vee x_1\bar{x}_3 \vee x_1x_2$$

Applying the second part of Qwine algorithm ("absorption"), the following will be obtained:

$$\bar{x}_1\bar{x}_2 \vee \bar{x}_2\bar{x}_3 \vee \bar{x}_1x_3 \vee x_2x_3 \vee x_1\bar{x}_3 \vee x_1x_2$$

**7b10.**

Search Zhegalkin polynomial in the following form:

$$f(x_1, x_2, x_3) = a_0 \oplus a_1x_1 \oplus a_2x_2 \oplus a_3x_3 \oplus a_{12}x_1x_2 \oplus a_{13}x_1x_3 \oplus a_{23}x_2x_3 \oplus a_{123}x_1x_2x_3$$

| x <sub>1</sub> | x <sub>2</sub> | x <sub>3</sub> | f(x <sub>1</sub> ,x <sub>2</sub> ,x <sub>3</sub> ) |
|----------------|----------------|----------------|----------------------------------------------------|
| 0              | 0              | 0              | 1                                                  |
| 0              | 0              | 1              | 1                                                  |
| 0              | 1              | 0              | 0                                                  |
| 0              | 1              | 1              | 1                                                  |
| 1              | 0              | 0              | 1                                                  |
| 1              | 0              | 1              | 1                                                  |
| 1              | 1              | 0              | 0                                                  |
| 1              | 1              | 1              | 1                                                  |

Substituting the right and left parts of the equation, sequentially all the possible values to x<sub>1</sub>, x<sub>2</sub> and x<sub>3</sub> variables, this will be obtained:

$$\begin{aligned} 1 &= a_0 \\ 1 &= 1 \oplus a_3, a_3 = 0 \\ 1 &= 1 \oplus a_2, a_2 = 0 \\ 1 &= 1 \oplus 1 \oplus a_{23}, a_{23} = 1 \\ 1 &= 1 \oplus a_1, a_1 = 0 \\ 1 &= 1 \oplus a_{13}, a_{13} = 0 \\ 1 &= 1 \oplus 1 \oplus a_{12}, a_{12} = 0 \\ 1 &= 1 \oplus 1 \oplus 1 \oplus a_{123}, a_{123} = 0 \end{aligned}$$

Putting the obtained values of the coefficients, Zhegalkin polynomial of the function will be obtained.

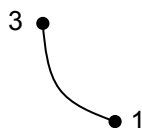
$$f(x_1, x_2, x_3) = 1 \oplus x_2 \oplus x_2x_3$$

**7b11.**

As the length of Cayley h(G)=(3,5,4,4,5,6,7,8) vertex is equal 8, therefore the number of tree nodes equals 8+2=10. The numbers of those nodes are:

$$(1,2,3,4,5,6,7,8,9,10)$$

From those numbers choose the number from the left which is missing in Cayley code. That number is 1. Connect the node of 1 number with number one node of the vertex by edge.

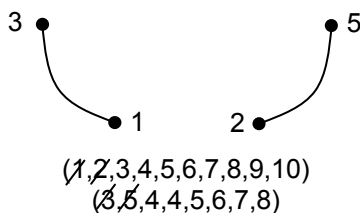


Delete 1 and 3 from the list of the number of nodes and vertex. The following will be obtained.

$$(\cancel{1}, 2, 3, 4, 5, 6, 7, 8, 9, 10)$$

$$(\cancel{3}, 5, 4, 4, 5, 6, 7, 8)$$

Do the same action after delete with the numbers of nodes and those numbers of nodes written in the code. This time connect the nodes of 2 and 5 numbers by edge and 2 and 5 numbers will be deleted from the list. This will be obtained:

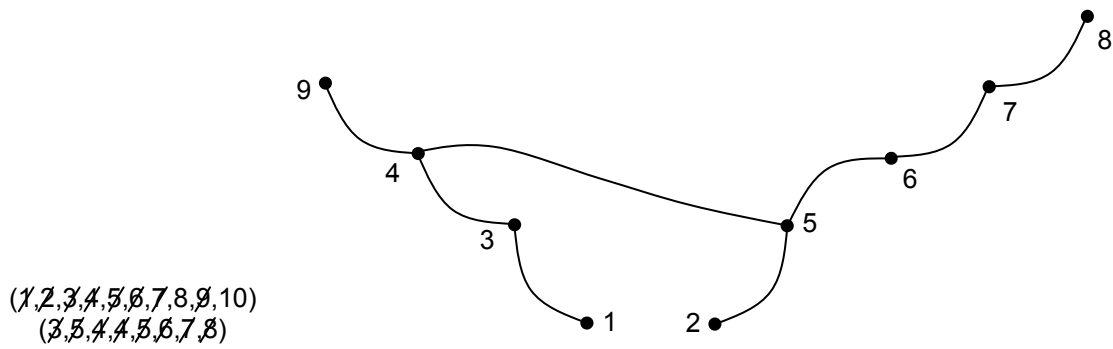


$$(\cancel{1}, \cancel{2}, 3, 4, 5, 6, 7, 8, 9, 10)$$

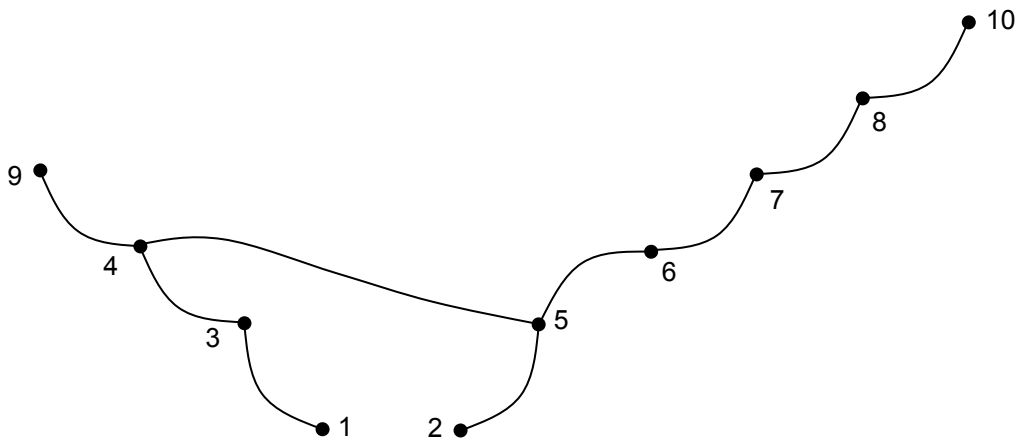
$$(\cancel{3}, \cancel{5}, 4, 4, 5, 6, 7, 8)$$

Continuing this action in the 8<sup>th</sup> step, this will be obtained

• 10



The last, final step is the connection of nodes with 8 and 10 numbers.



**7b12.**

| $x_1$ | $x_2$ | $x_3$ | $x_1^{x_2} \vee x_2^{x_3}$ |
|-------|-------|-------|----------------------------|
| 0     | 0     | 0     | 1                          |
| 0     | 0     | 1     | 1                          |
| 0     | 1     | 0     | 0                          |
| 0     | 1     | 1     | 1                          |
| 1     | 0     | 0     | 1                          |
| 1     | 0     | 1     | 0                          |
| 1     | 1     | 0     | 1                          |
| 1     | 1     | 1     | 1                          |

$x_1^{x_2} \vee x_2^{x_3} \notin \mathbf{S}$ , as the table is not antisymmetric.

$x_1 \bar{x}_2 \notin \mathbf{M}$ , as for (1,0) and (1,1) the monotony condition is violated.

$x_1 \rightarrow x_2 \notin \mathbf{T}_0$ , as  $0 \rightarrow 0 = 1$ .

$x_1 \bar{x}_2 \notin \mathbf{T}_1$ , as  $1 \cdot \bar{1} = 0$ .

$x_1 \bar{x}_2 \notin \mathbf{L}$ , as  $x_1 \bar{x}_2 = x_1 \oplus x_1 x_2$ .

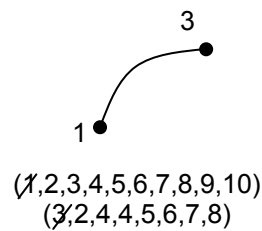
The system is complete.

**7b13.**

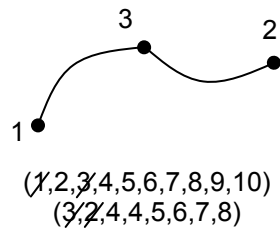
As the code length equals 8, the number of tree nodes equals 10. The numbers of those nodes and tree code are:

(1,2,3,4,5,6,7,8,9,10)  
(3,2,4,4,5,6,8,8)

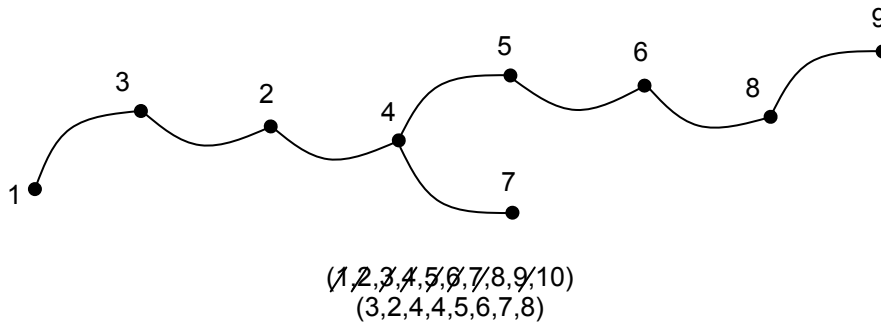
From those numbers choose the number from left to right which is missing in the code. It is 1. Connect the node of that number with number one node of the code from the left and delete those two numbers. The following will be obtained:



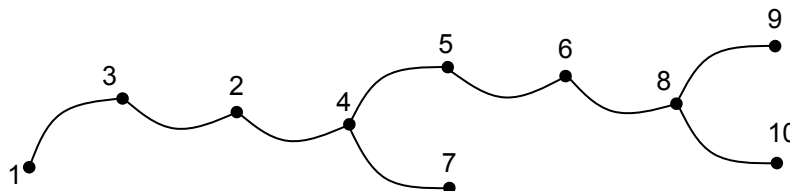
Apply the same process sequentially towards the obtained results. Once more the first number of the node from the left, which is missing in the code is 3. Connect the node of 3 number with the recurrent number of the node 3 and delete the two nodes. The following will be obtained:



Continuing this action until deleting all the numbers in the code, this is obtained:



The last, final step in the connection of not deleted nodes with 8 and 10 numbers in the end.



**7b14.**

To verify the wholeness of  $\{x_1^{x_3} \vee x_2^{x_1}, x_1 \rightarrow \bar{x}_2, x_1 \oplus x_2\}$  system it is enough to use Post theorem. It is clear that

$$x_1 \rightarrow \bar{x}_2 \notin T_0, \text{ as } 0 \rightarrow \bar{0} = 1;$$

$$x_1 \oplus x_2 \notin T_1, \text{ as } 1 \oplus 1 = 0;$$

$$x_1 \oplus x_2 \notin S, \text{ as the functions depending on two variables are not self-dual;}$$

$$x_1 \oplus x_2 \notin M, \text{ as } (0,1) \leq (1,1) \text{ whereas } 0 \oplus 1 = 1, 1 \oplus 1 = 0;$$

$$x_1 \rightarrow \bar{x}_2 \notin L, \text{ as } x_1 \rightarrow \bar{x}_2 = \bar{x}_1 \vee \bar{x}_2 = \overline{x_1 x_2} = 1 \oplus x_1 x_2, \text{ i.e. Zhegalkin polynomial contains sum of variables.}$$

Therefore, the system is complete.

**7b15.**

$$\xi_1 = 2, \xi_2 = 5, \xi_3 = 7, \xi_4 = 10, w=10.$$

As  $\xi_2 + \xi_3 = 5 + 7 = 12 > 10$  and  $\xi_4 \geq 10$ , the threshold function will be:

$$x_2 x_3 \vee x_4,$$

$x_1$  variable's activity of this function equals 0, as  $x_1$  is fictitious variable.

$$\omega_2^{x_2x_3 \vee x_4} = \|\overline{x_4}\| \omega_2^{x_2x_3} = \frac{1}{2} \cdot \|x_3\| = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}.$$

Due to symmetry

$$\omega_3^{x_2x_3 \vee x_4} = \frac{1}{4}.$$

$$\omega_4^{x_2x_3 \vee x_4} = \|\overline{x_2x_3}\| \omega_4^{x_4} = 1 - \|x_2x_3\| = 1 - \frac{1}{4} = \frac{3}{4}.$$

**7b16.**

To verify the wholeness of  $\{x_1x_2, x_1 \rightarrow x_2, x_1 \oplus x_2, x_1 \vee x_2\}$  system it is enough to use Post theorem. It is clear that

$$x_1 \rightarrow x_2 \notin T_0, \text{ as } 0 \rightarrow 0 = 1;$$

$$x_1 \oplus x_2 \notin T_1, \text{ as } 1 \oplus 1 = 0;$$

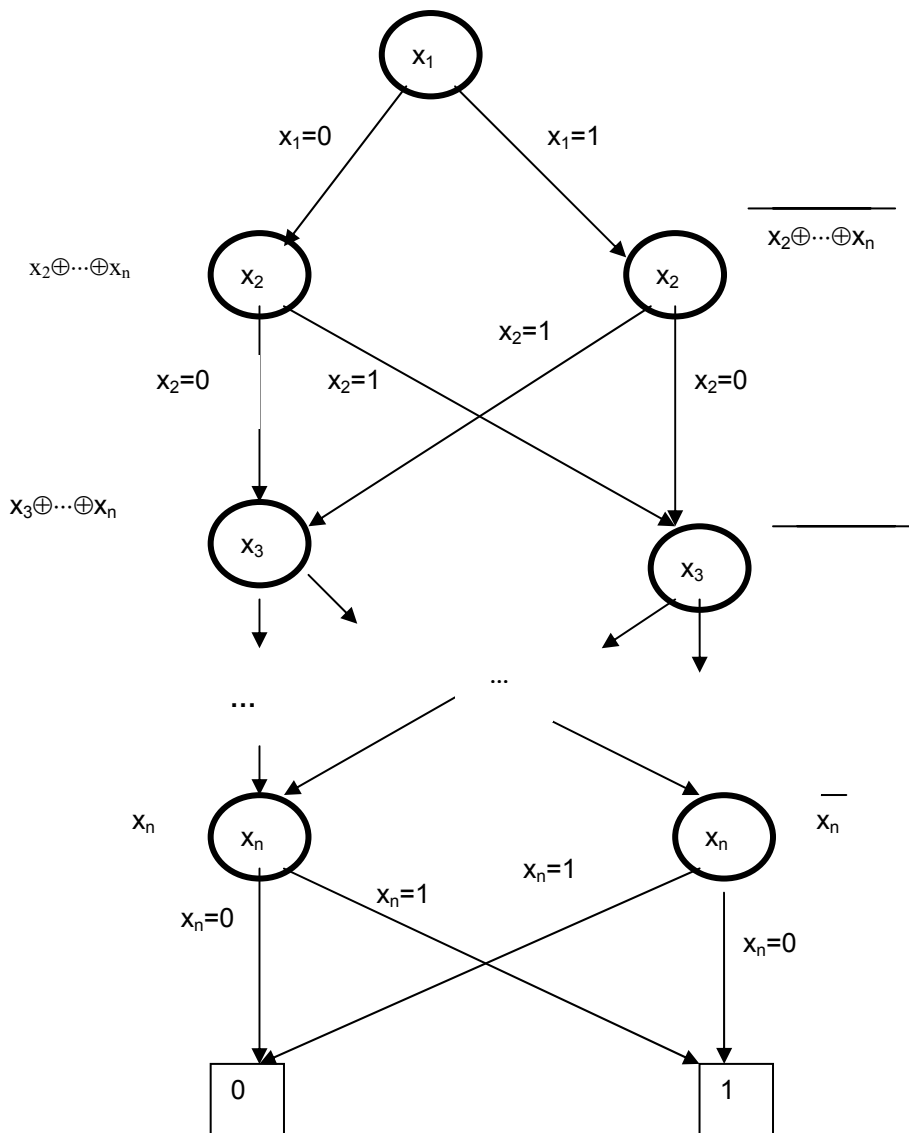
$$x_1 \oplus x_2 \notin S, \text{ as } \overline{\overline{x_1} \oplus \overline{x_2}} = x_1 \oplus x_2 + x_1 \oplus x_2;$$

$$x_1 \oplus x_2 \notin M, \text{ as } (0,1) \leq (1,1) \text{ whereas } 0 \oplus 1 = 1, 1 \oplus 1 = 0;$$

$$x_1 \rightarrow \overline{x_2} \notin L, \text{ as } x_1 \rightarrow x_2 \equiv x_1 \oplus x_2 = x_1x_2, \text{ i.e. Zhegalkin polynomial contains sum of variables.}$$

Therefore, the system is complete.

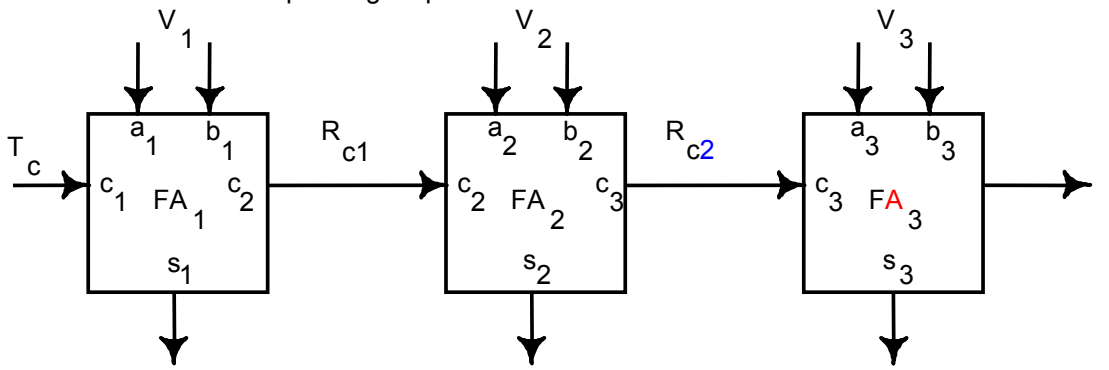
**7b17.**





**7b18.**

The figure depicts the corresponding modular form of an n-bit adder, as well as the input vectors of the main modules and the corresponding output vectors.



| $T_c$ | $V_1$ | $R_{c1}$ | $V_2$ | $R_{c2}$ | $V_3$ |
|-------|-------|----------|-------|----------|-------|
| 0     | 00    | 0        | 00    | 0        | 00    |
| 0     | 01    | 0        | 01    | 0        | 01    |
| 0     | 10    | 0        | 10    | 0        | 10    |
| 0     | 11    | 1        | 00    | 0        | 11    |
| 1     | 00    | 0        | 11    | 1        | 00    |
| 1     | 01    | 1        | 01    | 1        | 01    |
| 1     | 10    | 1        | 10    | 1        | 10    |
| 1     | 11    | 1        | 11    | 1        | 11    |

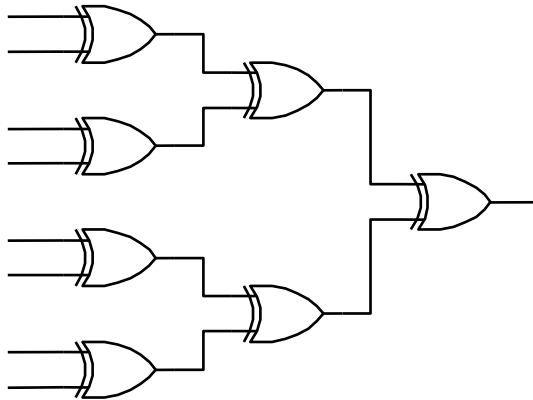
The input variables for module  $F_1$  are the only variable  $c_1$  of set  $T_c$  and the variables  $a_1, b_1$  of set  $V_1$ . For modules  $F_2, F_3$ , etc., the input variables are  $c_2, c_3$ , etc., of group  $R_c$ , as well as the variables  $a_2, b_2, a_3, b_3$ , etc., of groups  $V_2, V_3$ , etc. The output variable  $c_{i+1}$  of module  $F_i$  is an input for the input variable  $c_{i+1}$  directly connected with the input variable of module  $F_{i+1}$ . The variables of groups  $T_c$  and  $V_i$  are independent, and the variables of groups  $R_c$  are dependent (not independent). The variables of groups  $T_c$  and  $R_{c1}$  take exhaustively all 8 possible values. The variables of groups  $R_{ci}$  and  $V_{i+1}$  also take exhaustively all 8 possible values.

| $T_c$ | $V_1$ | $V_2$ | $V_3$ |
|-------|-------|-------|-------|
| 0     | 00    | 00    | 00    |
| 0     | 01    | 01    | 01    |
| 0     | 10    | 10    | 10    |
| 0     | 11    | 00    | 11    |
| 1     | 00    | 11    | 00    |
| 1     | 01    | 01    | 01    |
| 1     | 10    | 10    | 10    |
| 1     | 11    | 11    | 11    |

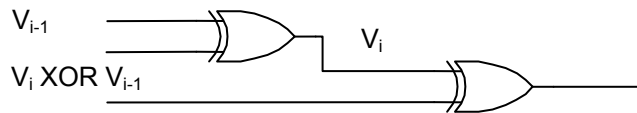
Thus, taking all 8 test vectors depicted in the figure it is possible to provide all 3 inputs for all modules  $F_i$  exhaustively all 8 test vectors which will exhaustively detect all possible faults on all input and output lines of module  $F_i$ .

**7b19.**

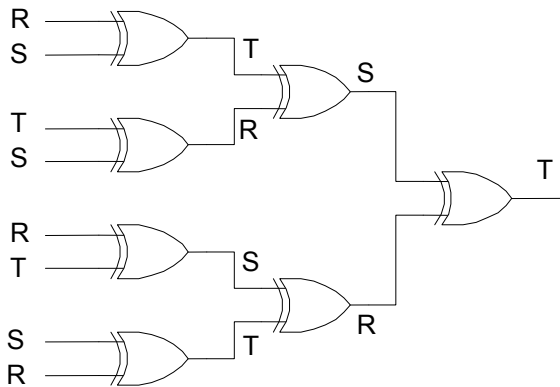
The figure depicts the corresponding circuit of an N-input parity tree.



In 1970, Bossen proved that is easily testable by means of only 4 test vectors. By taking the test consisting of 4 exhaustive tests for Modulo 2 logical element (gate XOR): { 00, 01, 10, 11}, and performing the following assignment for input vectors: R=1100, S=1010, T=0110, and after the following labeling in the parity tree it can be shown that for any  $V_i, V_j, V_k \in \{R, S, T\}$ ,  $V_i \neq V_j \neq V_k$  there is  $V_i \oplus V_j = V_k$ .



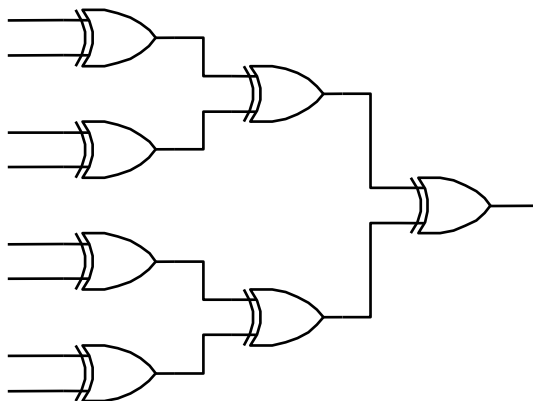
Then performing any labeling in the parity tree of the first figure, the next figure is obtained and it is possible to show that the whole parity tree can be tested by means of only 4 test vectors.



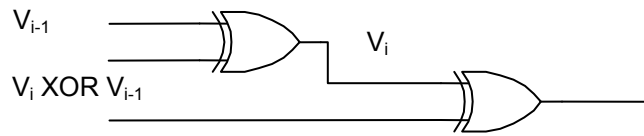
| Vector | R | S | T | S | R | T | S | R |
|--------|---|---|---|---|---|---|---|---|
| 1      | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 |
| 2      | 1 | 0 | 1 | 0 | 1 | 1 | 0 | 1 |
| 3      | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 0 |
| 4      | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

**7b20.**

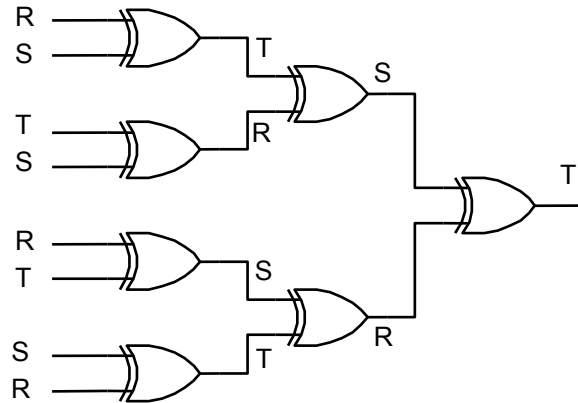
The figure depicts the circuit of N-input tree with negated Modulo 2 elements (gates NXOR).



By taking the exhaustive test consisting of 4 vectors for Modulo 2 element (gate NXOR): { 00, 01, 10, 11}, and performing the following assignment for input vectors: R=1010, S=0110, T=0011, and making the following labeling in our tree, the following is obtained:



and it is possible to show that for any  $V_i, V_j, V_k \in \{R, S, T\}$ ,  $V_i \neq V_j \neq V_k$  there is  $V_i \oplus V_j = V_k$ . Then by performing any labeling in the tree of the figure:



It is possible to show that by means of only 4 test vectors the whole tree can be tested.

| Vector | R | S | T | S | R | T | S | R |
|--------|---|---|---|---|---|---|---|---|
| 1      | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 |
| 2      | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 |
| 3      | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 4      | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |

**7b21.**

The number of all faults is equal

$$\sum_{k=2}^N \binom{m}{k} 2^m = 3^N - \binom{N}{0} 2^0 - \binom{N}{1} 2^1 = 3^N - 2N - 1.$$

Therefore, the number of all stuck-at-0 and stuck-at-1 faults will be  $3^{1000000} - 1999999$ .

**7b22.**

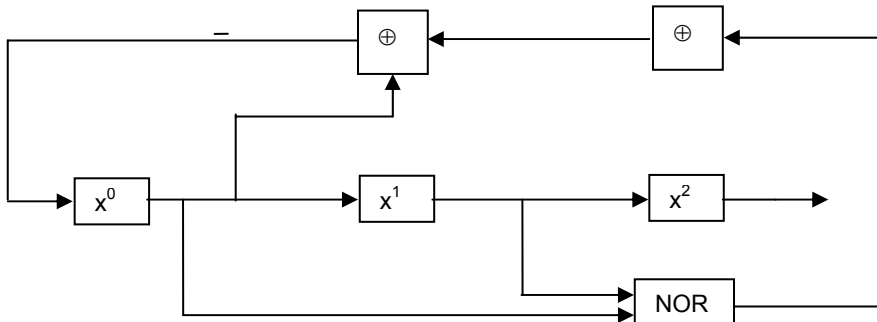
2 subsets are obtained, the corresponding cycles of which include the following sets:

1. (100) => (110) => (111) => (011) => (101) => (010) => (001) => (001) => (100)
2. (000) => (000)

**7b23.**

After adding NOR element, the obtained new circuit generates the only subset of the following patterns:

1. (100) => (110) => (111) => (011) => (101) => (010) => (001) => (001) => (000) => (100).



**7b24.**

First, it is needed to construct the truth table.

| $x_1$ | $x_2$ | $x_3$ | $x_1 \oplus x_3^{x_1 \vee x_2}$ |
|-------|-------|-------|---------------------------------|
| 0     | 0     | 0     | 1                               |
| 0     | 0     | 1     | 0                               |
| 0     | 1     | 0     | 0                               |
| 0     | 1     | 1     | 1                               |
| 1     | 0     | 0     | 1                               |
| 1     | 0     | 1     | 0                               |
| 1     | 1     | 0     | 1                               |
| 1     | 1     | 1     | 0                               |

Search Zhegalkin polynomial of mentioned function in the following form:

$$f = a_0 \oplus a_1 x_1 \oplus a_2 x_2 \oplus a_3 x_3 \oplus a_{12} x_1 x_2 \oplus a_{13} x_1 x_3 \oplus a_{23} x_2 x_3 \oplus a_{123} x_1 x_2 x_3$$

The solution of the problem is to find coefficients in the presented form.

Function equals 1 with values 0,0,0 of arguments  $x_1, x_2, x_3$ . Placing these values on right side of equation:  
 $1 = a_0$

For values 0,0,1 it will be

$$0 = 1 \oplus a_3 \text{ therefore: } a_3 = 1$$

Substituting the right and left parts of the equation, sequentially all the possible values to  $x_1, x_2$  and  $x_3$  variables, the following will be obtained:

$$\begin{aligned} 0 &= 1 \oplus a_2, & a_2 &= 1, \\ 1 &= 1 \oplus 1 \oplus a_{23}, & a_{23} &= 0, \\ 1 &= 1 \oplus a_1, & a_1 &= 0, \\ 0 &= 1 \oplus 1 \oplus a_{13}, & a_{13} &= 1, \\ 1 &= 1 \oplus 1 \oplus a_{12}, & a_{12} &= 1, \\ 0 &= 1 \oplus 1 \oplus 1 \oplus a_{23}, & a_{23} &= 0, \end{aligned}$$

Putting the obtained values of the coefficients, Zhegalkin polynomial of the function will be obtained.

$$f = 1 \oplus x_2 \oplus x_3 \oplus x_1 x_2 \oplus x_1 x_3 \oplus x_1 x_2 x_3$$

**7b25.**

There is some tree corresponding to 000101001111, mark it using the following symbol:



As the code cannot be divided into 2 parts, each containing equal number of 1s and 0s, the tree will have the following view:



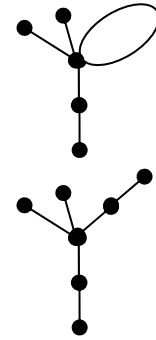
the ring code of which is 0010100111, which is derived from base code by removing 0 and 1 from each side.



and 01010011.

Repeating this procedure to new code, the following will be obtained respectively:

In the next step, the code is divided to 01, 01 and 0011 parts and this will be obtained:



In the last step, as 0011 cannot be divided into 2 parts, this will be obtained:

**7b26.**

$$\omega_1^{x_1x_2 \oplus (x_1 \vee x_2x_3)} = \|x_2x_3 \oplus (x_2 \oplus 1)\| = \|x_2x_3 \oplus \bar{x}_2\| = \|x_2x_3\| + \|\bar{x}_2\| = \frac{1}{4} + \frac{1}{2} = \frac{3}{4}$$

$$\omega_2^{x_1x_2 \oplus (x_1 \vee x_2x_3)} = \|x_1 \oplus (x_1 \oplus (x_1 \vee x_3))\| = \|x_1 \oplus x_1 \oplus (x_1 \vee x_3)\| = \|x_1 \vee x_3\| = \frac{1}{2} + \frac{1}{2} - \frac{1}{4} = \frac{3}{4}$$

$$\omega_3^{x_1x_2 \oplus (x_1 \vee x_2x_3)} = \|(x_1x_2 \oplus x_1) \oplus (x_1x_2 \oplus (x_1 \vee x_2))\| = \|(x_1x_2 \oplus x_1 \oplus x_1x_2 \oplus (x_1 \vee x_2))\| =$$

$$= \|(x_1 \oplus (x_1 \vee x_2))\| = \frac{1}{2} + \frac{3}{4} - 2\|(x_1 \oplus (x_1 \vee x_2))\| = \frac{1}{2} + \frac{3}{4} - 1 = \frac{1}{4}$$

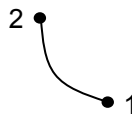
**7b27.**

As the code length equals 7, therefore the number of tree nodes equals 9. The numbers of those nodes and tree code are:

1, 2, 3, 4, 5, 6, 7, 8, 9:

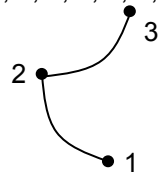
From those numbers choose the number from left to right which is missing in the code. It is 1. Connect the node of that number with number one node of the code from the left (which is 2) and delete those two numbers. The following will be obtained:

3, 3, 4, 5, 5, 8  
2, 3, 4, 5, 6, 7, 8, 9:

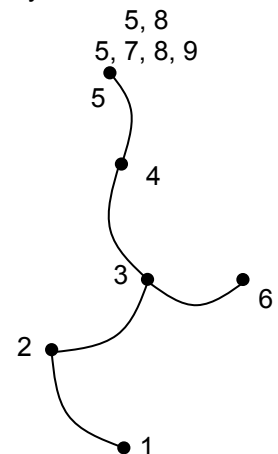
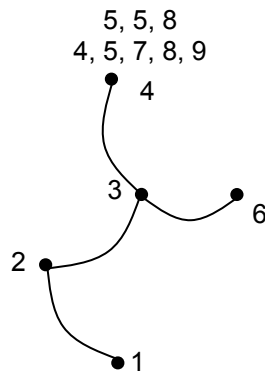
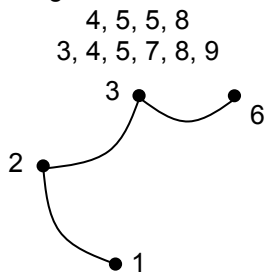


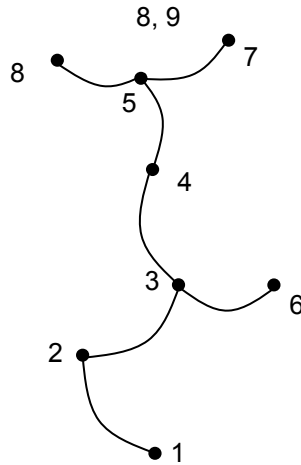
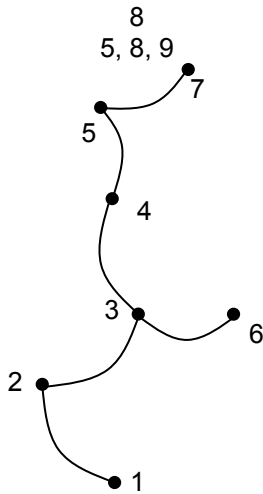
Apply the same process sequentially towards the obtained results, connect the node of 3 number with the node 3. The following will be obtained:

3, 4, 5, 5, 8  
3, 4, 5, 6, 7, 8, 9

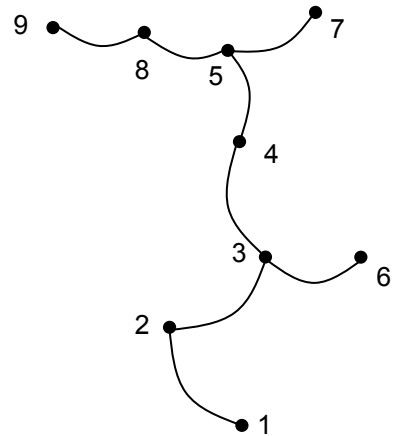


Continuing this action until deleting all the numbers in the code, this is sequentially obtained:





Finally, the remaining 8 and 9 nodes are connected to each other.



**7b28.**

Let F1 be the fault "line A stuck-at-1", F2 be "line B stuck-at-1" and F3 be "line Z\* be stuck-at-1". Then it is easy to check that  $F1 = F2 = F3$  since  $F1 = \{(0,0)\}$ ,  $F2 = \{(0,0)\}$ ,  $F3 = \{(0,0)\}$ .

**7b29.**

According to McCluskey, the number of input patterns for the pseudoexhaustive test is  $2^{n1} + 2^{n2} + \dots + 2^{n5} = 2^{12} + 2^{14} + 2^{16} + 2^{18} + 2^{20} = 2^{12}(1+4+16+64+256) = 341 \cdot 2^{12} = 1396736$ .

**7b30.**

Since  $T(F1) = \{(0,0)\}$ ,  $T(F2) = \{(0,0)\}$ ,  $T(F3) = \{(0,0)\}$ , then  $T(F1) = T(F2) = T(F3)$ .

**7b31.**

Denote by F1 the fault "line A stuck-at-0", F2 – "line B stuck-at-0" and F3 – "line Z stuck-at-0". Prove that single faults F1, F2 and F3 are equivalent.

Since  $T(F1) = \{(1,1)\}$ ,  $T(F2) = \{(1,1)\}$ ,  $T(F3) = \{(1,1)\}$ , then  $T(F1) = T(F2) = T(F3)$ .

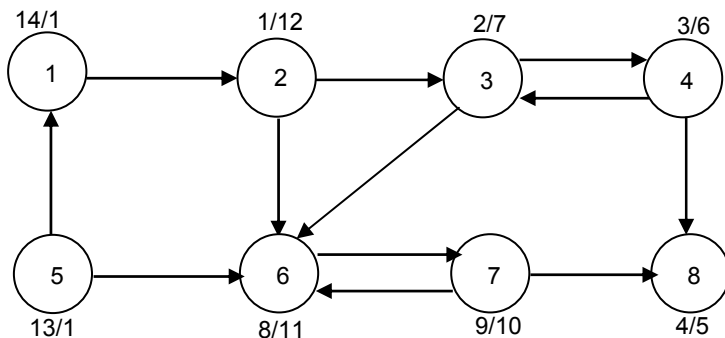
**7b32.**

Search strategy is the following:

- 1) Starting from the 2<sup>nd</sup> vertex, go deeper in any path as it is still possible.
- 2) Return by searching other paths.
- 3) Repeat the 1<sup>st</sup> and the 2<sup>nd</sup> steps until detecting all possible vertices.
- 4) If there are still undetected vertices, select one of them and repeat 1-3 steps.
- 5) Repeat 1-4 steps until detecting all the vertices of the graph.

Mark the numbers of detection and completion steps on vertices.

Answer



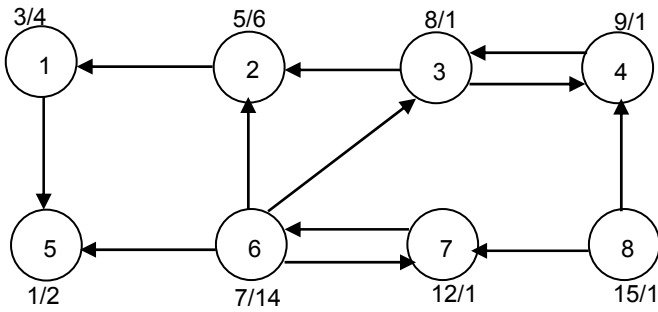
**7b33.**

Separate strongly connected components in the given directed graph.

Solution:

- 1) Search according to depth starting with any vertex.
- 2) Construct the transported graph of the given graph.
- 3) Search according to depth in transported graph. In each cycle, start from the vertex with the highest value of process completion.

Each search cycle will separate strongly connected components.  
 Answer: Observe the solution on the previous example.



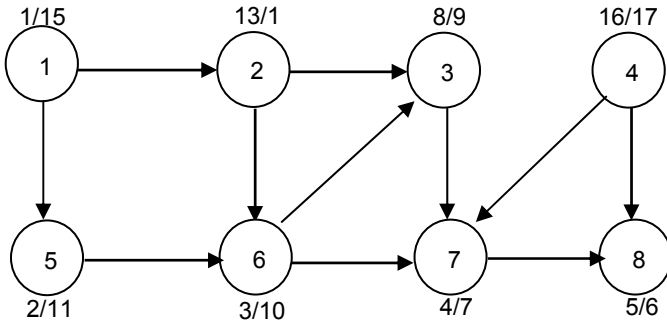
In the previous example, the completions of vertex processing according to decrease are the following: 5, 1, 2, 6, 7, 3, 4, 8.

Therefore the search will start from 5<sup>th</sup> vertex.

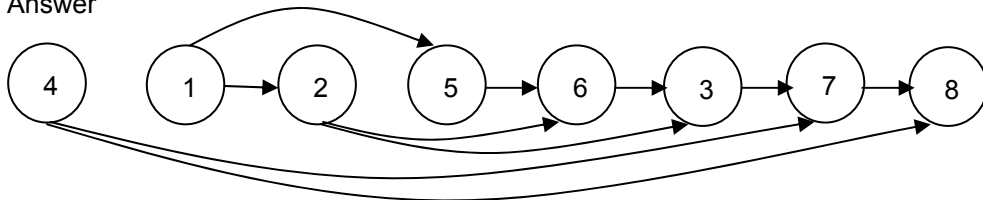
The strongly connected components will be a) 5; b) 1; c) 2; d) 3, 4, 6, 7; e) 8

**7b34.**

- 1) Search according to density, starting from any vertex.
- 2) Classify the vertices from left to right, according to the decrease of ends of vertex processing.



Answer



**7b35.**

The objective function will be  $f = 2f_1 + f_2$ .

Answer:

|   |   |   |   |   |   |    |      |
|---|---|---|---|---|---|----|------|
| 7 | 5 | 6 | 7 | 8 | 9 | 10 | 11   |
| 6 | 4 | 5 | 6 | 7 | 8 | 11 | 12   |
| 5 | 3 | 6 | 7 |   | 9 | 12 | 11 B |
| 4 | 2 | 3 | 4 |   | 8 | 11 | 10   |
| 3 | 1 | 2 | 3 | 6 | 7 |    | 9    |
| 2 | 0 | 1 | 2 | 5 | 6 | 7  | 8    |
| 1 | 1 | 2 | 3 | 6 | 7 |    | 9    |
|   | 1 | 2 | 3 | 4 | 5 | 6  | 7    |

**7b36.**

The matrix of orthogonal distances is built, calculating the orthogonal distance between contacts.

$$l_{a,b} = |2-8| + |9-8| = 7;$$

$$l_{ac} = 4;$$

$$l_{bc} = 9;$$

$$l_{ce} = 3;$$

$$l_{ad} = 13;$$

$$l_{bd} = 5;$$

$$l_{de} = 7;$$

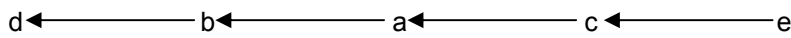
$$l_{ae} = 7;$$

$$l_{be} = 12;$$

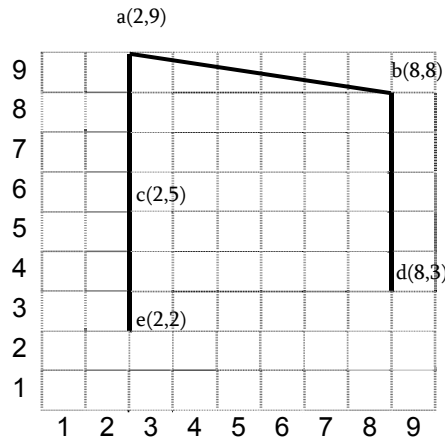
$$l_{cd} = 8;$$

The matrix will have the following view:

|   |    |     |     |     |     |
|---|----|-----|-----|-----|-----|
|   | a  | b   | c   | d   | e   |
| a | 0  | 7 v | 4 v | 12  | 7   |
| b | 7  | 0   | 9   | 5 v | 12  |
| c | 4  | 9   | 0   | 8   | 3 v |
| d | 12 | 5   | 8   | 0   | 7   |
| e | 7  | 12  | 3   | 7   | 0   |
|   | 2  | 3   | 1   |     |     |



Answer:



**7b37.**

The adjacency matrix and the solution process are the following:

|   |   |   |   |   |   |   |          |           |          |   |
|---|---|---|---|---|---|---|----------|-----------|----------|---|
|   | 1 | 2 | 3 | 4 | 5 | 6 | $f_0$    | $f_1$     |          |   |
| 1 | 0 | 1 | 0 | 2 | 1 | 0 | <u>4</u> | X         | X        |   |
| 2 | 1 | 0 | 1 | 0 | 0 | 2 | 4        | 2         | 2        |   |
| 3 | 0 | 1 | 0 | 1 | 0 | 0 | 2        | 2         | <u>0</u> | X |
| 4 | 2 | 0 | 1 | 0 | 0 | 0 | 3        | <u>-1</u> | X        |   |
| 5 | 1 | 0 | 0 | 0 | 0 | 2 | 3        | 1         | 1        |   |
| 6 | 0 | 2 | 0 | 0 | 2 | 0 | 4        | 4         | 4        |   |

Answer:

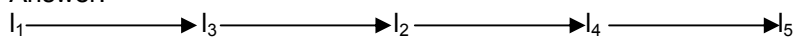
- 1<sup>st</sup> group - 1, 3, 4;
- 2<sup>nd</sup> group - 2, 5, 6.

**7b38.**

The adjacency matrix and the solution process are the following:

|       |       |       |       |       |       |          |       |          |           |       |
|-------|-------|-------|-------|-------|-------|----------|-------|----------|-----------|-------|
|       | $l_1$ | $l_2$ | $l_3$ | $l_4$ | $l_5$ | $f_0$    | $f_1$ | $f_2$    | $f_3$     | $f_4$ |
| $l_1$ | 0     | 0     | 1     | 1     | 0     | <u>2</u> | X     | X        | X         | X     |
| $l_2$ | 0     | 0     | 1     | 1     | 0     | 2        | 2     | <u>0</u> | X         | X     |
| $l_3$ | 1     | 1     | 0     | 0     | 1     | 3        | 1     | X        | X         | X     |
| $l_4$ | 1     | 1     | 0     | 0     | 1     | 3        | 1     | 1        | <u>-1</u> | X     |
| $l_5$ | 0     | 0     | 1     | 1     | 0     | 2        | 2     | 0        | <u>0</u>  |       |

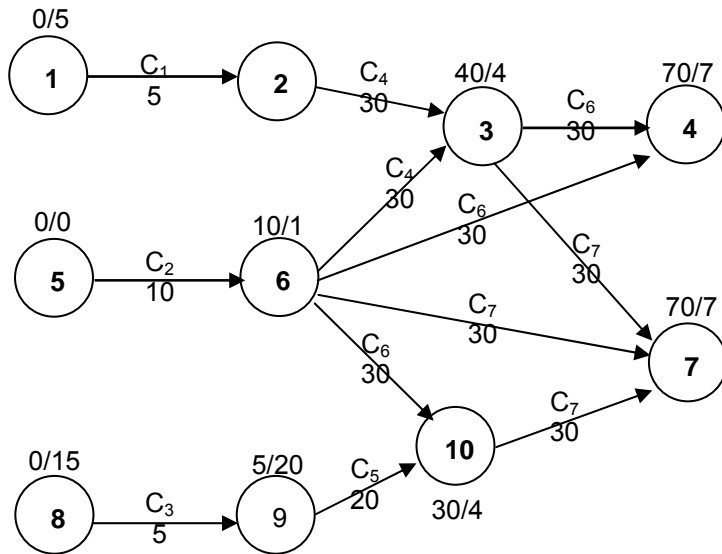
Answer:





7b39.

Timing graph and calculation process are the following:



Answer:

The critical paths are two and have 70 unit of delay.

a) 5 → 6 → 3 → 4 b) 5 → 6 → 3 → 7

7b40.

See the solution of 7b24.

7b41.

See the solution of 7b25.

7b42.

See the solution of 7b26.

7b43.

$f(x_1, x_2, x_3) = x_1 x_2 \vee x_3$ , as  $\xi_1 + \xi_2 = 7 > 5$  and  $\xi_3 = 6 > 5$ . Construct the table for perfect disjunctive normal form of the function.

| $x_1$ | $x_2$ | $x_3$ | $x_1$ | $x_2$ | $\vee$ | $x_3$ |
|-------|-------|-------|-------|-------|--------|-------|
| 0     | 0     | 0     | 0     |       |        |       |
| 0     | 0     | 1     | 1     |       |        |       |
| 0     | 1     | 0     | 0     |       |        |       |
| 0     | 1     | 1     | 1     |       |        |       |
| 1     | 0     | 0     | 0     |       |        |       |
| 1     | 0     | 1     | 1     |       |        |       |
| 1     | 1     | 0     | 1     |       |        |       |
| 1     | 1     | 1     | 1     |       |        |       |

Perfect disjunctive normal form of the function will be:

$$f(x_1, x_2, x_3) = \overline{x_1} \overline{x_2} x_3 \vee \overline{x_1} x_2 x_3 \vee x_1 \overline{x_2} x_3 \vee x_1 x_2 \overline{x_3} \vee x_1 x_2 x_3$$

7b44.

To check the completeness of the system, use Post theorem. It is obvious that  $\overline{x_1} \rightarrow x_2$  function preserves 0 constant  $\overline{0} \rightarrow 0 = 1 \rightarrow 0 = 0$ . The other functions of the system also preserve 0 constant:  $0 \cdot 0 = 0$  and  $0 \vee 0 = 0$ .

Therefore, the system is fully included in the class that preserves 0 constant functions. Hence, according to Post theorem, it follows that the system is not complete.

## 8. OBJECT-ORIENTED PROGRAMMING

### a) Test questions

|       |   |       |   |       |   |
|-------|---|-------|---|-------|---|
| 8a1.  | B | 8a26. | D | 8a51. | D |
| 8a2.  | B | 8a27. | C | 8a52. | C |
| 8a3.  | E | 8a28. | C | 8a53. | D |
| 8a4.  | D | 8a29. | E | 8a54. | D |
| 8a5.  | B | 8a30. | B | 8a55. | B |
| 8a6.  | C | 8a31. | E | 8a56. | A |
| 8a7.  | A | 8a32. | D | 8a57. | C |
| 8a8.  | B | 8a33. | B | 8a58. | C |
| 8a9.  | C | 8a34. | A | 8a59. | E |
| 8a10. | D | 8a35. | B | 8a60. | A |
| 8a11. | B | 8a36. | A | 8a61. | D |
| 8a12. | D | 8a37. | B | 8a62. | A |
| 8a13. | C | 8a38. | A | 8a63. | A |
| 8a14. | A | 8a39. | B | 8a64. | B |
| 8a15. | A | 8a40. | E | 8a65. | A |
| 8a16. | C | 8a41. | E | 8a66. | E |
| 8a17. | E | 8a42. | C | 8a67. | A |
| 8a18. | E | 8a43. | B | 8a68. | B |
| 8a19. | C | 8a44. | D | 8a69. | D |
| 8a20. | A | 8a45. | D | 8a70. | C |
| 8a21. | D | 8a46. | D |       |   |
| 8a22. | B | 8a47. | C |       |   |
| 8a23. | E | 8a48. | E |       |   |
| 8a24. | E | 8a49. | D |       |   |
| 8a25. | C | 8a50. | B |       |   |

## **b) Problems**

### **8b1.**

The length of the greatest common subsequence can be computed using the following program:

```
#include <stdio.h>
#define N 1000
#define max(a,b) ((a>b) ? a : b)
int L[N][N];
int main(){
 char a[N];
 char b[N];
 int i, j, n, m;
 int yes = 0;
 scanf("%s", a);
 scanf("%s", b);
 n = strlen(a);
 m = strlen(b);
 for(j = 0; j < m ; j++) {
 if(b[j] == a[0]) yes = 1;
 L[0][j] = yes;
 }
 yes = 0;
 for(i = 0; i < n; i++) {
 if(a[i] == b[0]) yes = 1;
 L[i][0] = yes;
 }
 for(i = 1; i < n; i++) {
 for(j = 1; j < m ; j++) {
 if(a[i] == b[j])
 L[i][j] = L[i-1][j-1] + 1;
 else
 L[i][j] = max(L[i-1][j], L[i][j-1]);
 }
 }
 printf("%d\n", L[n-1][m-1]);
 return 0;
}
```

### **8b2.**

AM(10,10)= 7368

### **8b3.**

Sum of digits =150

Q=3.752002426043100302699428993946639820.

### **8b4.**

```
Subs(n,k) :=
 If (n==k) then Ss={{1,2,3,...,n}};
 goto END;
 else A={1,...,k}
end;
Ss={};
p=k;
label P
Ss=Ss⊕A;
If (A(k)==n) then p=p-1
 else p=k
end;
If (p>=1) then i=k+1;
 label Q;
 i=i-1;
 If(i>=p) then A(i)=A(p)+i-p+1;
 goto Q
 else goto P;
end;
end;
label END;
Rerurn(Ss)
```

**8b5.**

Use Induction. The loop invariant is the following:  $x_j y_j^{z_j} = y_0^{z_0}$

**8b6.**

Use Induction.

**8b7.**

Use Induction.

**8b8.**

```
#include <iostream>
using namespace std;
int main(){
 int n;
 cin >> n;
 cout << n - (n/2 + n/3 + n/5 - n/6 - n/15 - n/10 + n/30) << endl;
 return 0;
}
```

**8b9.**

```
#include <iostream>
#include <fstream>
#include <cmath>
#include <algorithm>
#include <set>
#include <queue>
#include <stack>
#include <iomanip>

using namespace std;

struct matric {
 int a[2][2];
};

matric a1;

matric mul(matric a,matric b) {
 int i,j,k;
 matric h;
 for (i=0;i<2;i++)
 for (j=0;j<2;j++)
 h.a[i][j]=0;
 for (i=0;i<2;i++)
 for (j=0;j<2;j++)
 for (k=0;k<2;k++)
 {
 h.a[i][j]= ((__int64)h.a[i][j] + (__int64)a.a[i][k]*b.a[k][j])
 % 1000007;
 h.a[i][j]%=1000007;
 }
 return h;
}

matric stepen(matric a,int n) {
 if (n==1)
 return a;
 matric h=stepen(a,n/2);
 h=mul(h,h);
 if (n&1)
 h=mul(h,a1);
 return h;
}

int main() {
 int n;
 cin >> n;
 a1.a[0][0]=1;a1.a[0][1]=1;
 a1.a[1][0]=1;a1.a[1][1]=0;
 matric h1 = stepen(a1,n);
 cout << h1.a[0][0] << endl;
 return 0;
}
```

**8b10.**

```

#include <iostream>
#include <fstream>

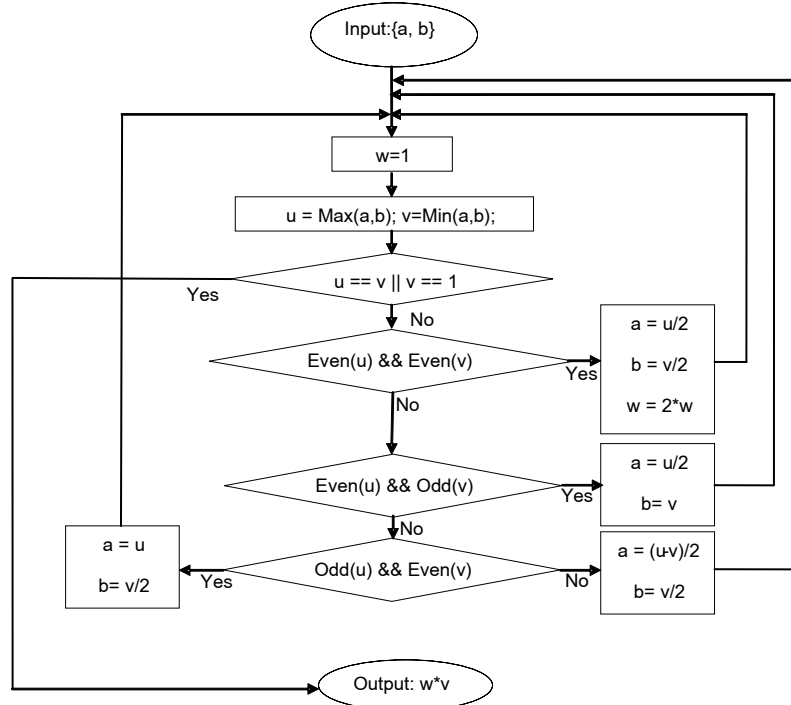
using namespace std;

ifstream in("15.in");
ofstream out("15.out");
int number, arr[5000], ans[5000];
int i, j, max;
int main()
{
 in >> number;
 for (i=0; i<number; i++)
 in >> arr[i];
 for (i=0; i<number; i++)
 {
 max=0;
 for (j=0; j<i; j++)
 if (arr[j]<arr[i] && max<ans[j])
 max=ans[j];
 ans[i]=max+1;
 }
 max=0;
 for (i=0; i<number; i++)
 if (ans[i]>max)
 max=ans[i];
 out << max << endl;
 return 0;
}

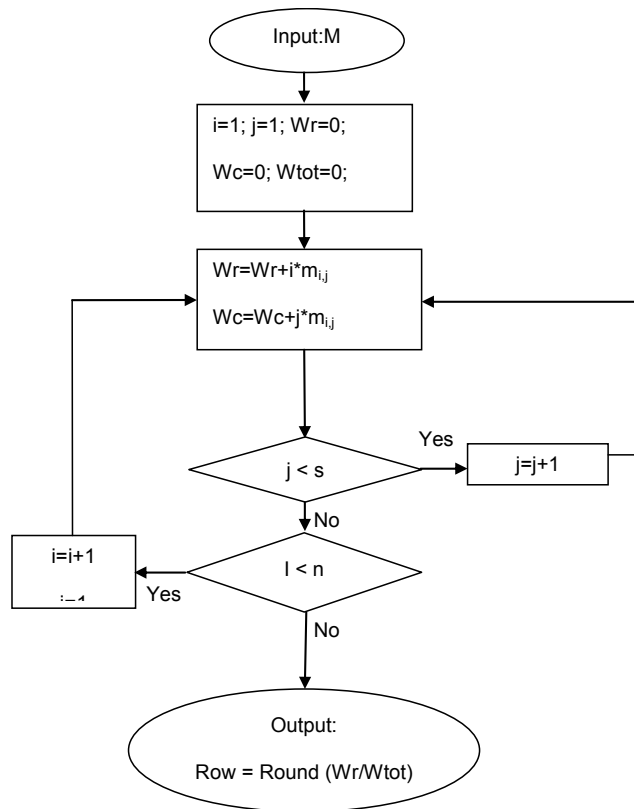
```

**8b11.**

Assume that detection Odd/Even as well as division of even integers by 2 are operations that are to be made with the last position of binary representations and hence do not use the deletion in common sense.



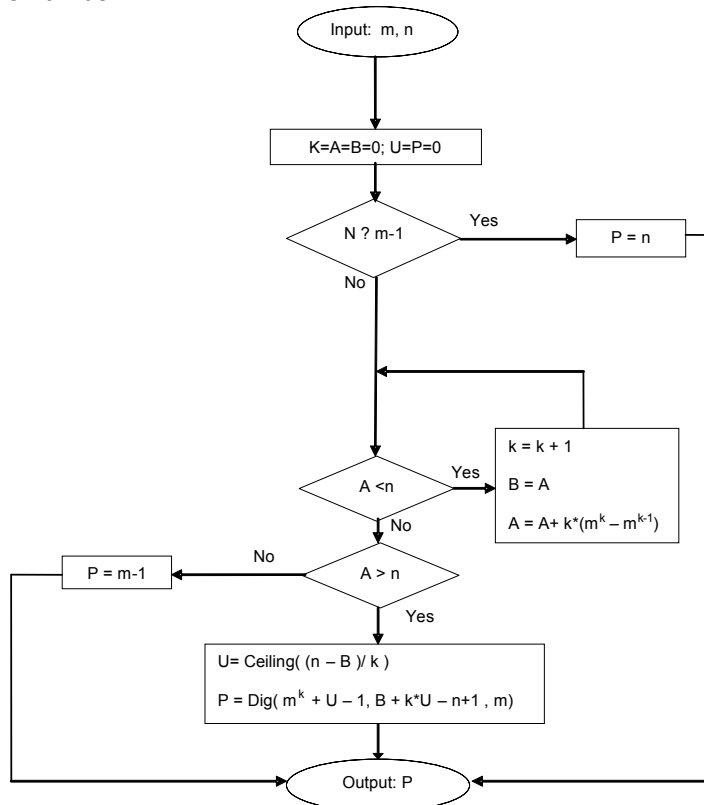
8b12.  
A.



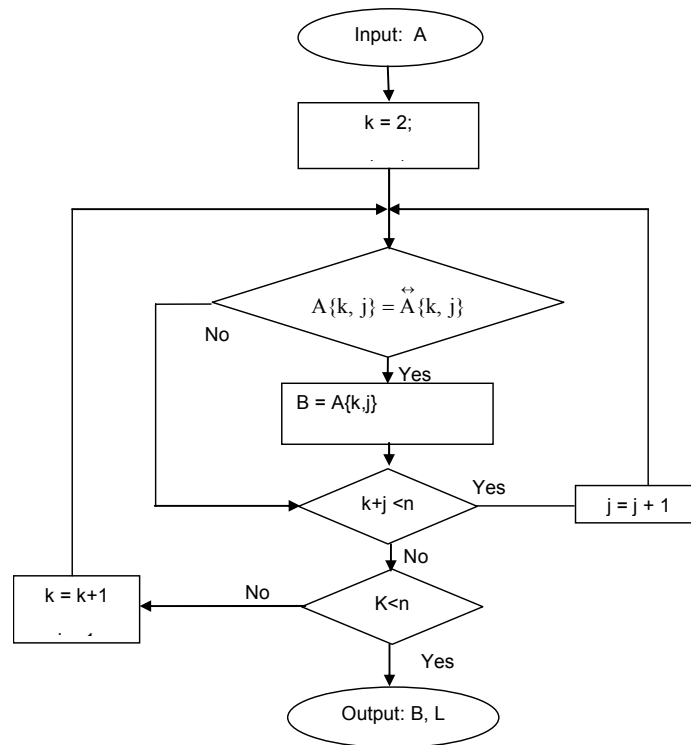
B.  $O(n*s)$ .

8b13.

$\text{Dig}(x,y,z) = \text{Mod}(\text{Floor}(x/z^{y-1}), z)$ , i.e  $\text{Dig}(x,y,z)$  is equal to  $y$ -th digit (from the right side) in  $z$ -ary representation of the number  $x$ .



8b14.



Here  $A\{k, j\}$  denotes a fragment of  $A$  having length  $k$  and startpoint  $j$ , and  $\tilde{A}\{k, j\}$  denotes the reflection (in mirror) of  $A\{k, j\}$ .

8b15.

```
#include <iostream>
#include <cmath>
```

```
int main()
{
 const int n = 1000;
 const int size = 200;
 int prime_numbers[size] = {0};
 int index = 0;

 for (int i = 2; i <= n; ++i) {
 int limit = (int)sqrt(i);
 int j = 2;
 for (; j <= limit; ++j) {
 if (0 == i % j) {
 break;
 }
 }
 if (j > limit) {
 prime_numbers[index] = i;
 ++index;
 }
 }

 for (int k = 0; k < index; ++k) {
 std::cout << prime_numbers[k] << " ";
 }
 std::cout << std::endl;

 return 0;
}
```

**8b16.**

```

#include <iostream>

int main(){
 const int n = 1000;
 const int size = 20;
 int perfect_numbers[size];
 int index = 0;

 for (int i = 2; i <= n; ++i) {
 int sum = 0;
 int limit = i/2;
 for (int j = 1; j <= limit; ++j) {
 if (0 == i % j) {
 sum += j;
 }
 }
 if (i == sum) {
 perfect_numbers[index] = i;
 ++index;
 }
 }

 for (int k = 0; k < index; ++k) {
 std::cout << perfect_numbers[k] << " ";
 }
 std::cout << std::endl;

 return 0;
}

```

**8b17.**

```

#include <iostream>

long reverse_number(long n);

int main(){
 std::cout << "Please enter the number: ";
 long n;
 std::cin >> n;
 long reverse_n = reverse_number(n);
 if (n == reverse_n) {
 std::cout << "Yes" << std::endl;
 } else {
 std::cout << "No" << std::endl;
 }

 return 0;
}

long reverse_number(long n){
 long reverse = 0;
 do {
 reverse = reverse * 10 + n % 10;
 n /= 10;
 } while (n != 0);

 return reverse;
}

```

**8b18.**

```

#include <iostream>

// Output a partition:
void output_partition(const int n, const int *x, const int how_many_partitions){
 std::cout << "Partition(" << how_many_partitions << ")" << " = ";
 for (int i = 1; i <= n; i++) {
 // Can't show negative numbers:
 if (x[i] > 0) {
 if (i == n)
 std::cout << x[i];
 }
 }
}

```



```

 else
 std::cout << x[i] << " + ";
 }
}
std::cout << std::endl;
}

// This is the function which generates the partitions of a given number "n".
void generate_partitions(int *x, const int n){
 int k, s = 0;
 int how_many_partitions = 0;
 for (k = 1; k < n; k++)
 x[k] = -1;
 k = 1;
 while (k > 0) {
 // Generated a solution, let's output it then:
 if (k == n) {
 how_many_partitions++; // Increase the number of generated
partitions
 x[n] = n - s;
 output_partition(n, x, how_many_partitions);
 k--;
 s = s - x[k];
 }
 else {
 // Check for another solution:
 if (((n - k + 1) * (x[k] + 1)) <= n - s) {
 x[k]++;
 if (x[k] >= x[k - 1])
 {
 s = s + x[k];
 k++;
 }
 }
 else {
 x[k] = -1;
 k--;
 s = s - x[k];
 }
 }
 }
}

int main() {

 char str[100];
 int number;
 std::cout << "n= " ;
 while(true) {
 std::cin >> str;
 if (sscanf(str, "%d", &number) != 1 || number < 2){
 std::cout << "Please enter number >= 2!" <<std::endl;
 std::cout << "n = ";
 }
 else{
 break;
 }
 };

 int *x = new int[number + 1];
 if (x == NULL)
 throw std::bad_alloc("");
 x[0] = 0;
}

```

```

std::cout << "Partitions of " << number << ":\n\n";
generate_partitions(x, number);

delete [] x;
std::cin.ignore();
std::cin.get();
return 0;
}

```

### 8b19.

a.

```

pick n,m;
n_factorial = 1;
m_factorial = 1;
n_m_factorial = 1;
for (count1 = 2, n)
 n_factorial = n_factorial * count1;
end for
for (count2 = 2, m)
 m_factorial = m_factorial * count2;
end for
for (count2 = 2, n-m)
 m_n_factorial = m_n_factorial * count2;
end for
return (m_factorial * n_factorial / m_n_factorial);

```

b.

```

pick n,m;

m_factorial = 1;
n_m_factorial = 1;
temp = 2;
for (count1 = 2, m)
 m_factorial = m_factorial * count1;
end for
n_factorial = m_factorial
for (count2 = m, n)
 m_n_factorial = m_n_factorial * temp;
 temp++;
 n_factorial = n_factorial * count2;
end for
return (m_factorial * n_factorial / m_n_factorial);

```

c.

$$C_1(n,m) = n * m(n-m)$$

$$C_2(n,m) = m(n-m)$$

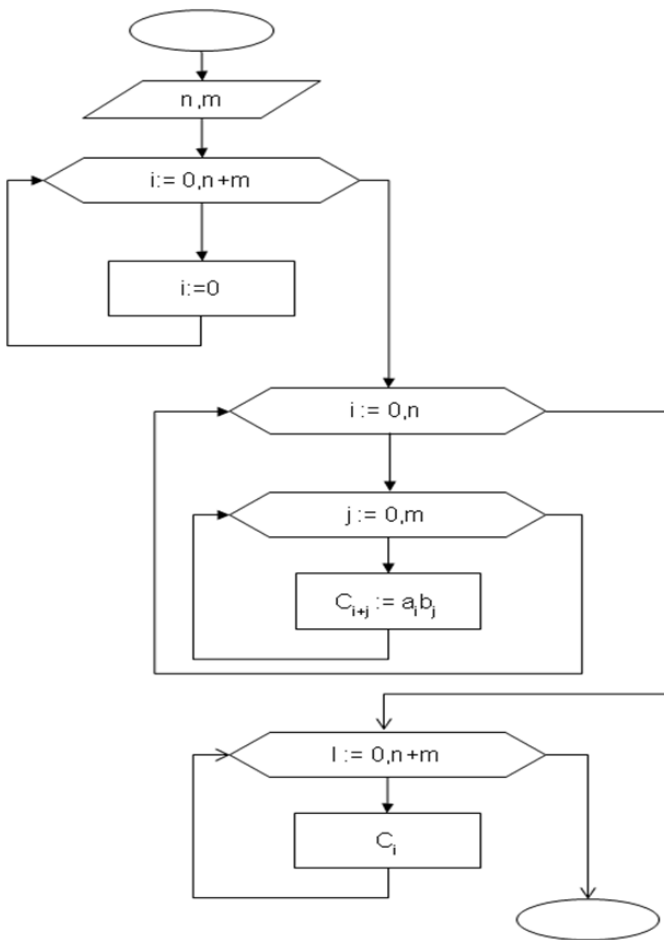
d.

For the simplest algorithm ( $n=100$ ,  $m=90$ ), 202 multiplication and division actions will be required. For the given algorithm, 112 multiplication and division actions will be required.

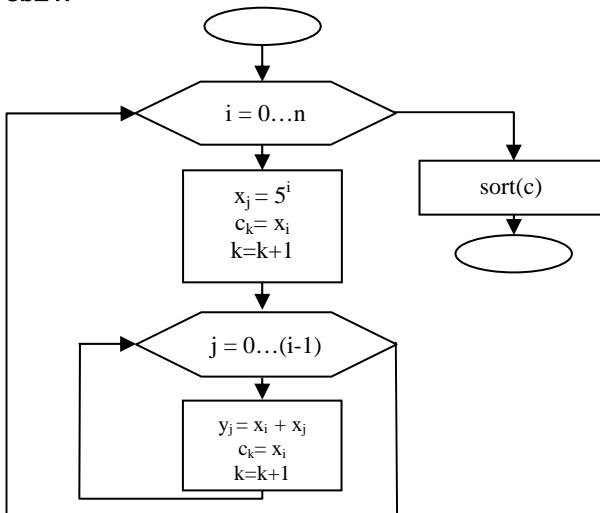
### 8b20.

$$a_i, i = \overline{0, n} \quad b_j, j = \overline{0, m}$$

$$C_k = \sum_{\substack{i+j=k \\ 0 \leq i \leq n \\ 0 \leq j \leq m}} \prod a_i b_j, k = \overline{0, n+m}$$



8b21.



8b22.

```

#include <iostream>
struct rect{
 rect()
 : left(), bottom(), right(), top()
 {}
 double left;
 double bottom;
 double right;
 double top;
};

std::ostream& operator <<(std::ostream& out, const rect& r){
 out << "(" << r.left << ", " << r.top << ") ("

```

```

 out << "(" << r.left << ", " << r.top << ") ("
 << r.right << ", " << r.bottom << ")";
 return out;
}

std::istream& operator >>(std::istream& in, rect& r){
 in >> r.left >> r.top >> r.right >> r.bottom;
 return in;
}

void intersection(const rect& a, const rect& b, rect& r){
 r.left = std::max(a.left, b.left);
 r.bottom = std::max(a.bottom, b.bottom);
 r.right = std::min(a.right, b.right);
 r.top = std::min(a.top, b.top);
}

bool is_valid(const rect& r){
 return (r.left <= r.right) && (r.bottom <= r.top);
}

int main(){
 rect a, b;
 std::cin >> a >> b;
 rect r;
 intersection(a, b, r);
 if (is_valid(r)) {
 std::cout << r << std::endl;
 } else {
 std::cout << "Has no intersection!" << std::endl;
 }
 return 0;
}

```

### 8b23.

```

#include <iostream>
int main() {
 unsigned long long n;
 std::cin >> n;
 // The number of intersection of diagonals when no 3 intersect in one
 // point is equal to 4-combinations of a n, as every 4 point introduce
 // a new intersection point.
 unsigned long long s = n;
 s = s * (n - 1) / 2;
 s = s * (n - 2) / 3;
 s = s * (n - 3) / 4;
 std::cout << s << std::endl;
 return 0;
}

```

### 8b24.

```

#include <iostream>
#include <vector>

struct point{
 point(const double& a, const double& b)
 : x(a), y(b)
 {}
 point()
 : x(0), y(0)
 {}

 double x;
 double y;
};

typedef std::vector<point> polygon;

int sign(const double& x){
 return x == 0 ? 0 : (x > 0 ? 1 : -1);
}

```

```

}

double cross_product(const point& a, const point& b, const point& c){
 return (b.x - a.x) * (c.y - b.y) - (c.x - b.x) * (b.y - a.y);
}

bool is_convex(const polygon& p){
 if (p.size() < 3) {
 return false;
 }
 const size_t n = p.size();
 int s = sign(cross_product(p[n - 1], p[0], p[1]));
 if (s == 0) {
 return false;
 }
 for (size_t i = 0; i < n - 2; ++i) {
 if (s * cross_product(p[i], p[i + 1], p[i + 2]) <= 0) {
 return false;
 }
 }
 return s * cross_product(p[n - 2], p[n - 1], p[0]) > 0;
}

int main(){
 size_t n;
 std::cin >> n;
 polygon p(n);
 for (size_t i = 0 ; i < n ; ++i) {
 std::cin >> p[i].x >> p[i].y;
 }
 if (is_convex(p)) {
 std::cout << "Yes" << std::endl;
 } else {
 std::cout << "No" << std::endl;
 }
 return 0;
}

```

### 8b25.

```

#include <iostream>
int main()
{
 unsigned long long m, n;
 std::cin >> m >> n;
 // The number of rectangles is equal to 2-combinations of a (m + 1)
 // multiplied with 2-combinations of a (n + 1).
 unsigned long long s = m + 1;
 s = s * m / 2;
 s = s * (n + 1) * n / 2;
 std::cout << s << std::endl;
 return 0;
}

```

## 9. NANOELECTRONICS

### a) Test questions

- 9a1. E
- 9a2. C
- 9a3. E
- 9a4. C
- 9a5. E
- 9a6. D
- 9a7. B

## b) Problems

### 9b1.

It is possible to charge nanoparticles 1) under light illumination with photon energy, sufficient for photoelectric effect, 2) in case of the exchange of charge with a solvent that is controlled, in particular, by acidity level of pH environment.

Minimum charge of nanoparticles is equal modulo charge of an electron, and the maximum actually is not limited, but in practice rarely exceeds 1-2 electron charge in an electrically neutral overall colloidal solution, as a result of dynamic equilibrium with the ions in solution. The collision of particles and, consequently, the formation of agglomerates is possible if the kinetic energy of their thermal motion exceeds the potential energy of the Coulomb repulsion. For spherical silicon nanocrystals, having density of Si equal to  $\rho = 2 \text{ g/cm}^3$ ,

can compute mass  $M = \frac{4}{3}\pi R^3 \rho$  and estimate the minimum speed  $V_0$ , at which collision is possible:

$$\frac{MV_0^2}{2} + \frac{MV_0^2}{2} = \frac{q^2}{4\pi\epsilon_0\epsilon R},$$

where  $\epsilon_0$  – dielectric constant,  $\epsilon$  – permittivity of environment.

Then for nanoparticle with  $R = 1 \text{ nm}$  and charge  $q = 2e$  ( $e = 1.6 \cdot 10^{-19} \text{ Kl}$ ) in benzol ( $\epsilon = 2.3$ ) the following will be

$$\text{obtained } V_0 = \frac{q}{2\sqrt{2\pi\epsilon_0\epsilon RM}} = \frac{q}{4\pi R^2 \sqrt{\frac{2}{3}\epsilon_0\epsilon\rho}} \approx 155 \text{ m/s.}$$

Estimate the average thermal speed  $V_T$  from the following relation:

$$\frac{MV_T^2}{2} = \frac{3}{2}kT,$$

where  $k$  – Boltzmann constant,  $T$  – temperature in Kelvin degrees.

$$\text{Then } V_T = \sqrt{\frac{3kT}{M}} = \frac{3}{R} \sqrt{\frac{kT}{4\pi R\rho}} \approx 40 \text{ m/s for } T = 30 \text{ K, which is much smaller than the above calculated } V_0,$$

and it means a conflict of nanoparticles in benzene and their subsequent agglomeration is unlikely.

At the same time for the colloidal solution of similar nanocrystals in water ( $\epsilon = 80$ ),  $V_0 \approx 26 \text{ m/c} < V_T$  is obtained which means high probability of constants of nanoparticles and, consequently, their agglomeration.

The probability of agglomeration, obviously depends on the size of nanoparticles and increases with increasing  $R$  due to a stronger dependence on the parameter values  $V_0$ .

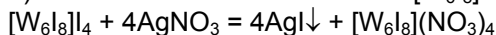
In very dilute colloidal solutions in the above 2 partial approximation, the possibility of contact of nanoparticles in a collision does not depend on the concentration of particles. However, with the growth of the latter, the probability of collisions is obviously increasing, and thus increases the probability of agglomeration. Moreover, given the dependence of the effective permittivity on the concentration of nanoparticles, with the increase of the latter the probability of collision of charged particles may change, increasing, in particular, for silicon nanocrystals ( $\epsilon = 12$ ) in benzene.

In accordance with the above analysis, the possibility of collision of nanoparticles, obviously, depends on temperature and increases with increasing  $T$ , due to increased  $V_T$ , which should lead to an increase in the probability of agglomeration. At the same time, in case of temperature increase the agglomerates can be destroyed by thermal motion of particles. All this leads to a nonmonotonic dependence of the probability of agglomeration of the temperature.

### 9b2.

1) Sectional area equals to  $\pi r^2$ . For nanoparticles of radius, this value is  $3.14 \times (50 \times 10^{-9} \text{ m})^2 = 7.85 \times 10^{-15} \text{ m}^2$ . Then, according to the written formula  $v = (2 \times 0.72 \times 0.4 \text{ H} / (1170 \text{ kg/m}^3 \times 10^7 \times 7.85 \times 10^{-15} \text{ m}^2))^{1/2} = 79.2 \text{ m/s}$

2) Nanocluster has ionic structure  $[\text{W}_6\text{I}_8]^{4+}(\text{I})_4$ . Only outer-iodine can be precipitated by silver nitrate.



Cation is octahedron of atoms of tungsten, on each edge of which iodine atom is located.

3) Any reasonable means are accepted. In particular, consider the option of breaking a compact mercury into nanoparticles with large surface area and their subsequent dispersion in the molten sulfur (this will ensure a complete course of the reaction) to form insoluble in water and most acids, mercuric sulphide, which can be used as component paints.

4) The calculation as per the given formula gives  $(E_g)^2 = 1.468 \times 10^{-37} \text{ J}^2$ , hence  $E_g = 3.83 \times 10^{-19} \text{ J}$ . This corresponds to the wavelength  $\lambda = hc/E_g = 6.62 \times 10^{-34} \text{ J}\cdot\text{s} \times 3 \times 10^8 \text{ m/s} / (3.83 \times 10^{-19} \text{ J}) = 5.2 \times 10^{-7} \text{ m} = 520 \text{ nm}$ , which corresponds to the green color.

**9b3.**

1)  $m_1$  – body mass of a truck (with or without cargo) is a constant component, and the wheels are composed of two parts: a cylindrical surface and hemispheres. The mass of the cylindrical part is directly proportional to the radius, and the mass of hemisphere - square of the radius. Since perimeter of the wheel, and subsequently the radius is proportional to the number  $N$ , the following is obtained:

$m_2N$  – the mass of the cylindrical part of the wheels;

$m_3N^2$  – the mass of hemispheres;

2) In cross-section the wheel of nano-truck consists of a regular  $N$ -gon. The side of the  $N$ -gon is equal to the diameter of a circle inscribed in a hexagon with side  $a = 1.4 \text{ \AA}$ , or the larger side of an equilateral triangle with an angle  $120^\circ$ :

$$b = \sqrt{a^2 + a^2 + 2 \cdot a \cdot a \cdot \cos 120^\circ} = a\sqrt{3}$$

During the rotation of such wheel, the position of its center oscillates from a minimum height equal to the radius of the circle inscribed in the  $N$ -gon with side  $b$ , up to the maximum height equal to the radius of the circle:

$$h_{\min} = r = \frac{b}{2 \operatorname{tg}(\pi / N)} = \frac{a\sqrt{3}}{2 \operatorname{tg}(\pi / N)}$$

$$h_{\max} = R = \frac{b}{2 \sin(\pi / N)} = \frac{a\sqrt{3}}{2 \sin(\pi / N)}$$

Hence the height of the jump is:

$$h = h_{\max} - h_{\min} = \frac{a\sqrt{3}}{2} \left( \frac{1}{\sin(\pi / N)} - \frac{1}{\operatorname{tg}(\pi / N)} \right) = \frac{a\sqrt{3}(1 - \cos(\pi / N))}{2 \sin(\pi / N)}$$

$$E = mgh = \frac{(m_1 + m_2N + m_3N^2)[1 - \cos(\pi / N)]ga\sqrt{3}}{2 \sin(\pi / N)}$$

3) To find the minimum solve the equation:

$$\frac{dE}{dN} = 0$$

$$\begin{aligned} \frac{dE}{dN} &= \left( \frac{(m_1 + m_2N + m_3N^2)[1 - \cos(\pi / N)]ga\sqrt{3}}{2 \sin(\pi / N)} \right)' = \\ &= \frac{(m_2 + 2m_3N)[1 - \cos(\pi / N)]ga\sqrt{3}}{2 \sin(\pi / N)} + \\ &+ \frac{(m_1 + m_2N + m_3N^2)(\sin^2(\pi / N) - \cos(\pi / N)[1 - \cos(\pi / N)])ga\sqrt{3}}{2 \sin^2(\pi / N)} \cdot \left( -\frac{\pi}{N^2} \right) = \\ &= \frac{(m_2 + 2m_3N)[1 - \cos(\pi / N)]ga\sqrt{3}}{2 \sin(\pi / N)} - \\ &- \frac{\pi(m_1 + m_2N + m_3N^2)[1 - \cos(\pi / N)]ga\sqrt{3}}{2N^2 \sin^2(\pi / N)} = 0 \end{aligned}$$

Next, use the asymptotic formulas for trigonometric functions:

$$\frac{(m_2 + 2m_3N) \frac{\pi^2}{2N^2} ga\sqrt{3}}{2(\pi / N)} - \frac{\pi(m_1 + m_2N + m_3N^2) \frac{\pi^2}{2N^2} ga\sqrt{3}}{2N^2(\pi / N)^2} = 0$$

$$\frac{(m_2 + 2m_3N) \frac{\pi^2}{2N^2}}{2(\pi / N)} - \frac{\pi(m_1 + m_2N + m_3N^2) \frac{\pi^2}{2N^2}}{2N^2(\pi / N)^2} = 0$$



$$\frac{(m_2 + 2m_3N)\pi}{4N} - \frac{\pi(m_1 + m_2N + m_3N^2)}{4N^2} = 0$$

$$\frac{m_2 + 2m_3N}{N} - \frac{m_1 + m_2N + m_3N^2}{N^2} = 0$$

$$\frac{-m_1 + m_3N^2}{N^2} = 0$$

Find N:

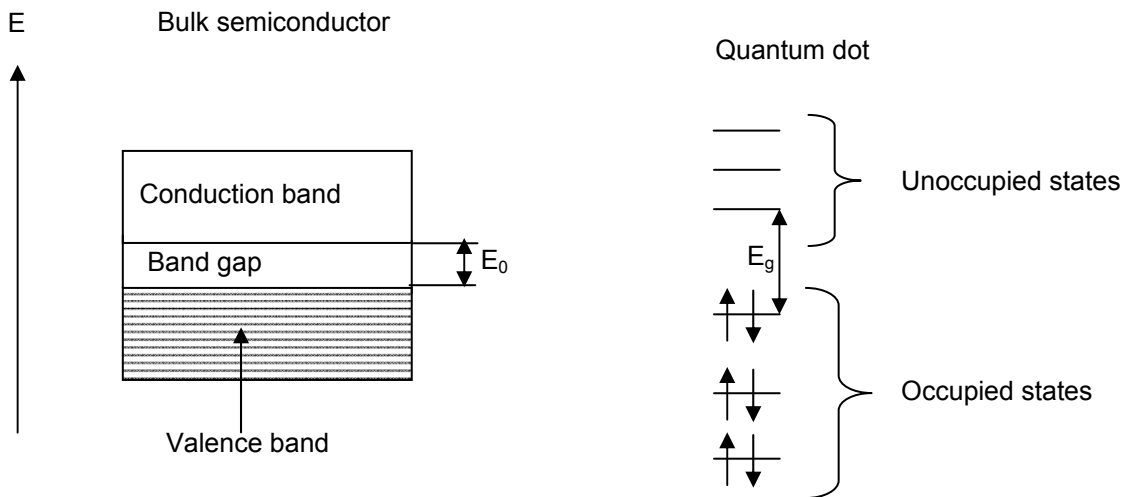
$$N = \sqrt{\frac{m_1}{m_3}} = \sqrt{\frac{10000}{25}} = 20$$

Find the energy:

$$E = \frac{\left( \frac{10000 + 700 \cdot 20 + 25 \cdot 20^2}{6.02 \cdot 10^{23}} \cdot 10^{-3} \text{ kg} \right) [1 - \cos(\pi/20)] \cdot 9.8 \frac{\text{H}}{\text{kg}} \cdot 1.4 \cdot 10^{-10} \text{ m} \cdot \sqrt{3}}{2 \sin(\pi/20)} = 5.2 \cdot 10^{-33} \text{ J}$$

9b4.

1)



2) Hole is an excited quantum state of multi-electron system, characterized by the thing that one of the single-electron states is unoccupied (from physical encyclopedia). Hole is a point from where an electron went and which can be represented as a quasiparticle with a positive charge equal to the charge of an electron.

3) The luminescence is the radiation of atoms, molecules, ions and other more complex particles, resulting from an electronic transition in these particles during their return from excited to normal state.

4) It is obvious that the addition for the crystal with 1cm radius associated with the quantum behavior would be negligible, in this case  $E_g = E_0 = 2.88 \times 10^{-19} \text{ J}$ . The following is obtained for the crystal with 1nm radius as per the given formula:  $E_g^2 = 8.29 \times 10^{-38} + (6.26 \times 10^{-68}) / (1.09 \times 10^{-31}) = 8.29 \times 10^{-38} + 5.74 \times 10^{-37} = 6.57 \times 10^{-37} \text{ J}^2$  (All calculations are in SI units). Hence  $E_g = 8.1 \times 10^{-19} \text{ J}$ .

5) Visible light has a range of 400-750 nm. The smaller the wavelength, the greater the energy, the smaller the radius of the nanoparticle. That is, the minimum size of the nanoparticle will be responsible of the luminescence light with a wavelength of 400 nm, which corresponds to the energy  $E_g = h\nu = hc/\lambda = (6.62 \times 10^{-34} \text{ J}\cdot\text{s}) \times (3 \times 10^8 \text{ m/s}) / (4 \times 10^{-7} \text{ m}) = 4.97 \times 10^{-19} \text{ J}$ . Transforming the expression to find  $E_g$ , this is obtained:  $r^2 = (E_0 \times h^2) / [2 \times (E_g^2 - E_0^2) \times m] = 1.26 \times 10^{-85} / 3.58 \times 10^{-68} = 3.52 \times 10^{-18} \text{ m}^2$ , hence  $r = 1.88 \times 10^{-9} \text{ m}$  or 1.88 nm.

6)  $\text{Cd}(\text{C}_{17}\text{H}_{33}\text{COO})_2 + \text{SeP}(\text{C}_8\text{H}_{17})_3 = \text{CdSe} + \text{PO}(\text{C}_8\text{H}_{17})_3 + (\text{C}_{17}\text{H}_{33}\text{CO})_2\text{O}$

7) The atmosphere of argon is needed to prevent oxidation of raw and end products. The solvent is chosen high-boiling and inert with respect to the quantum dots. Heating during the synthesis is necessary for obtaining well-crystallized one-dimensional quantum dots. The reagents are selected so as to ensure solubility in appropriate solvents and to eliminate chemical interaction with it. Also reagents must be easy to obtain and store, and have as much molecular weight as possible. The proposed method for the second part of the question is completely unacceptable.

a) The boiling temperature of water is 100 ° C lower than the optimum temperature of synthesis.

b) It is known that cadmium salts are considerably hydrolyzed by a cation and have acid reaction medium,

and selenides - anion, hence, their solutions have alkaline reactions. When pouring the solutions, mutually reinforcing hydrolysis will occur. And when it is considered that the reaction is supposed to occur in boiling water, and at pouring the solution dilution of each of them will happen, then, remembering that the heating and dilution just significantly accelerate the hydrolysis, one can precisely say that the main products of the reaction in this case would be useless hydroxide cadmium in the bottom of the vessel and the poisonous gas Hydrogen selenide in the laboratory.

8) Quantum dots based on cadmium selenide are already widely used in the following areas:  
a) LED lamp with the main characteristics of an order of magnitude superior to traditional incandescent and mercury lamps;

b) As a component of sensitive sensor devices as the intensity of the luminescence of quantum dots is sensitive to the presence of minimal amounts of vapors of certain substances (amines, arenes) and minimum amounts of certain bacteria, including harmful;

a) Quantum dots of cadmium selenide doped with magnetic components (e.g., iron) can shift the luminescence in the near infrared range, where weakly absorb water and hemoglobin. It is used in magnetic resonance imaging of internal organs and tissues.

### 9b5.

In this task, the apparent contradiction arises from the thing that the arguments of the first friend the concept of phase and group velocity are confused. In their formulas it is written phase velocity  $U_\phi$  and group velocity  $U_{gr}$ , but he did not distinguish these speeds and marked them with the same letter  $v$ . Because of this, "veiled" error occurred.

The phase velocity is included in the formula of the period of the wavelength:  $T = \frac{\lambda}{U_\phi}$ , that is why the phase

$$\text{velocity is: } \omega = \frac{2\pi}{T} = \frac{2\pi U_\phi}{\lambda}.$$

At the same time, group velocity appears in the expression for the pulse:  $\vec{p} = m\vec{U}_{rp}$ .

Since the phase and group velocities of the electron are not equal to each other in a chain of equalities below there is an error (a violation of equality is indicated by exclamation marks):

$$\omega = \frac{2\pi}{T} = \frac{2\pi v}{\lambda} = \frac{2\pi v p}{h} = \frac{p v}{\hbar} = \frac{p m v}{\hbar m} \neq \frac{\hbar^2 k^2}{\hbar m} = \frac{\hbar k^2}{m},$$

On the left of the numerator is the product of pulse on the mass at the phase velocity, and on the right of the numerator is the square of the pulse, and these expressions are not equal to each other. Therefore, the formula obtained by the first friend is wrong.

The second friend got the correct formula for connection of frequency with wave vector, contained in a number of textbooks. However, he also made an inaccuracy in reasoning. The thing is that in the formula

$E = \hbar \omega$  appears as full energy, and the second friend wrote the expression  $E = \frac{p^2}{2m}$ , representing the

kinetic energy in the nonrelativistic approximation. To be more precise, the relativistic expression should be written for the total energy:

$$E = mc^2 = \sqrt{m_0^2 c^4 + p^2 c^2}$$

where  $m_0$  – the rest mass of an electron, and  $m$  - its total mass. Expanding the right side of in Taylor series, there is:

$$\hbar \omega = m_0 c^2 + \frac{p^2}{2m_0} + \dots$$

where ellipsis identifies a number of the terms of a higher order. Considering  $p = \hbar k$ , there is:

$$\omega = \frac{m_0 c^2}{\hbar} + \frac{\hbar k^2}{2m_0} + \dots$$

Thus, the frequency and wave vector are related by the above written formula which differs from formula of the second friend. The difference lies in the presence of a large term in the right side of the rest energy, and corrections, which in the nonrelativistic limit can be considered small (indicated by an ellipsis). However, in practice the frequency of the de Broglie wave of the electrons is not measured directly in experiments, and only difference between the frequencies corresponding energy difference is measured. Therefore, as energy,

the frequency of de Broglie waves can be measured not from the absolute zero, but from an arbitrary zero level. This allows choosing as the origin of frequency value  $\frac{m_0 c^2}{\hbar}$ . Then the formula, obtained by the second friend, is true for nonrelativistic approximation:

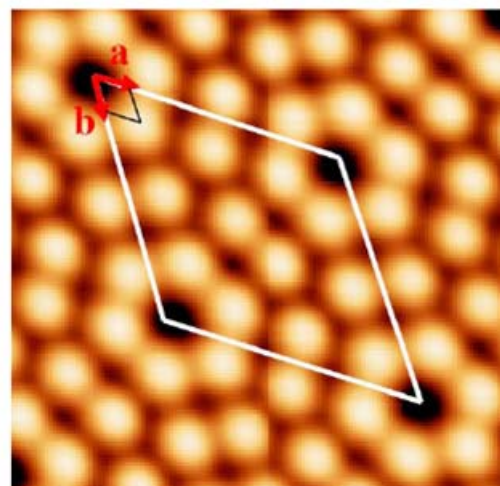
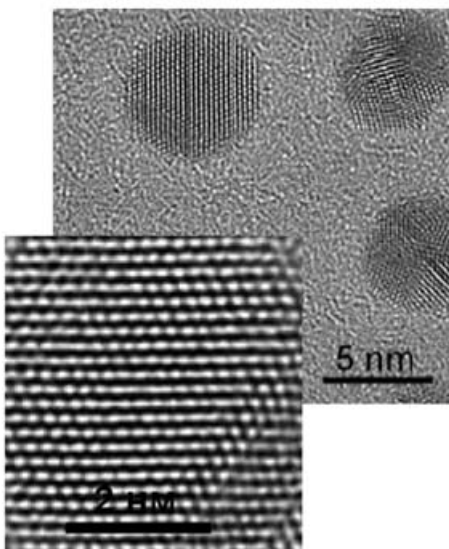
$$\omega = \frac{\hbar k^2}{2m}$$

**9b6.**

First, explain in more detail, what is the proposed "paradox". Recall what the concept of wave-particle duality is:

"All microscopic objects have both wave and corpuscular properties. Their movement in space must be described by the wave theory. The corresponding wave field is distributed in space. However, when measuring the microparticle, space is registered to a certain point as a single entity with all the characteristics of the particle (mass, charge, energy, etc.). The measurement result is probabilistic in nature, to predict where the particle will be detected with certainty unit is impossible. One can only talk about the probability of an event and this probability is ultimately determined by the wave field, which describes the motion of a particle in space.

Simply speaking, "measurement" is an instant photograph on which the electron (or some other particle) is recorded as one point in space, as a point particle with characteristic mass, charge and other characteristics. Therefore, it would seem, not a continuous "electron cloud" is seen on the "photographs" of microworld, but a single point or several points, the position of which is determined randomly with some probability distribution.



Why a continuous cloud is seen? The answer is quite simple: actually it is not dealt with instant photography, but with "photography with great exposure." As known, instant photographs (in the literal sense of the word) do not exist, this is only an idealization, but the real photograph always has the final extract (final imaging time). The real "dimension" in this case is the interaction of atoms with a probe microscope (the scanning probe microscopy), or with an external electron beam (transmission electron microscopy - TEM). Without going into details of the interaction of an atom with a measuring device, one can identify a general property of all the microscopes discussed in such types of tasks: typical time during which images are formed (like the ones in the figure), much larger than the characteristic atomic time, therefore such images are formed statistically as a result of the myriad interactions of atoms with a microscope.

For example, in transmission electron microscopy, image is formed by multiple electron beam, which interact with atoms of the sample and then are registered by the receiver. Each electron separately carries little information and cannot form an image. The same can be said about other types of microscopy. In the AFM, image is formed by processing a large number of cantilever oscillations, but even a period of one oscillation (which already represents a very complex process) is much larger than the characteristic atomic time. Therefore, measuring the interaction potential of the cantilever with an atom is a "shot with great exposure", and this potential is formed statistically from one of the elementary acts of the electromagnetic interaction, which, in terms of quantum field theory, is the exchange of quanta of electromagnetic field - photons.

What is the characteristic atomic time, and how can it be assessed? This is the time during which phase of the wave function (wave field, as discussed above), describing the electron shell of the atom, manages to change to the order of  $2\pi$ . To put it simply, this is the time during which significant changes occur in atom. If an electron is found at some point (more precisely, in a sufficiently small neighborhood), then the wave field, describing this electron is localized (concentrated) in this neighborhood. For the wave field to "blur" again and take all the characteristic volume of the atom, it takes time of atomic. The atomic time is estimated by:

$$t_{at} \approx \frac{\hbar}{E_{at}}$$

where  $\hbar = 1,05 \cdot 10^{-34}$  J·s – Planck's constant with feature,  $E_{at}$  – characteristic energy of an electron in an atom (taken modulo). The above written formula can be obtained from Heisenberg's uncertainty relation, written for the energy and time:

$$\Delta E \cdot \Delta t \geq \hbar$$

It can also be obtained from the consideration that the phase of the wave function contains term  $\omega t$ , where  $\omega$  – cyclic frequency, and therefore the phase changes by  $2\pi$  in time

$$T = \frac{2\pi}{\omega} = \frac{2\pi\hbar}{E}$$

A characteristic electron energy can be estimated as the energy of an electron in the ground state of the simplest atom - hydrogen atom. This value is called the Rydberg (denoted by Ry) and is equal to  $E_{at} = Ry = 13,6 \text{ eV} = 2,18 \cdot 10^{-18}$  J. An estimate for the atomic time is obtained:

$$t_{at} \approx 5 \cdot 10^{-17} \text{ c.}$$

Obviously, this is an extremely short time compared to the time of imaging, given as examples in the problem. For comparison, the maximum oscillation frequency of the cantilever is of the order of several megahertz, which corresponds to the period of oscillation  $T \sim 10^{-6}$  s. In scanning electron microscopy, image is formed as a result of multiple interactions of electrons with the sample matrix and the receiver, each of which has a length greater than the atomic time. Also note that the minimum laser pulse duration is of the order of several femtoseconds, which is also much larger than the atomic time.

### 9b7.

1. Since gold has a high conductivity ( $\sim 4,3 \cdot 10^7$  cm) and rather thin contacts can be obtained from it (plastic deformation at not very high temperatures). For tungsten - high conductivity ( $\sim 1,2 \cdot 10^7$  Cm), developed method of obtaining thin needles for STM.

2. Physicists have observed the effect of quantum conductance. This is seen from the piecewise linear nature of the CVC. Reducing the size of the conductor leads to a decrease in the levels that determine the conductivity. Thus, not completely filled band appears below the Fermi level, but a set of subbands which are separated by "forbidden" minibands.

The specific conductivity of tungsten  $G_w = 18200000$  Cm/m. Subsequently, specific resistance  $\rho_w = 1/G_w = 5,4945 \cdot 10^{-8}$  Ohm\*m.

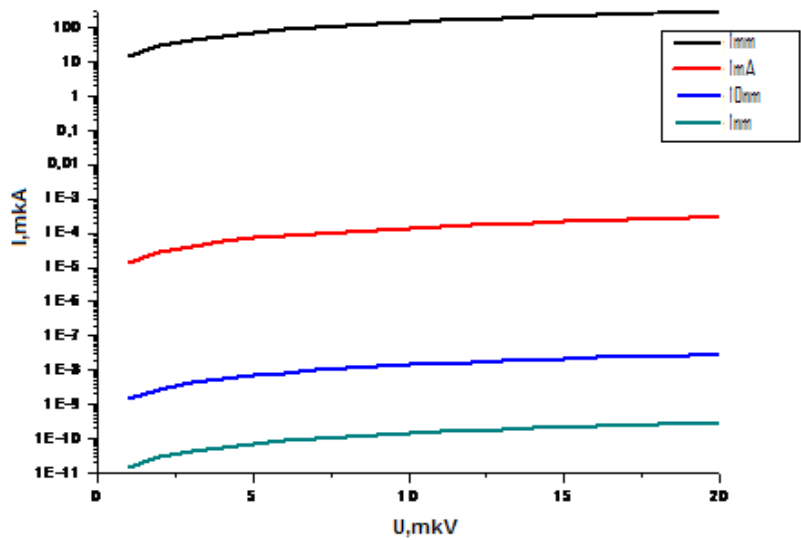
Formulas for calculation:

$$S_{sec} = \pi \cdot r^2 = (\pi \cdot d^2)/4; R = \rho \cdot l/S_{sec}.$$

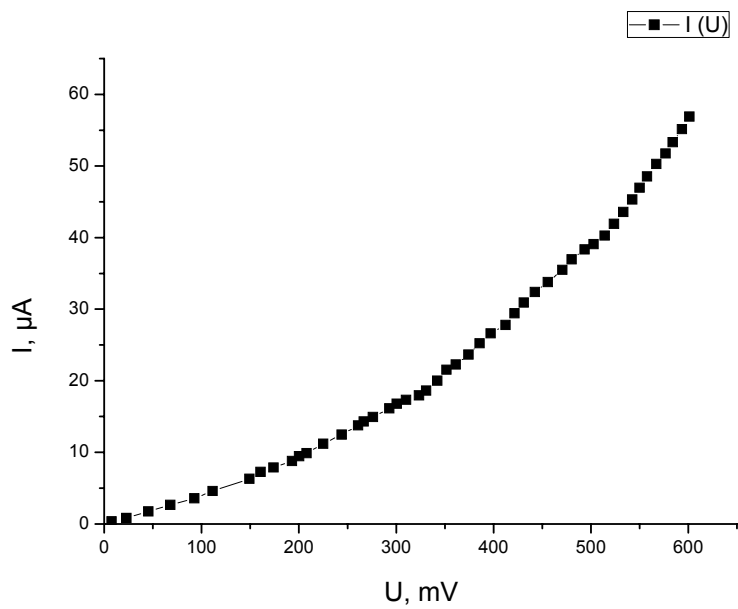
Let  $l=1$ m.

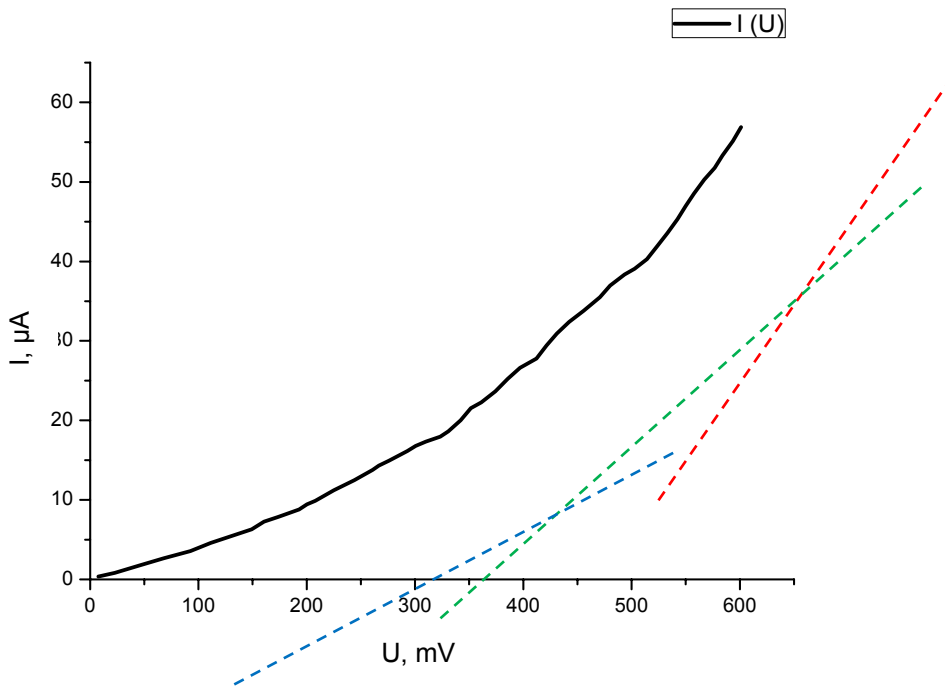
| Diameter | Diameter, m       | $S_{sec}, m^2$        | R, Ohm            |
|----------|-------------------|-----------------------|-------------------|
| 1 mm     | $1 \cdot 10^{-3}$ | $7,85 \cdot 10^{-7}$  | $7 \cdot 10^{-2}$ |
| 1 um     | $1 \cdot 10^{-6}$ | $7,85 \cdot 10^{-13}$ | $7 \cdot 10^4$    |
| 10 nm    | $1 \cdot 10^{-8}$ | $7,85 \cdot 10^{-17}$ | $7 \cdot 10^8$    |
| 1 nm     | $1 \cdot 10^{-9}$ | $7,85 \cdot 10^{-19}$ | $7 \cdot 10^{10}$ |

Using formula  $U = I \cdot R$ , plot the following graph.



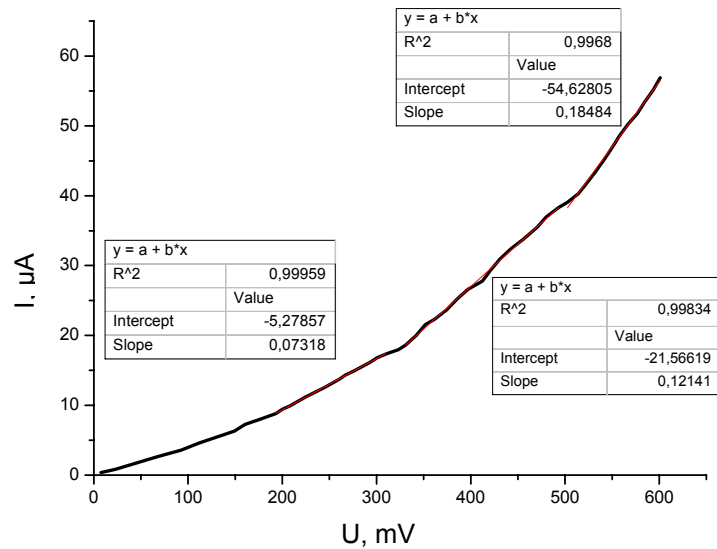
1. Plot the chart of dependence of conduction on the applied voltage as per data given in the table.



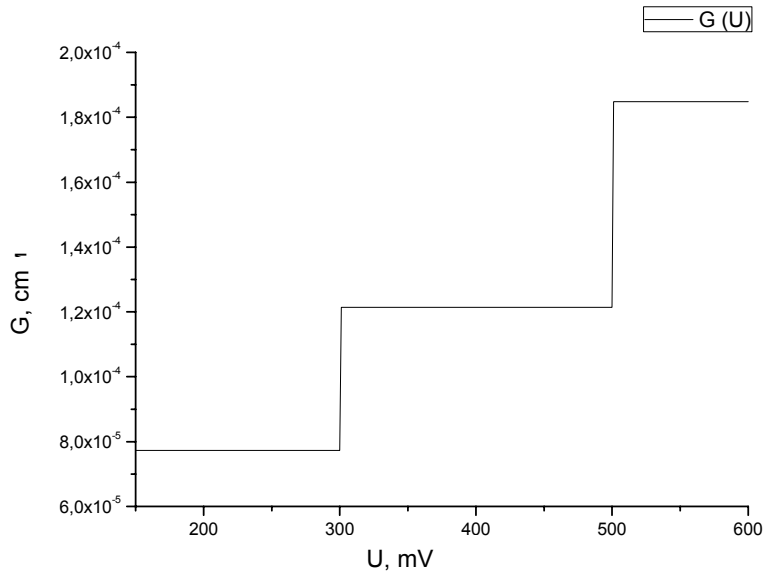


At least 3 regions with different angles of slope of the curve can be clearly distinguished on the presented chart. Calculate the angles for 3 regions:

- From 150 to 300 mV -  $0,07318 \text{ uA/mV} = 7,7318 \cdot 10^{-5} \text{ cm} = G_0$ ;
- From 300 to 500 mV -  $0,12141 \text{ uA/mV} = 12,141 \cdot 10^{-5} \text{ cm} = 1,5 \cdot G_0$ ;
- From 500 to 600 mV -  $0,18484 \text{ uA/mV} = 18,484 \cdot 10^{-5} \text{ cm} = 2,3 \cdot G_0$  (for the given case it must be  $2 \cdot G_0$ , however, due to errors in the calculation of the tangent, somewhat conservative value is obtained).



Next, construct graph  $G(U)$  as per calculated data:



Constant  $G_0$  is usually applied to the above described effect. What is this constant called? What is its dimension and value in the SI system? And for what is this value currently used? (3 points)

$G_0 = 2e^2/h$  - quantum of conductance.

$e = 1,6 \cdot 10^{-19}$  Kl,  $h = 6,6 \cdot 10^{-34}$  J\*s, consequently  $G_0 = 7,75 \cdot 10^{-5}$  cm.

It (or rather its inverse) is used to calibrate the resistance and since 1990 has been the benchmark for measuring the resistance.

### 9b8.

Answer 1: It is known that films of mesoporous silicon have great specific surface, and its limit reaches the value of about 800 m<sup>2</sup>/g. The presence of the developed surface of porous silicon causes the presence of her huge number of defects - dangling silicon bonds, which in turn are trapping centers for FCC. The fact that in the process of electrochemical etching of crystalline silicon, dopant atoms are not "washed" with the silicon atoms must also be taken into account. Therefore, reducing the concentration of FCCs in the meso-PC cannot be attributed to the "removal" of the substance.

Answer 2: Using expression for crystalline silicon it can be written:

$$N_{sthz}(c-Si) = \frac{A\alpha_{c-Si}n_{c-Si}}{\lambda^2\tau},$$

For porous silicon:

$$N_{FCC}(PC) = \frac{A\alpha_{PC}n_{PC}}{\lambda^2\tau(1-p)},$$

where  $A = 4\pi^2c^3\varepsilon_0m^{*2}/e^2 = const$ ,  $p = 0.6$ ,  $\alpha_{c-Si}$ ,  $\alpha_{meso-PC}$  - absorption coefficient for crystalline and porous silicon, measured at a wavelength of  $\lambda$ ,  $n_{c-Si} = 3.4$ ,  $n_{meso-PC} = 1.7$  - refractive index for crystalline and porous silicon, respectively. Using the above written formulas to calculate the concentration of free charge carriers in mesoporous silicon, the following is obtained:

$$N_{FCC}(PC) = \frac{\alpha_{meso-PC}n_{meso-PC}N_{FCC}(c-Si)}{(1-p)\alpha_{c-Si}n_{c-Si}},$$

where  $N_{c-Si}$  is equal to  $10^{20}$  cm<sup>-3</sup>.

Substituting numerical values, the following is obtained:

$$N_{FCC}(PC) \approx \frac{130 \text{ cm}^{-1} * 1.7 * 10^{20} \text{ cm}^{-3}}{(1-0.6) * 620 \text{ cm}^{-1} * 3.4} \approx 2.6 * 10^{19} \text{ cm}^{-3}$$

Answer 3: During thermal oxidation of silicon nanocrystals, concentration in their FCCs decreases. This is due to the increasing number of defects - dangling silicon bonds that are formed during the oxidation of the samples. It is known that these dangling bonds are trapping centers for FCC.

Answer 4: Microporous silicon films are obtained by electrochemical etching of lightly doped silicon substrates. The concentration of FCCs in these substrates is approximately  $10^{16} \text{ cm}^{-3}$ . It is also known in case of reducing the size of silicon nanocrystals to a few nanometers (as in the case of microporous silicon), the electronic spectrum of charge carriers undergoes significant changes due to quantum confinement effect. It can therefore be stated that this material is almost completely depleted of charge carriers in equilibrium.

**9b9.**

The cantilever is represented as a rectangular beam, which bends under the effect of force  $F$ , applied along the normal to the free end of the beam. The beam theory of bending was developed by Bernoulli and Euler. It is clear that the upper surface due to deformation is accustomed, and the upper one stretches. For simplicity, beam is replaced by a segment which is bent under load and at each point the curvature is proportional to the moment of external forces. For small deflections of the curvature is almost equal to the second derivative.

So, the interval in the undisturbed position is described by  $Z(x) = 0$ , and when the load has a certain dependence  $Z(x)$ , which is to be found.

Thus  $EJ \frac{d^2Z}{dx^2} = F(L - X)$ .

The coefficient of proportionality  $EJ$  is called the flexural rigidity and is equal to the product of the modulus of elasticity of the material of the beam on the moment of inertia.

Modulus of elasticity is given in the problem, and the moment of inertia is given by  $J = \frac{wt^3}{12}$ . So there is a differential equation with boundary conditions:

$$\begin{cases} EJ \frac{d^2Z}{dx^2} = F(L - X) \\ Z(x = 0) = 0 \\ \frac{dZ}{dx}(x = 0) = 0 \end{cases}$$

Solution of this system:  $Z = \frac{F}{2EJ} \left( Lx^2 - \frac{x^3}{3} \right)$  or  $Z' = \frac{F}{EJ} \left( Lx - \frac{x^2}{2} \right)$

For  $x = L$  there is the real height of the object:  $Z(L) = \frac{4FL^3}{Ewt^3}$ .

Generally speaking, the signal of deflection in AFM is proportional to cantilever deflection, that is,  $Z'$ , but in solving the problem it could also be simplified and assumed that the signal is proportional to height. Both options were considered correct solutions.

Estimate as per the order of the values obtained.

Typical force at work on AFM has the order of  $F = 1 \text{ nN}$ . According to the formula, this corresponds to

$$Z = \frac{4 * 10^{-9} * 8 * 10^6 * 10^{-18}}{2 * 10^{10} * 40 * 10^{-6} * \frac{1}{8} * 10^{-18}} = \frac{32 * 10^{-3}}{10^5} = 32 * 10^{-8} \approx 300 \text{ nm}$$

Let's see what happens if the beam is put in the middle of the cantilever:

$$Z = \frac{12 * 10^{-9}}{2 * 2 * 10^{10} * 40 * 10^{-6} * \frac{1}{8} * 10^{-18}} * \left( 200 * 10000 - \frac{1000000}{3} \right) * 10^{-18} = 10^7 * 10^{-14} = 10^{-7} = 100 \text{ nm}$$

That is, there was a great mistake in 60%!

When calculating through derivative, the error is 25%.

The difference between the answers is large, but as first of all it is important to do calculations in the first question, they are both counted as correct.

Since  $Z'$  is linearly dependent on the force, the same relative error will be obtained at different forces (i.e., heights).

Questions 2 and 3. The data in the topographic images are usually formed as follows. Feedback monitors changes in signal deflection and generates such a signal to move the piezoelectric ceramic to compensate the deviation of all time. Piezoceramic is calibrated and its move (exactly known in nm) gives a high-rise image.

Thus, the incorrect position of the beam on the cantilever does not directly lead to a distortion of the height of objects on topographic images, but also significantly reduces signal/noise ratio. It is more critical for small objects (DNA, thin films, etc.).

Questions 4. To minimize this error, a laser system must be properly set up. But if not sure, one can accurately adjust the laser by choosing a shorter and more rigid cantilever.



**9b10.**

When hitting a hard surface, the stream breaks up into many drops of different diameters. Due to surface tension, the potential energy of the stream, that crashed on the drops, increases. To estimate, assume that for the formation of drops of a certain diameter at impact it is necessary that the kinetic energy of a selected volume of the drop in the stream be greater than the potential energy of the surface tension of the drop after its discharge from the stream. According to the law of conservation of energy:

$$\frac{m_0 V^2}{2} = S\sigma + \frac{m_0 V'^2}{2}$$

where S – surface area of drops,  $m_0$  – mass of drop,  $V'$  - the speed of drop after impact.

The process of drop formation of a certain diameter has a threshold as per kinetic energy (kinetic energy of a drop after impact is zero). Thus, the condition of the threshold for drop formation:

$$4\pi\sigma R^2 = \frac{2}{3}\pi\rho R^3 V^2$$

$$V = \sqrt{\frac{6\sigma}{\rho R}}$$

For drops with a diameter of 100 nm, the following is obtained:

$$V = \sqrt{\frac{6\sigma}{\rho R}} \approx 92 \text{ m/s}$$

which is less than the speed of sound in air. Consequently, this stream will be heard.

For drops with a diameter of 10 nm:

$$V = \sqrt{\frac{6\sigma}{\rho R}} \approx 290 \text{ m/s}$$

which is also slightly less than the speed of sound.





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